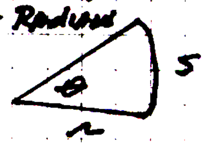
ROTATION, MOMENT OF INERTIA, ENERGY

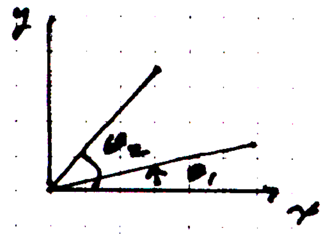
REVOLUTION = 360° = 2π RADIANS

1 RADIAN = 57.3°



$s = r\theta$

ANGULAR DISPLACEMENT



$\Delta x = x_2 - x_1$ (LINEAR)

$\Delta \theta = \theta_2 - \theta_1$ (ANGULAR)

LINEAR VELOCITY

$$v_{avg} = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

ANGULAR VELOCITY

$$\omega_{avg} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta \theta}{\Delta t}$$

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta \theta}{\Delta t} = \frac{d\theta}{dt}$$

LINEAR ACCELERATION

$$a_{avg} = \frac{v_2 - v_1}{t_2 - t_1} = \frac{\Delta v}{\Delta t}$$

ANGULAR ACCELERATION

$$\alpha_{avg} = \frac{\omega_2 - \omega_1}{t_2 - t_1} = \frac{\Delta \omega}{\Delta t}$$

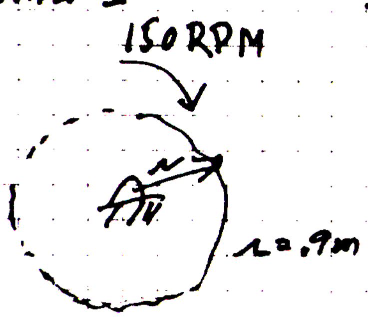
$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t} = \frac{d\omega}{dt}$$

$$s = r\theta$$

$$\frac{ds}{dt} = r\frac{d\theta}{dt} = r\omega$$

$$\vec{v}(r) = r\vec{\omega}$$

EXAMPLE I



find VELOCITY ON THE edge OF THE WHEEL

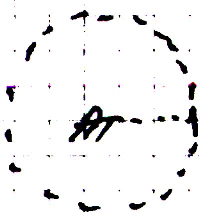
$$V = r\omega$$

$$= (0.9)\left(150\,\frac{\text{Rev}}{\text{MIN}} \times \frac{1\ \text{min}}{60\ \text{SEC}} \times \frac{2\pi\ \text{RAD}}{1\ \text{Rev}}\right) = 14.1\ \text{m/s}$$

RELATIONSHIPS BETWEEN LINEAR & ANGULAR MOTION

LINEAR	ANGULAR
x	θ
v	ω
a	α
$v = v_0 + at$	$\omega = \omega_0 + \alpha t$
$x - x_0 = v_0 t + \frac{1}{2} a t^2$	$\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2$
$v^2 = v_0^2 + 2a(x - x_0)$	$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$
$x - x_0 = \frac{1}{2}(v_0 + v)t$	$\theta - \theta_0 = \frac{1}{2}(\omega_0 + \omega)t$

EXAMPLE II (#12) A DISK ROTATES WITH CONSTANT ACCELERATION. IN 5 SEC IT ROTATES 25 RADIANS. DURING THAT TIME FIND:



$\omega_0 = 0$

a) THE ANGULAR ACCELERATION

$$\theta = \cancelto{0}{\omega_0 t} + \frac{1}{2}\alpha t^2$$

$$25 = \frac{1}{2}\alpha(5)^2 \qquad \alpha = 2.0 \text{ RAD}/s^2$$

b) THE AVERAGE ANGULAR VELOCITY

$$\omega_{Avg} = \frac{\theta_f - \theta_i}{t_f - t_i} = \frac{25 - 0}{5 - 0} = 5 \text{ RAD/SEC}$$

c) THE INSTANTANEOUS ω AFTER 5 SEC

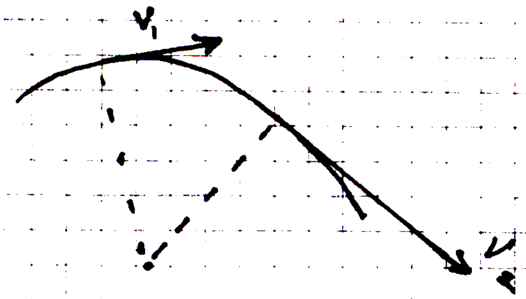
$$\omega = \cancelto{0}{\omega_0} + \alpha t = 2.0(5) = 10 \text{ RAD/SEC}$$

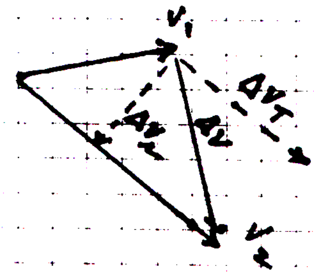
d) IF α REMAINS CONSTANT THROUGH WHAT ADDITIONAL ANGLE WILL THE DISK TURN IN THE NEXT 5 SEC

$$\theta_{TOTAL} = \cancelto{0}{\omega_0} + \frac{1}{2}\alpha t^2 = \frac{1}{2}(2)(10)^2 = 100 \text{ RADIANS}$$

$$\theta_{\text{IN THE LAST 5 SEC}} = \theta_{TOT} - \theta_{INITIAL} = 100 - 25 = 75 \text{ RAD}$$

ACCELERATION IN ROTATIONAL MOTION





ΔV_T INCREASES THE MAGNITUDE OF THE VELOCITY VECTOR

$$\frac{\Delta V_T}{\Delta t} = a_T = r\alpha \quad \text{TANGENTIAL ACCEL}$$

ΔV_R CHANGES THE DIRECTION OF THE VELOCITY VECTOR

$$\frac{\Delta V_R}{\Delta t} = a_R = \frac{V^2}{R} = \frac{(R\omega)^2}{R} = R\omega^2$$

$$a_R = R\omega^2$$

EXAMPLE III AN ASTRONAUT IS BEING TESTED IN A
(#23) CENTRIFUGE. IT HAS A RADIUS OF 10m & AS IT STARTS IT ROTATES ACCORDING TO $\theta = 0.3t^2$
FOR $t = 5$ SEC FIND:

a) ASTRONAUT'S ω
b) " LINEAR VELOCITY v
c) " TANGENTIAL ACCEL
d) " RADIAL ACCEL

a) $\omega = \frac{d\theta}{dt} = \frac{d}{dt}(0.3t^2) = 2(0.3)t\Big|_{t=0}^{t=5} = 2(0.3)(5-0)$
$= 3.0$ RAD/SEC

b) $v = r\omega = 3.0(10) = 30$ m/SEC
(SHOW RIGHT HAND RULE APPLICATION)

c) $a_T = r\alpha$

$\alpha = \frac{d\omega}{dt} = \frac{d}{dt}\left(0.6t\right) = 0.6 \text{ rad/s}^2$

$a_T = r\alpha = 10(0.6) = 6 \text{ m/s}^2$

d) $a_R = R\omega^2 = 10(3.0)^2 = 90 \text{ m/s}^2$

KINETIC ENERGY

$KE = \frac{1}{2}mv^2$ BUT $v = R\omega$

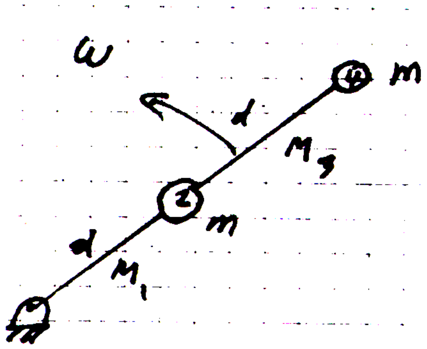
$KE = \frac{1}{2}m(r\omega)^2 = \frac{1}{2}\underbrace{mr^2}_{I}\omega^2 = \frac{1}{2}I\omega^2$

$\boxed{KE = \frac{1}{2}I\omega^2}$ $I = \Sigma m_i r_i^2$

I IS TERMED THE ROTATIONAL INERTIA OR MOMENT OF INERTIA

REFER TO PAGE 227 OF THE TEXT FOR TYPICAL ROTATIONAL INERTIA'S

EXAMPLE IV (# 37)



IN TERMS OF THE SYMBOLS GIVEN AT THE LEFT CALCULATE THE COMBINATIONS I & KE

$I = I_1 + I_2 + I_3 + I_4$

$= \left(\frac{1}{12}Md^2 + M\left(\frac{1}{2}d\right)^2\right) + md^2$

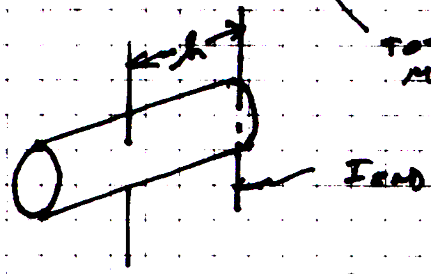
$+ \left(\frac{1}{12}Md^2 + M\left(\frac{3}{2}d\right)^2 + m(2d)^2 = \frac{8}{3}Md^2 + 5md^2\right.$

PARALLEL AXIS THEOREM

$$I = I_{C\,of\,M} + M h^2$$

M: TOTAL MASS

h: DISTANCE FROM C of M TO THE AXIS IN QUESTION



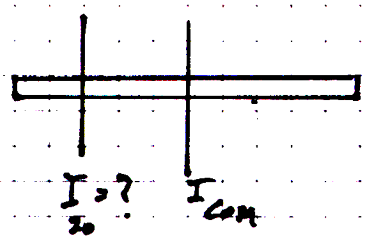
$$I_{cm} = \frac{1}{12} M L^2$$

$$I_{END} = I_{C\,of\,M} + M(h^2) = \frac{1}{12} M L^2 + M\left(\frac{L}{2}\right)^2$$
$$= \frac{1}{12} M L^2 + M \frac{L^2}{4} = \frac{1}{3} M L^2$$

$$\boxed{I_{END} = \frac{1}{3} M L^2}$$

EXAMPLE X (# 39)

CALCULATE I OF A METER STICK WITH $m = 0.56\,kg$ ABOUT AN AXIS ⊥ TO THE STICK & LOCATED AT THE 20 CM MARK (USE THE STICK AS A THIN ROD)



FOR A THIN ROD

$$I_{\text{THIN ROD}} = \frac{1}{12} M L^2$$

$$I_{20} = I_{com} + M h^2$$
$$= \underbrace{\frac{1}{12}(.56)(1)^2}_{0.0467} + \underbrace{(.56)(.5 - .2)^2}_{0.0504} = 0.0971\,kg \cdot m^2$$