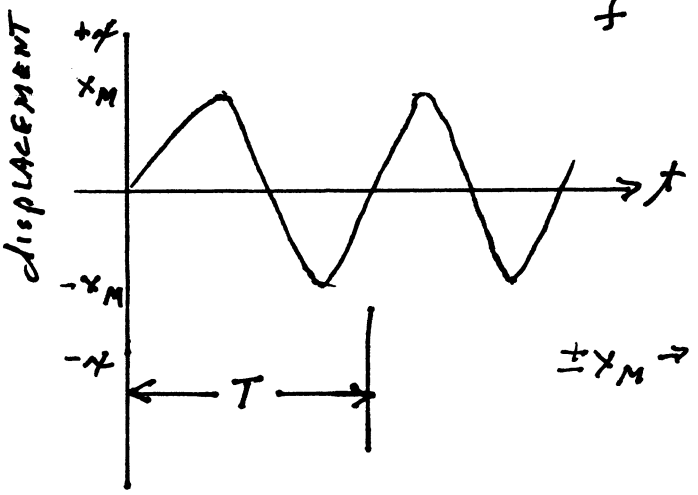
OSCILLATIONS

PERIODIC OR HARMONIC MOTION IS ANY MOTION THAT REPEATS ITSELF AT REGULAR INTERVALS OF TIME

1 HERTZ = 1 Hz = 1 OSCILLATION/SEC

FREQUENCY = # OF OSCILLATIONS/SEC

PERIOD $T = \frac{1}{f}$ = TIME PER OSCILLATION

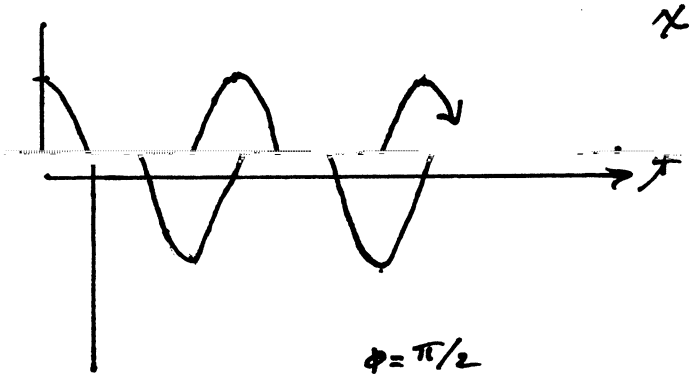


$x = x_M \sin \underbrace{\omega t}_{\text{RADIANS}}$

$\pm x_M \rightarrow$ AMPLITUDE

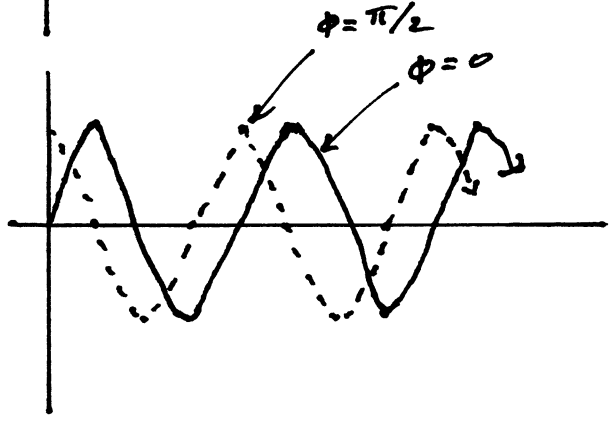
$\omega \frac{\text{RAD}}{\text{SEC}} \times \frac{\text{OSCIL}}{2\pi \text{ RAD}} = f \frac{\text{OSCIL}}{\text{SEC}}$

$x = x_M \sin 2\pi f t$



$x = x_M \sin\left(\omega t + \frac{\pi}{2}\right)$

Φ – PHASE ANGLE



SO IN GENERAL

$x = x_M \sin(\omega t + \phi)$

OR

$x = x_M \cos(\omega t + \phi')$

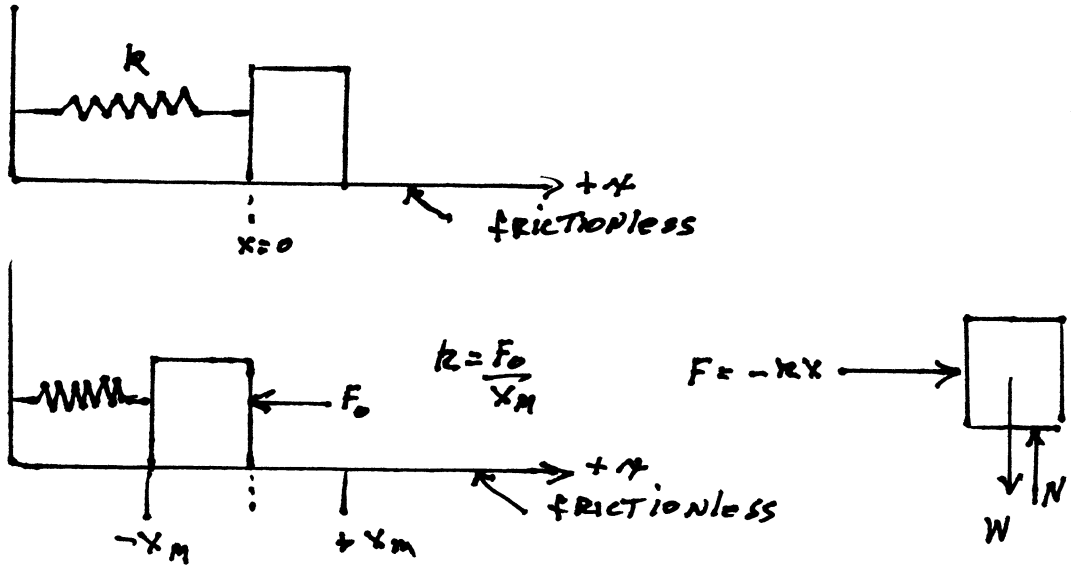
$$x(t) = x_M \cos(\omega t + \phi)$$

$$v(t) = \frac{dx(t)}{dt} = -\omega x_M \sin(\omega t + \phi)$$

$$a(t) = \frac{dv(t)}{dt} = -\omega^2 \underbrace{x_M \cos(\omega t + \phi)}_{x(t)}$$

$$a(t) = -\omega^2 x(t)$$

A LINEAR SIMPLE HARMONIC OSCILLATOR



FROM THE HOOKE'S LAW RELATION $F = -kx(t)$

FROM NEWTON'S 2ND LAW $F = ma(t) = -m\omega^2 x(t)$

SO THAT $k = m\omega^2$

$$\omega = \sqrt{k/m} \quad \& \quad \omega = 2\pi f \quad f = \frac{1}{2\pi}\sqrt{k/m}$$

$$T = 2\pi\sqrt{m/k}$$

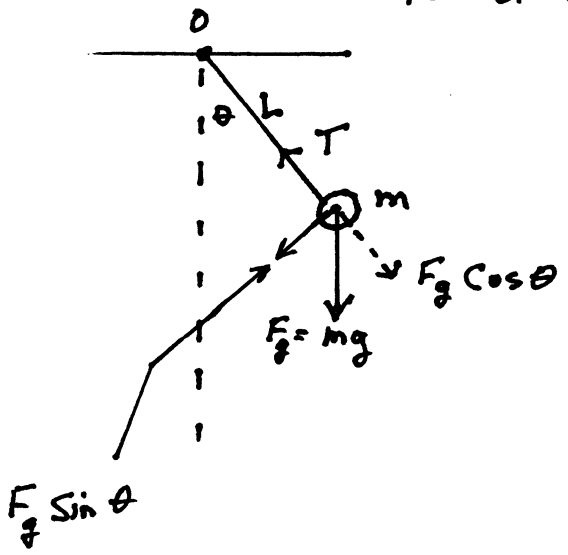
ENERGY IN S.H.M.

$$U(t) = \frac{1}{2} k x^2 = \frac{1}{2} k x_m^2 \cos^2(\omega t + \phi)$$

$$K(t) = \frac{1}{2} m v^2 = \frac{1}{2} m \omega^2 x_m^2 \sin^2(\omega t + \phi)$$

THE MECHANICAL ENERGY $E = U + K = \frac{1}{2} k x_m^2$

PENDULUMS



$\circlearrowleft$ $\tau_o = -L(F_g \sin\theta)$

$\Sigma\tau = I\alpha$

$-L F_g \sin\theta = I\alpha$

FOR SMALL θ $\sin\theta \approx \theta$

$\alpha = -\frac{mgL}{I}\theta$ BUT $I = mL^2$

$\alpha = -\frac{mgL\theta}{mL^2} = -\frac{g}{L}\theta$

COMPARING WITH 16-8 $a = -\omega^2 x$

$\omega = \sqrt{g/L}$ $T = 2\pi\sqrt{L/g}$ FOR SMALL θ

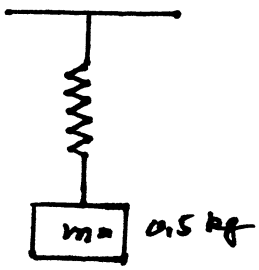
PROBLEM 16-2 A BLOCK-SPRING SYSTEM TAKES 3/4 SEC TO BEGIN REPEATING ITS MOTION

a) $T = ?$ TIME / CYCLE = 3/4 SEC $T = 3/4$ SEC

b) $f = 1/T = 1/3/4 = 1.33$ HZ

c) $\omega = ?$ $\omega = 2\pi f = 6.28(1.33) = 8.37$ RAD/S

16-3



$X_M = 35CM$ THE OSCILLATOR REPEATS ITS MOTION IN 1/2 SEC

a) find T T = 1/2 SEC (THE REPITITION TIME)

b) find f $f = \frac{1}{T} = \frac{1}{1/2} = 2\ HZ$

c) find w $w = 2\pi f = 2\pi(2) = 12.56\ RAd/s$

d) k = ? $w = \sqrt{k/m}$ $k = w^2 m = (12.56)^2(\frac{1}{2}) = 78.9\ N/m$

e) $V_{MAX} = ?$ $V_m = w X_M = 12.56(.35) = 4.40\ m/s$

f) F_{MAX} ON THE BLOCK BY THE SPRING =

$F_{MAX} = k X_{MAX} = 78.9(.35) = 27.6\ N$