SYSTEMS OF PARTICLES

CENTER OF MASS — CM

THE CM OF A SYSTEM OF PARTICLES IS THE POINT THAT MOVES AS THOUGH ALL THE MASS (OF THE PARTICLES) WERE CONCENTRATED THERE

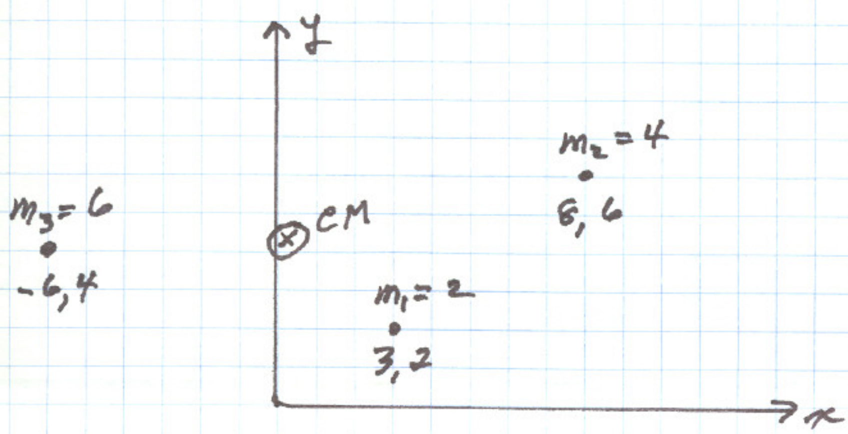
$$\sum_{\substack{\text{ALL THE}\\\text{PARTICLES}}} F = m_{CM}\, a_{CM}$$

$$X_{CM} = \frac{1}{M}\sum_{i=1}^{N} x_i m_i \qquad Y_{CM} = \frac{1}{M}\sum_{i=1}^{N} y_i m_i$$

$$Z_{CM} = \frac{1}{M}\sum_{i=1}^{N} z_i m_i$$

$$M = \sum_{i=1}^{N} m_i$$

EXAMPLE I



$$X_{CM} = \frac{1}{2+4+6} \times (3m_1 + 8m_2 + (-6)m_3)$$

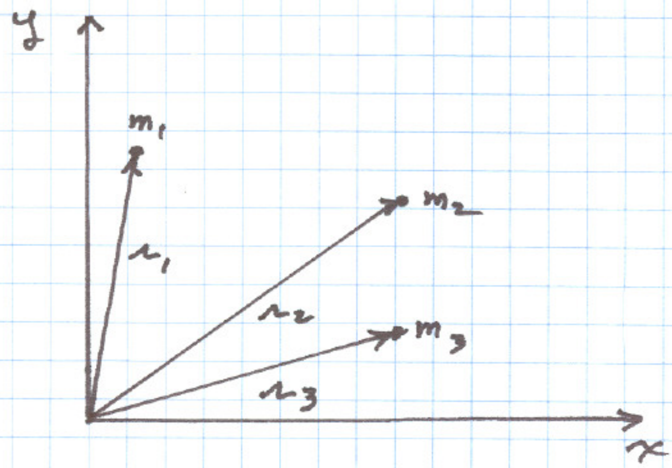
$$= \frac{3(2) + 8(4) - 6(6)}{12} = \frac{6 + 32 - 36}{12} = 2$$

$$X_{CM} = \frac{2}{12} = \frac{1}{6} = 0.166$$

$$Y_{CM} = \frac{1}{2+4+6} \times (2m_1 + 6m_2 + 4m_3)$$

$$= \frac{2(2) + 6(4) + 4(6)}{12} = \frac{4 + 24 + 24}{} = \frac{52}{12}$$

$$Y_{CM} = \frac{52}{12} = 4\tfrac{1}{3} = 4.33$$



$$\vec{r}_{CM} = \frac{1}{M} \sum_{i=1}^{N} m_i \vec{r}_i \qquad \vec{r}_i = x_i \vec{i} + y_i \vec{j} + z_i \vec{k}$$

$$\vec{r}_{CM} = \frac{1}{M} \left\{ m_1 \vec{r}_1 + m_2 \vec{r}_2 + \cdots m_N \vec{r}_N \right\}$$

$$\vec{v}_{CM} = \frac{1}{M} \left\{ m_1 \vec{v}_1 + m_2 \vec{v}_2 + \cdots m_N \vec{v}_N \right\}$$

$$\vec{a}_{CM} = \frac{1}{M} \left\{ m_1 \vec{a}_1 + m_2 \vec{a}_2 + \cdots m_N \vec{a}_N \right\}$$

$$\sum F_{ext,x} = M a_{CM,x}$$

$$\sum F_{ext,y} = M a_{CM,y}$$

$$\sum F_{ext,z} = M a_{CM,z}$$

MOMENTUM

$$\vec{p} = m\vec{v}$$

$$\frac{d\vec{p}}{dt} = m\frac{d\vec{v}}{dt} + \cancelto{0}{\vec{v}\frac{dm}{dt}} = m\vec{a} = \Sigma \vec{F}_{EXT}$$

$$\Sigma \vec{F}_{EXT} = \frac{dp}{dt}$$

IF $\Sigma F_{EXT} = 0$ $\quad \frac{d\vec{p}}{dt} = 0 \quad$ $p =$ CONSTANT

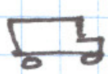
$$\left.\begin{aligned} p_{ix} &= p_{fx} \\ p_{iy} &= p_{fy} \\ p_{iz} &= p_{fz} \end{aligned}\right\} \text{CONSERVATION OF MOMENTUM WHEN } \Sigma \vec{F}_{EXT} = 0$$

EXAMPLE II

$m_1 = 2400$ kg

$\vec{v} = 80 \frac{km}{HR}$

find the $\vec{v}_{CM}$



$m_2 = 1600$ kg

$\vec{v} = 60 \frac{km}{HR}$

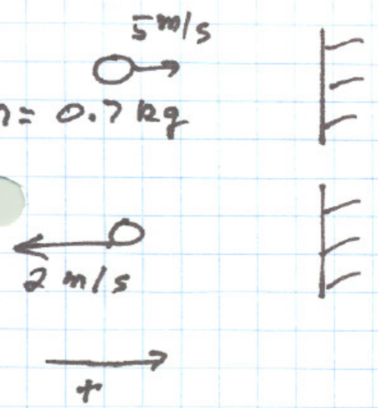
$$v_{CM} M = v_1 m_1 + v_2 m_2$$

$$v_{CM}(2400 + 1600) = 80(2400) + 60(1600)$$

$$v_{CM} = \frac{288{,}000}{4000} = 72 \frac{km}{HR}$$

EXAMPLE III - A BALL HITS A WALL AND BOUNCES BACK in the OPPOSITE DIRECTION

find THE CHANGE IN LINEAR MOMENTUM



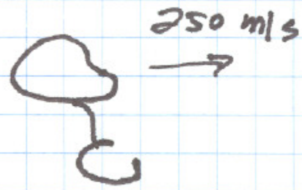
$$\Delta p = p_f - p_i = 0.7(-2) - 0.7(+5)$$

$$= -1.4 - 3.5 = -4.9 \ \frac{kg\text{-}m}{s}$$

$$\overleftarrow{\Delta p} = 4.9 \ \frac{kg\text{-}m}{s}$$

EXAMPLE IV

ROCKET SLED



$m = 2900\ kg$

Determine the FINAL Velocity of the SLED & Scoop

$$\Sigma F_x = \frac{dp_x}{dt} \qquad \text{BUT } F_x = 0$$

$$p_f = p_i$$

$$v_f(2900 + 920) = 2900(250)$$

$$v_f = 190\ m/s$$

$1\# = 4.45\,N \qquad 242\,kg \times \frac{9.8\,m}{s^2} \Big/ 4.45\,N/\# = 527\#$

EXAMPLE V

R.R. FLATCAR

$v_0 = 0 \quad 2140\ kg$

SUMO WRESTLER

242 kg

5.3 m/s

$v_f = ?$

$$\Sigma F_{ext} = \frac{dp}{dt}$$

$$\Sigma F_{ext} = 0 \qquad \frac{dp}{dt} = 0 \qquad p = \text{CONSTANT}$$

$$p_f = p_i$$

$$p_i = 2140(0) + 242(5.3)$$

$$p_f = v_f(2140 + 242)$$

$$v_f = \frac{242(.5)}{242 + 2140} = 0.54\ m/s$$