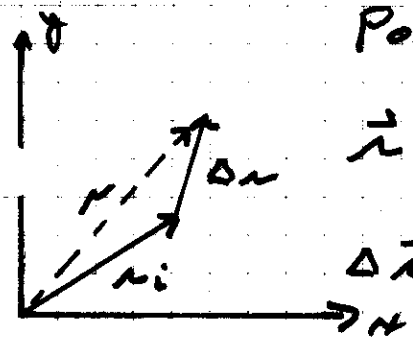
Position & Displacement



$\vec{r} = \vec{r}_i + \Delta\vec{r}$ WHERE $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$

$\Delta\vec{r} = (x - x_i)\vec{i} + (y - y_i)\vec{j} + (z - z_i)\vec{k}$

AVG VELOCITY $\vec{\bar{v}} = \frac{\Delta x}{\Delta t}\vec{i} + \frac{\Delta y}{\Delta t}\vec{j} + \frac{\Delta z}{\Delta t}\vec{k}$

INSTANTANEOUS VELOCITY $\vec{v} = \frac{dx}{dt}\vec{i} + \frac{dy}{dt}\vec{j} + \frac{dz}{dt}\vec{k}$

LIM $\Delta t \to 0$ $\vec{v} = v_x\vec{i} + v_y\vec{j} + v_z\vec{k}$

EXAMPLE A PARTICLE IS INITIALLY AT $\vec{r}_i = 5\vec{i} - 6\vec{j} + 2\vec{k}$
& 10 SEC LATER $\vec{r} = -2\vec{i} + 8\vec{j} - 2\vec{k}$
FIND THE AVG VELOCITY DURING THE 10 SEC

$\Delta\vec{r} = \vec{r} - \vec{r}_i = (-2 - 5)\vec{i} + (8 - (-6))\vec{j} + (-2 - 2)\vec{k}$

$\Delta\vec{r} = -7\vec{i} + 14\vec{j} - 4\vec{k}$

$\vec{\bar{v}} = \frac{\Delta\vec{r}}{\Delta t} = \frac{-7}{10}\vec{i} + \frac{14}{10}\vec{j} - \frac{4}{10}\vec{k}$

$\vec{\bar{v}} = -0.7\vec{i} + 1.4\vec{j} - 0.4\vec{k}$

ACCELERATION

$$\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \frac{\Delta \vec{v}}{\Delta t}$$

INSTANTANEOUS ACCELERATION

LIM $\Delta t \to 0$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\vec{i} + \frac{dv_y}{dt}\vec{j} + \frac{dv_z}{dt}\vec{k}$$

$$\vec{a} = a_x\vec{i} + a_y\vec{j} + a_z\vec{k}$$

EXAMPLE - A PARTICLE MOVES SO THAT

$$\vec{r} = \vec{i} + 4t^2\vec{j} + t\vec{k}$$

WRITE EXPRESSIONS FOR $\vec{v}$ & $\vec{a}$

$$\vec{v} = \frac{dx}{dt}\vec{i} + \frac{dy}{dt}\vec{j} + \frac{dz}{dt}\vec{k}$$

$$\frac{d(1)}{dt} = 0 \qquad \frac{d}{dt}(4t^2) = 8t \qquad \frac{d}{dt}(t) = 1$$

$$\vec{v} = 8t\vec{j} + \vec{k}$$

$$\vec{a} = \frac{dv_x}{dt}\vec{i} + \frac{dv_y}{dt}\vec{j} + \frac{dv_z}{dt}\vec{k}$$

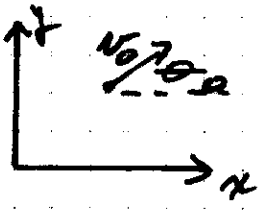
$$= 0 + 8\vec{j} + 0$$

$$\vec{a} = 8\vec{j}$$

PROJECTILE MOTION

FROM CHAPTER 2 $v_x = v_{0x} + a_x t$

$v_y = v_{0y} + a_y t$



W/O AIR RESIS $a_y = -g$ $\quad a_x = 0$

$v_x = v_0 \cos\theta_0$

$v_y = v_0 \sin\theta_0$

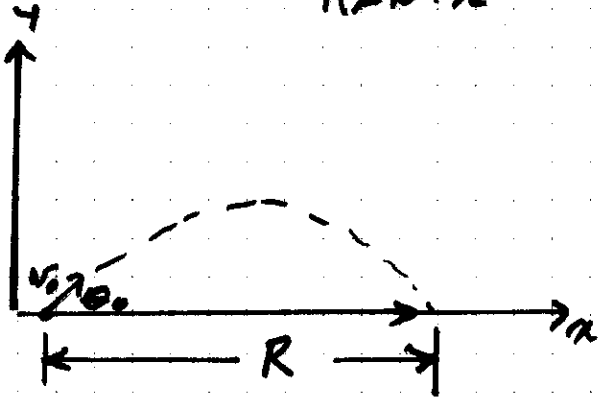
POSITION: $x = x_0 + v_0 \cos\theta_0 \, t$ $\quad y = y_0 + \underbrace{v_0 \sin\theta_0}_{v_{0y}} t - \frac{1}{2} g t^2$

VELOCITY: $v_x = v_{0x} + a_x t$ ($a_x = 0$)

$v_y = v_{0y} - g t$

$v_y^2 = (v_0 \sin\theta_0)^2 - 2g(y - y_0)$

RANGE

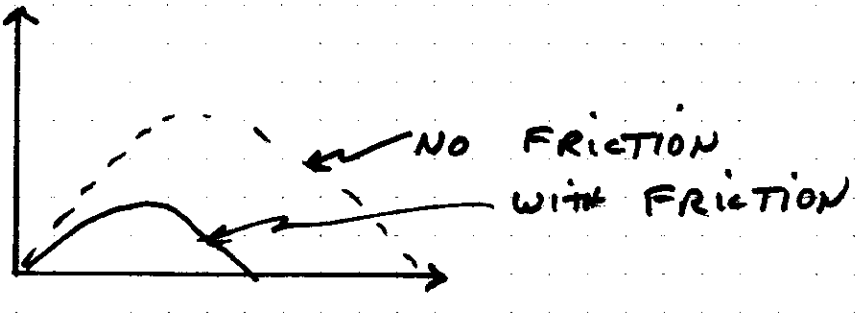


$$R = \frac{v_0^2}{g} \sin 2\theta_0$$

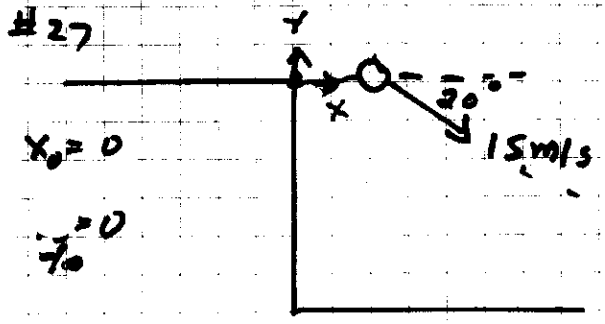
WITH AIR RESISTANCE

$x - x_0 = v_0 \cos\theta_0 \, t - k v^2$

$y - y_0 = v_0 \sin\theta_0 \, t - \frac{1}{2} g t^2 - k v^2$



#27



V_{0x} is +

V_{0y} is −

a) find THE HORIZONTAL DISPLACEMENT ATER 2.3 SEC

$$x = \cancel{x_0}^{=0} + V_0 \cos\theta_0 t = 15 \cos 20 (2.3) = 32.4 \text{ m}$$

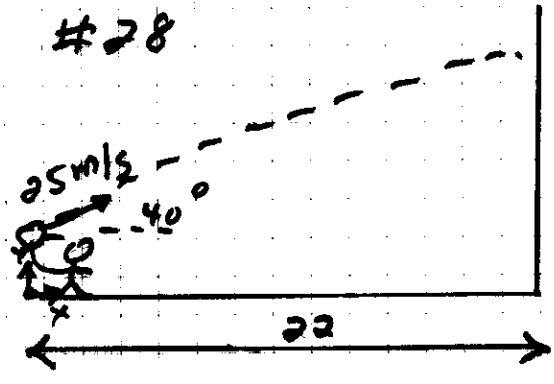
b) find THE VERTICAL DISPLACEMENT AFTER 2.3 SEC

$$y = \cancel{y_0}^{0} + V_0 \sin\theta_0 t - \tfrac{1}{2} g t^2$$

$$= -15 \sin 20 (2.3) - \tfrac{1}{2} (9.8)(2.3)^2$$

$$= -11.8 - 25.9 = -37.7 \text{ M}$$

#28



PUT COORD AT ORIGIN.

THE BALL IS AT

$X_0 = 0 \quad Y_0 = 0$

a) HOW LONG DOES IT TAKE THE BALL TO REACH THE WALL

$$x = \cancel{x_0}^{0} + V_0 \cos 40\, t$$

$$22 = 25 \cos 40\, t$$

$$t = 1.15 \text{ SEC}$$

b) FIND THE ELEVATION OF THE BALL AT IMPACT

$$y = \cancel{y_0}^{0} + V_0 \sin 40\, t - \tfrac{1}{2} g t^2$$

$$+25 \underset{18.5}{\sin 40} (1.15) - \tfrac{1}{2} (9.8) \underset{6.5}{(1.15)^2} = 12.0 \text{ m}$$

c) FIND THE HORIZ & VERT COMPONENTS OF THE VELOCITY AT THE BALL AT IMPACT

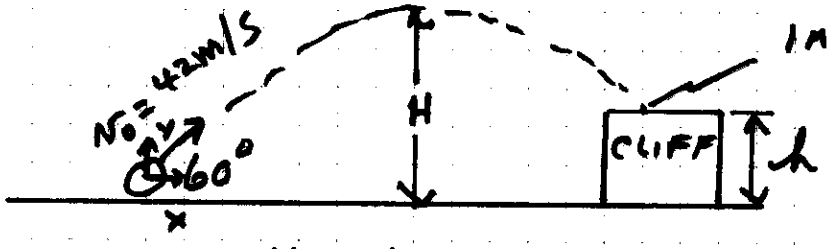
$$v_x = v_{0x} = 25 \cos 40 = 19.15 \text{ m/s}$$

$$v_y = v_{0y} - gt = \underset{16.07}{25 \sin 40} - \underset{11.27}{9.8(1.15)} = +4.8 \text{ m/s}$$

d) WHEN THE BALL HITS THE WALL HAS THE BALL PASSED THE HIGHEST POINT OF ITS TRAJECTORY ?

SINCE $v_y = +4.8$ m/s IT IS STILL MOVING HIGHER SO IT HAS NOT REACHED THE MAX ELEVATION

#31



IMPACT IN 5 ½ SEC

$x_0 = y_0 = 0$

a) FIND THE HEIGHT OF THE CLIFF

$$y = v_0 \sin 60 (t) - \tfrac{1}{2} g t^2$$

$$= \underset{200}{42 \sin 60 (5.5)} - \underset{-146}{\tfrac{1}{2}(9.8)(5.5)^2} = 52 \text{ m}$$

b) FIND THE VELOCITY OF THE STONE AT IMPACT

$$v_{HORIZ} = v_0 \cos 60 = 21 \text{ m/s}$$

$$v_y = \underset{36.37}{v_{0y}} - gt = \underset{-53.9}{42 \sin 60} - 9.8(5.5) = 17.53 \text{ m/s}$$

$$v_{IMPACT} = \sqrt{21^2 + 17.53^2} = 27.35 \text{ m/s}$$

c) FIND THE MAX HEIGHT REACHED ABOVE THE GROUND

AT THE MAX HEIGHT $v_y = 0$

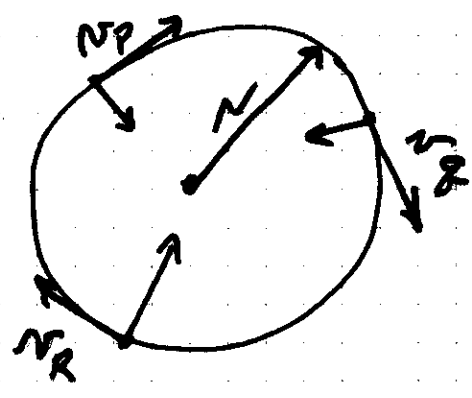
$$0 = v_{0y} - gt \qquad t_{MAX} = \frac{v_{0y}}{g} = \frac{42 \sin 60}{9.8} = 3.71 \text{ s}$$

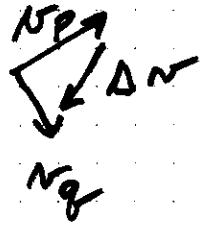
$$y = H = \cancel{y_0}^{0} + v_{0y} t - \frac{1}{2} g t^2$$

$$= \underset{135}{42 \sin 60}(3.71) - \underset{-67.4}{\frac{1}{2}(9.8)(3.71)^2} = 67.6 \text{ m}$$

UNIFORM CIRCULAR MOTION

PARTICLE MOVES WITH CONSTANT SPEED AROUND A CIRCLE





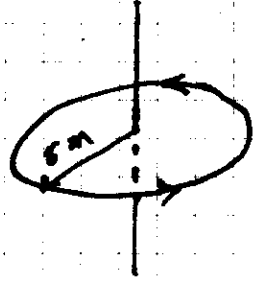
$$\vec{v}_Q = \vec{v}_P + \Delta \vec{v}$$

PERIOD - TIME TO COMPLETE ONE REVOLUTION

$$\rightarrow \quad T = \frac{\text{CIRCUMFERENCE}}{\text{VELOCITY}} = \frac{2\pi r}{v}$$

ACCELERATION $\Rightarrow$ CHANGE IN VELOCITY MAGNITUDE &/OR DIRECTION

$$\rightarrow \quad \text{CENTRIPETAL ACCEL} \quad a = \frac{v^2}{r} \quad \text{TOWARD THE CENTER}$$



AN ASTRONAUT ROTATES IN A HORIZONTAL CENTRIFUGE OF 5m RADIUS

a) FIND HIS SPEED IF THE CENTRIPETAL ACCEL = 7.0g's

$$a = \frac{v^2}{R} \qquad v = \sqrt{7gR} = (7(9.8)(5))^{V_2} = 18.5 \text{ m/s}$$

b) HOW MANY RPM ARE REQUIRED TO PRODUCE THIS a

$$\frac{(\#\text{ OF REV}) \times (\text{CIRCUMFERENCE})}{\text{VELOCITY}} = \text{TIME} = t$$

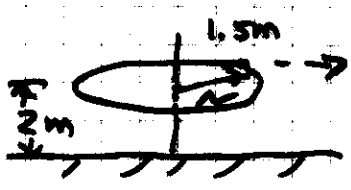
$$\frac{N}{v} \quad \frac{2\pi R}{} = t$$

$$\frac{N 2\pi R}{v} = t \qquad RPM = \frac{N}{t} = \frac{v}{2\pi R}$$

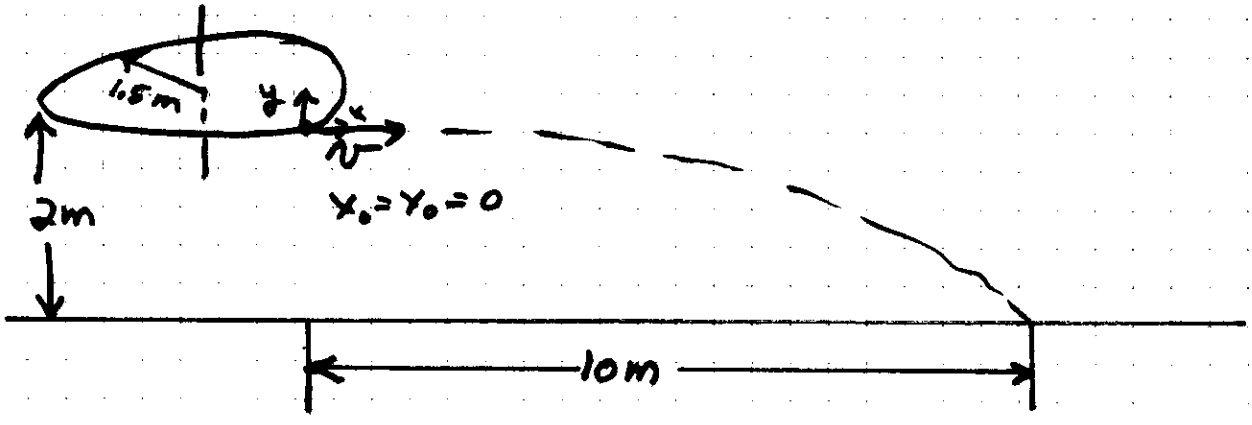
$$\frac{N}{T} = \frac{18.5}{2\pi(5)} = 0.589 \frac{\text{REV}}{\text{SEC}} \times \frac{60 \text{ SEC}}{\text{min}} = 35.4 \text{ RPM}$$

c) WHAT IS THE PERIOD OF THE MOTION

$$T = \frac{\text{CIRCUM}}{\text{VELOCITY}} = \frac{6.28(5)}{18.5} = 1.7 \text{ SEC}$$

#71 

A BOY WHIRLS A STONE IN A HORIZ CIRCLE 2 m ABOVE GROUND BY MEANS OF A STRING 1.5 m LONG. THE STRING BREAKS & THE STONE FLIES OFF HORIZ & STRIKES THE GROUND 10 m AWAY. FIND THE a_R OF THE STONE IN CIRCULAR MOTION



$$y = \cancel{y_0}^{\,0} + v_{0y}t - \tfrac{1}{2}gt^2$$

$$-2 = 0 + (0)t - \tfrac{1}{2}gt^2 \qquad t = \sqrt{\frac{2(2)}{g}} = 0.639 \text{ sec}$$

$$x = \cancel{x_0}^{\,0} + v_{0x}t$$

$$10 = v_{0x}(.639) \qquad v_{0x} = v_0 = 15.64 \text{ m/s}$$

$$a_c = \frac{v^2}{r} = \frac{15.64^2}{1.5} = 163 \text{ m/s}^2$$