Oscillations

Periodic or harmonic motion is any motion that repeats itself at regular intervals of time.

1 Hertz = 1 Hz = 1 Oscillation/sec

Frequency = # of oscillations/sec

Period \( T = \frac{1}{f} \) = Time per oscillation

\[ x = x_M \sin \left( \frac{2\pi t}{T} \right) \]

\( x \) = Amplitude

\( \pm x_M \)

So in general

\[ x = x_M \sin (\omega t + \phi) \]

or

\[ x = x_M \cos (\omega t + \phi') \]
\[ x(t) = x_0 \cos (\omega t + \phi) \]
\[ v(t) = \frac{dx(t)}{dt} = -\omega x_0 \sin (\omega t + \phi) \]
\[ a(t) = \frac{dv(t)}{dt} = -\omega^2 x_0 \cos (\omega t + \phi) \]
\[ a(t) = -\omega^2 x(t) \]

**A Linear Simple Harmonic Oscillator**

\[ F = -kx(t) \]

**From Newton's 2nd Law**

\[ F = ma(t) = -m\omega^2 x(t) \]

So that

\[ k = m\omega^2 \]

\[ \omega = \sqrt{\frac{k}{m}} \quad \omega = 2\pi f \quad f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \]

\[ T = 2\pi \sqrt{\frac{m}{k}} \]
ENERGY IN S.H.M.

\[ U(x) = \frac{1}{2} k x^2 = \frac{1}{2} k x_m^2 \cos^2 (\omega t + \phi) \]

\[ K(x) = \frac{1}{2} m v^2 = \frac{1}{2} m (\omega x_m)^2 \sin^2 (\omega t + \phi) \]

THE MECHANICAL ENERGY

\[ E = U + K = \frac{1}{2} k x_m^2 \]

Pendulum

\[ T = -L (F_y \sin \theta) \]

\[ \Sigma F_y = I \alpha \]

\[-L F_y \sin \theta = I \alpha \]

For small \( \theta \), \( \sin \theta \approx \theta \)

\[ \frac{\alpha - mg L \theta}{I} \quad \text{but} \quad I = mL^2 \]

\[ \alpha \approx -\frac{mg L \theta}{mL^2} = -\frac{g \theta}{L} \]

Comparing with 16.8 \( \alpha = -w^2 \theta \)

\[ \omega = \sqrt{g/L} \quad T = 2\pi \sqrt{\frac{L}{g}} \quad \text{for small} \ \theta \]

Problem 16.2 A block-spring system takes 3/4 sec to begin repeating its motion

a) \( T = ? \) Time/cycle = 3/4 sec \( T = 3/4 \) sec

b) \( f = \frac{1}{T} = \frac{1}{3/4} = 1.33 \) Hz

c) \( \omega = ? \) \( \omega = 2\pi f = 6.28(1.33) = 8.37 \) rad/s
16-3

\[ X_M = 35 \text{ cm} \]

The oscillator repeats its motion in \( \frac{1}{2} \text{ sec} \)

\[ T = \frac{1}{2} \text{ sec} \]

(a) Find \( T \)

(b) Find \( f \)

\[ f = \frac{1}{T} = \frac{1}{\frac{1}{2}} = 2 \text{ Hz} \]

(c) Find \( \omega \)

\[ \omega = 2\pi f = 2\pi (2) = 12.57 \text{ rad/s} \]

(d) \( k = ? \)

\[ k = \frac{\omega^2 m}{m} = (12.57)^2 \left( \frac{1}{2} \right) = 78.9 \text{ N/m} \]

(e) \( v_{\text{max}} = ? \)

\[ v_{\text{max}} = \omega X_M = 12.57 \times 35 = 440 \text{ m/s} \]

(f) \( F_{\text{max}} \) on the block by the spring:

\[ F_{\text{max}} = k X_{\text{max}} = 78.9 \times 3.5 = 276 \text{ N} \]