WORK & KINETIC ENERGY

$KE = \frac{1}{2} m v^2$ $\quad [kg][m/s]^2 = \frac{kg\, m^2}{s^2} = 1$ JOULE

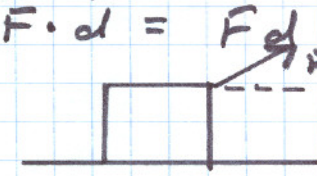
WORK = FORCE x displacement in the DIRECTION OF THE FORCE

$[N][m] = \left[\frac{kg \cdot m}{s^2}\right] \times [m] = 1$ JOULE

- WK - WORK DONE ON AN OBJECT

+ WK " " BY " "

$WK = W = \vec{F} \cdot \vec{d} = F d \cos\theta$

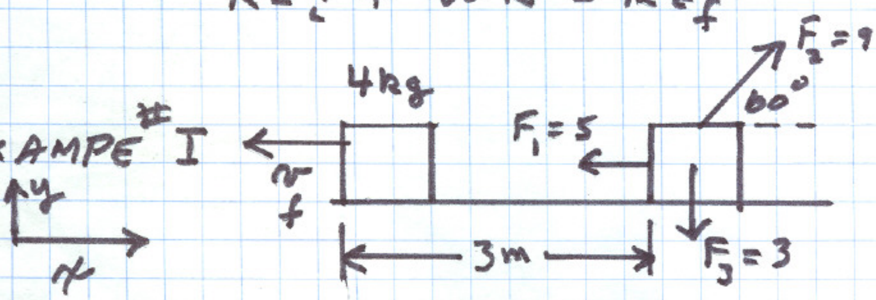


$W = (F\cos\theta)\, d$

WORK - KE THEOREM

$KE_i + WK = KE_f$

EXAMPE # I



$v_i = 0$

find

a) W

b) v_f

$F_{NET} = -5 + 9\underbrace{\cos 60}_{1/2} = -5 + 4.5 = -0.5N$

$W = F_{NET} \times disp = (-.5)(-3) = 1.5$ Joules $\quad \theta = 0°$

$KE_i = \frac{1}{2}(4)(v_i = 0)^2 = 0 \quad KE_f = \cancel{KE_i}^{0} + W = 1.5$

$\frac{1}{2}(4) v_f^2 = 1.5 \quad v_f = 0.866$ m/s

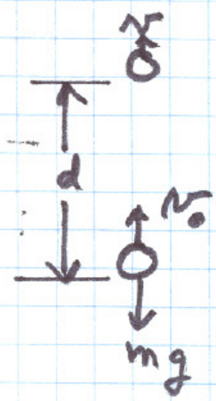
WK DONE BY A WEIGHT

$$WK = \vec{F} \cdot \vec{d} = mgd\cos\theta$$

FOR A RISING OBJECT mg ↓ BUT d ↑ $\theta = 180$ $\cos\theta = -1$

$W_g = -mgd$ ⟶ ABSORBING ENERGY

$W_g = mgd$ WHEN mg ↓ $\theta = 0$ ⟶ RELEASING ENERGY

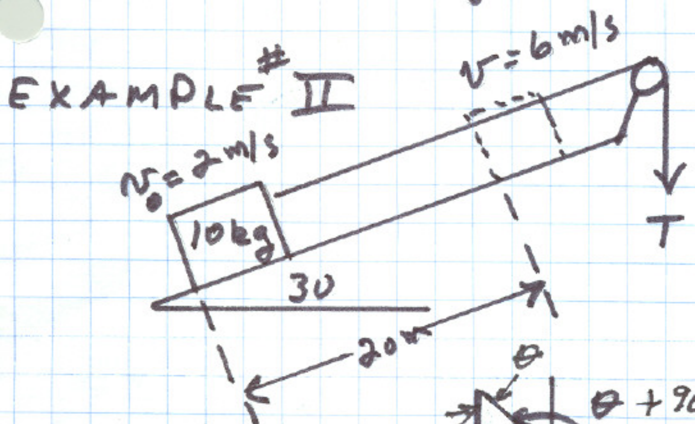


Lo KE high W_g

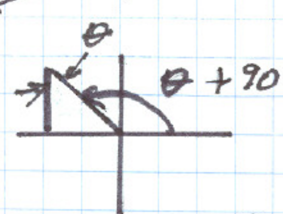
hi KE low W_g

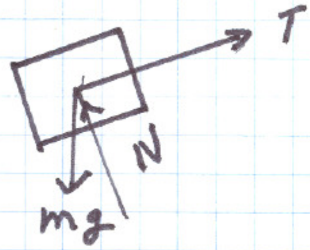
$$KE_F = KE_i + W_{APP} + W_g$$

EXAMPLE # II



a) find the WK done BY THE TENSION T





$\cos\theta = -\sin\theta$

$\cos(90+\theta) = \cos 90 \cos\theta - \sin 90 \sin\theta$ (cos 90 = 0)

$\cos(90+\theta) = -\sin\theta$

$\cos 120 = -.5$ $\quad -\sin 30 = -.5$

$$KE_F = KE_i + W_{APP} + W_g$$

$$\tfrac{1}{2}(10)6^2 = \tfrac{1}{2}(10)2^2 + W_{APP} + mg(20)\cos(90+\theta)$$

$$180 = 20 + W_{APP} + 10g(20)\underbrace{\sin 30}_{-1/2}$$

-980

$$W_{APP} = 180 - 20 + 980$$
$$= 1140 \text{ JOULES}$$

b) find THE ACCELERATION

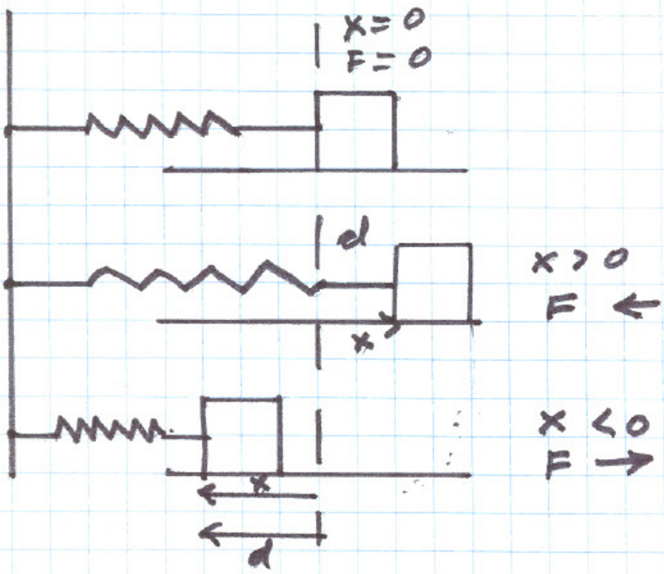
$$v_f^2 = v_i^2 + 2a(\Delta S)$$

$$6^2 = 2^2 + 2a(20) \qquad a = \frac{36-4}{2(20)} = 0.8\ m/s^2$$

$$-mg\sin 30 + T = ma = 10(.8) = 8\ N$$

$$T = 8 + 49 = 57\ N \qquad 57 \times 20 = 1140 = W_{APP}$$

WORK DONE BY A SPRING FORCE



HOOKE'S LAW FOR A SPRING

$$F = -kx$$

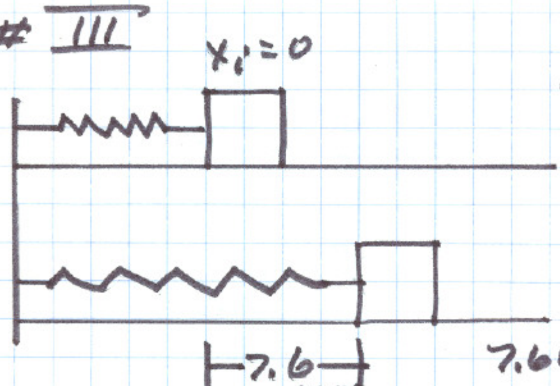
F - FORCE EXERTED BY THE SPRING

$$WK_s = \int_{x_i}^{x_f} F\,dx = \int_{x_i}^{x_f} (-kx)\,dx$$

$$W_s = \frac{1}{2}k(x_i^2 - x_f^2) = -\frac{1}{2}k(x_f^2 - x_i^2)$$

$$KE_f = KE_i + W_{APP} + W_g + W_s$$

EXAMPLE # III



$$k = 15\frac{N}{CM} \times 100\frac{CM}{m} = 1500\ N/m$$

$$7.6\,mm \times \frac{CM}{10\,mm} \times \frac{m}{100\,CM} = 7.6 \times 10^{-3}\ m$$

a) find the WK THE SPRING FORCE DOES ON THE BLOCK AS THE BLOCK MOVES TO THE RIGHT

$$W_s = -\tfrac{1}{2} k \left(x_f^2 - x_i^2\right)$$
$$= -\tfrac{1}{2}(1500)\left((7.6\times10^{-3})^2 - 0^2\right) = -0.0433 \text{ J}$$

(WORK DONE ON THE SPRING)

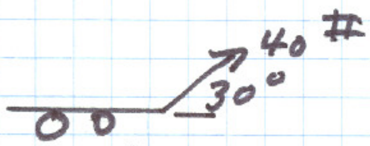
b) find the ADDITIONAL WORK TO STRETCH THE SPRING FROM 7.6 mm TO 15.2 mm

$$W_s = -\tfrac{1}{2}(1500)\left\{(15.2\times10^{-3})^2 - (7.6\times10^{-3})^2\right\} = -0.13 \text{ J}$$

POWER is THE RATE OF DOING WORK

$$P = \frac{dW}{dt} = \frac{d}{dt}(F\cdot x) = F\frac{dx}{dt} = \vec{F}\cdot\vec{v}$$

EXAMPLE # IV



$v_0 = 6$ mph

$$6\,\frac{\text{miles}}{\text{HR}} \times 5280\,\frac{\text{FT}}{\text{mile}} \times \frac{1\text{ HR}}{60\text{ min}} = 528\text{ FT/min}$$

a) find the WORK DONE IN 10 min

$$\text{WK} = F\times\text{disp} = 40\cos 30 \times 528\,\frac{\text{FT}}{\text{min}} \times 10\text{ min}$$
$$= 1.83\times10^5\ \#\ \text{FT}$$

b) WHAT is THE AVG POWER IN HP

$$\text{POWER} = \frac{\text{WORK}}{\text{TIME}} = \frac{1.83\times10^5\ \#\ \text{FT}}{10\text{ min}\times 60\,\frac{\text{SEC}}{\text{min}}} \times \frac{1\text{ HP}\cdot\text{SEC}}{550\text{ FT}\ \#} = 0.55\text{ HP}$$

CONSERVATION OF ENERGY

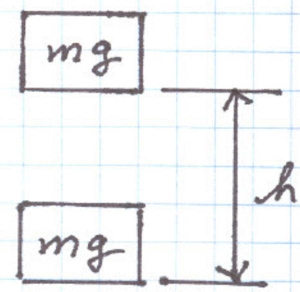
THE POTENTIAL ENERGY U IS ASSOCIATED WITH THE ARRANGEMENT OF OBJECTS THAT EXERT FORCES ON ONE ANOTHER

FOR THE EARTH ORBIT SYSTEM

$$\Delta U_g \stackrel{\Delta}{=} -WK$$

OR

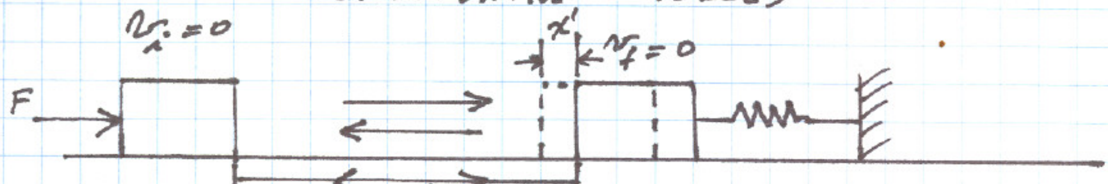
$$\Delta U_g = -mg\,h\cos\theta$$



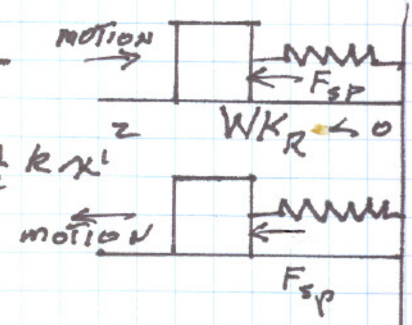
$WK = mg|h|\cos\theta$

$(\theta = 0)$

CONSERVATIVE FORCES

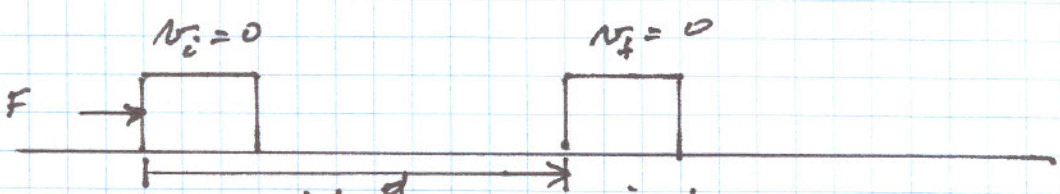


FRICTIONLESS PLANE



$WK = F_{sp}x$ $\quad {}_{sp} = \frac{1}{2}kx^2$

NON CONSERVATIVE FORCES



Coefficient OF FRICTION $\mu = 0.4$

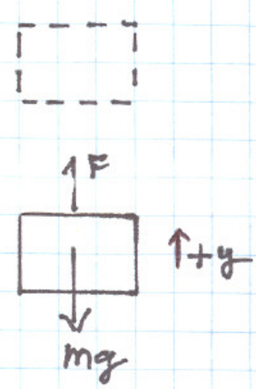
WORK $= Fd \Rightarrow$ HEAT

GRAVITATIONAL P.E.

$$\Delta U = -WK = -\int F(y)\,dy = -\int_{y_i}^{y_f} (-mg)\,dy$$

$$= mg(y_f - y_i)$$

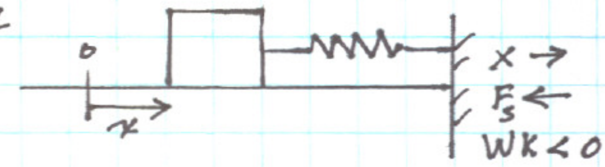
$$\boxed{\Delta U = mg\,\Delta y}$$



$F = -mg$

ELASTIC POTENTIAL ENERGY

FOR A SPRING $F = -kx$



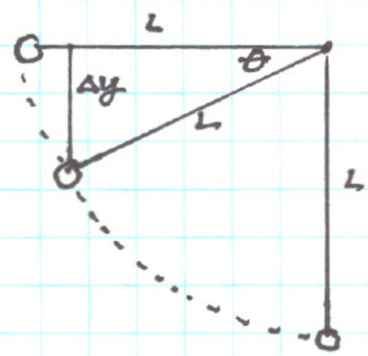
$$\Delta U_{sp} = -\int F\,dx = -\int(-kx)\,dx$$

$$\Delta U_{sp} = \frac{k}{2}\left(x_f^2 - x_i^2\right)$$

MECHANICAL ENERGY

TOTAL ENERGY $E = KE + PE = \text{CONSTANT} = mgL$

$PE = mgL$



$$KE + PE = mgL$$

$$\frac{1}{2}mv^2 + mg(L - \Delta y) = mgL$$

$$\frac{1}{2}mv^2 + mgL - mg\Delta y = mgL \qquad \text{BUT } \Delta y = L\sin\theta$$

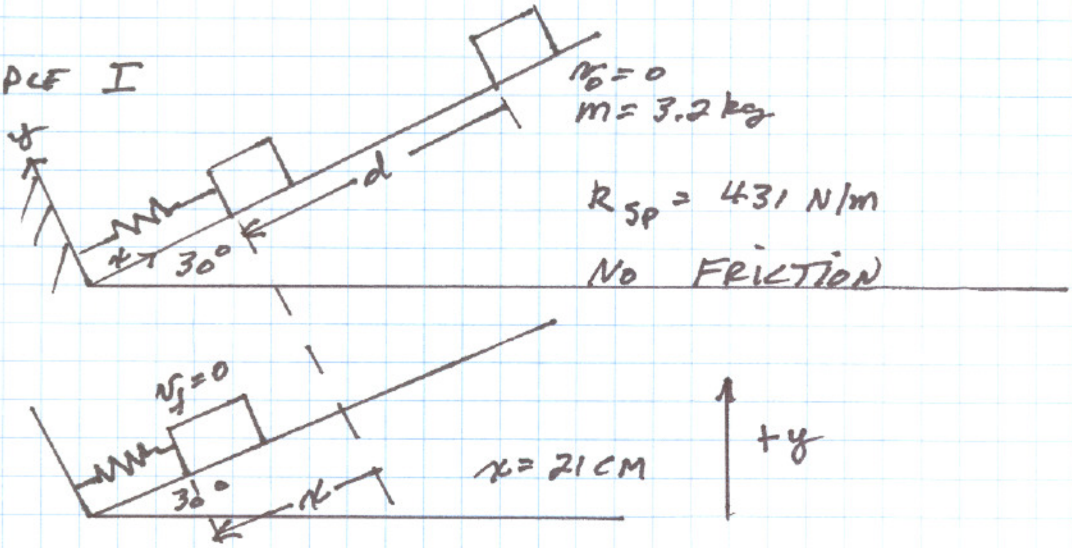
$$\frac{1}{2}v^2 + gL - gL\sin\theta = gL$$

$$\frac{1}{2}v^2 = gL\sin\theta$$

$$v = \sqrt{gL\,2\sin\theta}$$

$$WK_{\text{NET EXTERNAL FORCES}} = \Delta KE + \Delta U_g + \Delta U_{sp}$$

EXAMPLE I



a) THE SPRING IS COMPRESSED 21 CM BY THE BLOCK COMING DOWN THE PLANE. FIND THE DISTANCE d THE BLOCK MOVES DOWN THE PLANE BEFORE IMPACTING THE SPRING

$$\cancel{W_{App}}^{\,0} = \cancel{\Delta KE}^{\,0} + \Delta U_g + \overbrace{\Delta U_{sp}}^{\frac{1}{2}k(x_f^2 - x_i^2)}$$

$$0 = \underbrace{-mg(x+d)\sin 30}_{\Delta U_g = mg\,\Delta y,\ \Delta y < 0} + \frac{1}{2}k(x_f^2 - 0^2)$$

(k = 431, x_f = 0.21 m)

$$0 = -3.2(9.8)(.21+d)(.5) + \frac{1}{2}(431)(.21)^2$$

$$d = 0.39 \text{ m}$$

b) WHAT IS THE VELOCITY OF THE BLOCK JUST BEFORE IMPACTING THE SPRING

$$\cancel{W_{App}}^{\,0} = \Delta KE + \Delta U_g + \cancel{\Delta U_{sp}}^{\,0}$$

$$0 = \frac{1}{2}m(v^2 - 0^2) + mg(y_f - y_i)$$

$$0 = \frac{\cancel{m}}{2}v^2 - \cancel{m}g\,d\sin 30$$

$$v^2 = 2(9.8)(.39)\left(\frac{1}{2}\right)$$

$$v = 1.95 \text{ m/s}$$