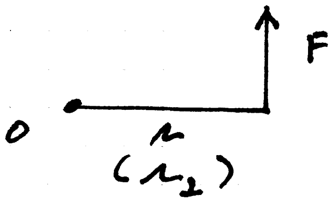
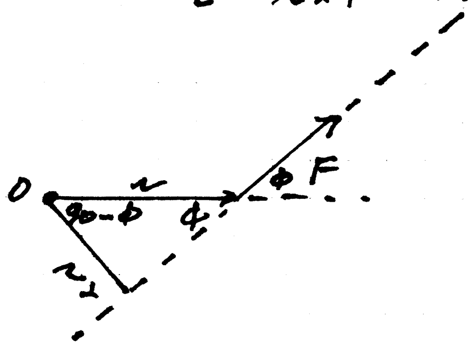
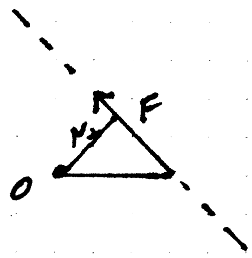
TORQUE, NEWTON'S 2ND LAW, POWER, ROLLING

TORQUE IS THE PRODUCT OF FORCE TIMES THE PERPENDICULAR DISTANCE BETWEEN THE ROTATION AXIS & THE LINE OF ACTION OF THE FORCE



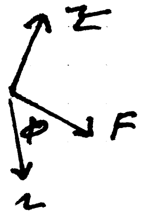
$\vec{\tau} = \vec{r} \times \vec{F} = F r \sin\phi$



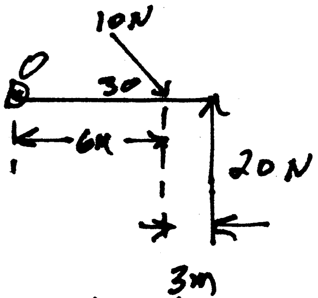


$\tau = F(r_\perp) = F(r\cos(90-\phi))$
$= F r \sin\phi$
$= F r_\perp$

DIRECTION ASSOCIATED WITH TORQUE $\curvearrowright_{cw}$ OR $\curvearrowleft_{ccw}$



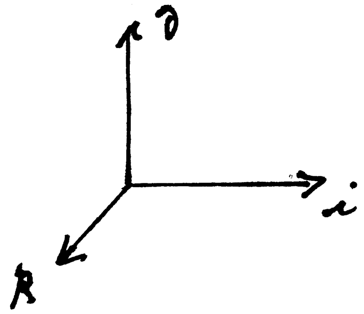
EXAMPLE I



$\Sigma \tau_0 = -10\sin 30(6) + 20(6+3)$

$-30 + 180 = +150\ NM$

LINEAR MOTION	ANGULAR MOTION
x	θ
F	τ
a	α
m	I
$\Sigma F = ma$	$\Sigma \tau = I\alpha$

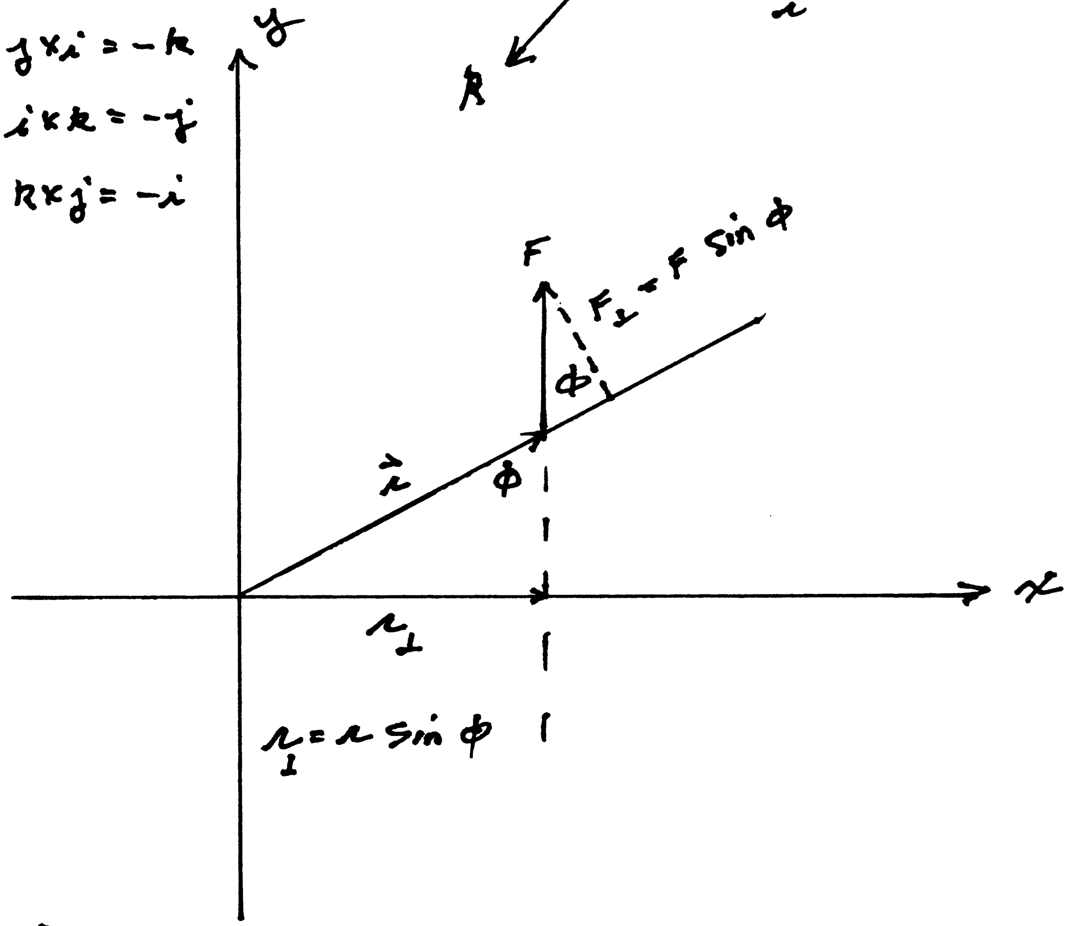


$i \times j = |i||j| \sin 90^\circ = 1\,k$

$i \times j = k \qquad j \times i = -k$

$k \times i = j \qquad i \times k = -j$

$j \times k = i \qquad k \times j = -i$

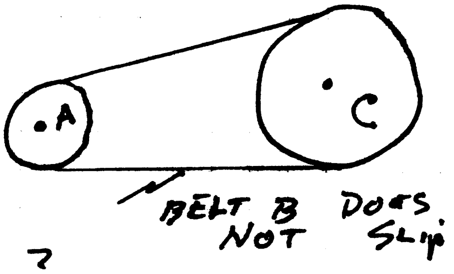


$\tau = \vec{r} \times \vec{F} = rF \sin \phi = r_\perp F = F_\perp r$

EXAMPLE III (#79)

$r_A = 10\,CM$

$r_C = 25\,CM$



$\omega_A \quad 0 \longrightarrow ?$
$\alpha = 1.6\ RAD/s^2$

find Time FOR WHEEL C TO REACH A Speed of 100RPM if $\omega_{0C} = 0$

SINCE THE BELT B DOES NOT SLIP

$$\left\{\begin{matrix}\text{POINT ON THE RIM OF} \\ \text{WHEEL C}\end{matrix}\right\} a_{T \text{ of } C} = a_{T \text{ of } A} \left\{\begin{matrix}\text{POINT ON THE} \\ \text{RIM OF WHEEL A}\end{matrix}\right\}$$

$$\alpha_A r_A = \alpha_C r_C$$

$$\alpha_C = \left(\frac{r_A}{r_C}\right)\alpha_A = \frac{10}{25}(1.6) = 0.64\ RAD/s^2$$

$$\omega_C = \omega_{0C}^{\nearrow 0} + \alpha_C t$$

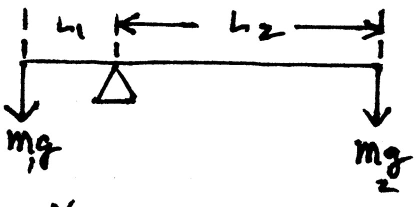
$$100\,\frac{REV}{min} \times \frac{2\pi\ RAD}{1\,REV} \times \frac{1\,min}{60\,SEC} = 10.5\ RAD/s$$

$$\frac{\omega_C}{\alpha} = t = \frac{10.5}{0.64} = 16.3\ SEC$$

EXAMPLE IV #57

$L_1 = 20\,CM$

$L_2 = 60\,CM$



find the initial ACCELERATION of the MASS ON THE LEFT

$$\circlearrowleft^{+} \quad \Sigma \tau = I\alpha$$

$$\tau = mgL_1 - mgL_2 = I\alpha$$

$$I_1 = \sum_{i=1}^{N} m_i r_i^2 = mgL_1$$

$$I_2 = \qquad = mgL_2$$

$$\cancel{m}gL_1 - \cancel{m}gL_2 = \cancel{m}(L_1^2 + L_2^2)\alpha$$

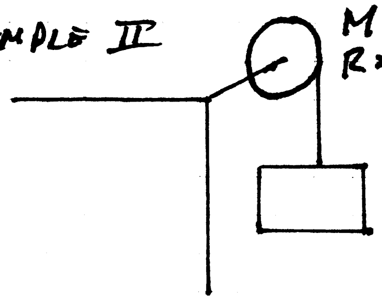
$$I = I_1 + I_2 = m(L_1^2 + L_2^2)$$

$$\alpha = \frac{g(L_1 - L_2)}{L_1^2 + L_2^2} = \frac{9.8(.2 - .6)}{.6^2 + .2^2}$$

$8.65\ RAD/Sec^2$

$a_1 = 8.65(0.2) = 1.7\ m/s^2 \uparrow \qquad a_2 = 8.65(.6) = 6.9\ m/s^2 \downarrow$

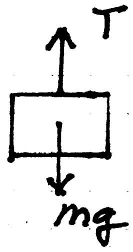
EXAMPLE II



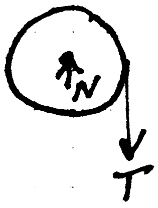
$M = 2.5\ kg$
$R = 20\ CM$

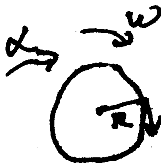
SAMPLE PROBLEM (11-7)

$m = 1.2\ kg \quad \downarrow +$



$I_{DISK} = \frac{1}{2} M R^2$





$v = R\omega$

$a = R\alpha$

$\Sigma \Gamma = I\alpha$

$TR = \frac{1}{2} M R^2 \alpha$

$T = \frac{1}{2} M R \alpha$

$\Sigma F = ma$

$mg - T = ma$

$mg - T = R\alpha m$

$mg - \frac{1}{2} M R\alpha = m R\alpha$

$mg = \alpha (mR + \frac{1}{2} MR)$

$\alpha = \dfrac{mg}{R(m + \frac{M}{2})}$

$= \dfrac{1.2(9.8)}{0.2(1.2 + \frac{2.5}{2})} = \dfrac{11.76}{0.49}$

$= 24$ RADIANS / SEC^2

$T = \frac{1}{2}(2.5)(0.2)(24)$

$= 6$ NEWTONS

WORK & ROTATIONAL KE

$\text{POWER} = \dfrac{dW}{dt} = Fv$

BUT $F \rightarrow \tau \qquad v \rightarrow \omega$

$P = \tau\omega$

ALSO THE CONNECTION BETWEEN WK & KE IS

$\Delta KE = KE_f - KE_i = \frac{1}{2} I\omega_f^2 - \frac{1}{2} I\omega_i^2 = WK$

SINCE $WK = \Delta KE + \cancel{\Delta U_g}^{\,0} + \cancel{\Delta U_{sp}}^{\,0}$