GRAVITATION (CHAP 14)

THE MILKY WAY GALAXY CONTAINS BILLIONS OF PLANETS & STARS INCLUDING OUR SUN & SOLAR SYSTEM

PLANET	SUN	MERCURY	VENUS	EARTH	MARS	JUPITER	SATURN	URANUS	NEPTUNE	PLUTO
SYMBOL	☉	☿	♀	⊗	♂	♃	♄	♅	♆	♇
PERIOD	—	88 DAYS	225 DAYS	365¼ DAYS	687 DAYS	11.9 YRS	29.5 YRS	84 YRS	165 YRS	248.4 YRS

UNIVERSAL LAW OF GRAVITATION

$$F = G\frac{m_1 m_2}{r^2} \qquad G = 6.67\times10^{-11}\ N\cdot m^2/kg^2$$

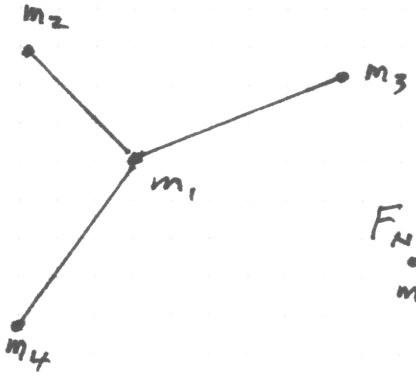
$r_{MASSES} << r_{SEPARATION}$

$r_e = 6.37\times10^6\ m$

$r_m = 1.74\times10^6\ m$

$r_{em} = 3.82\times10^8\ m$

$m_e = 5.98\times10^{24}\ kg$



$$F_{NET\ ON\ m_1} = F_{12} + F_{13} + F_{14}$$

NOW LET US CONSIDER HOW GRAVITY VARIES WITH ALTITUDE

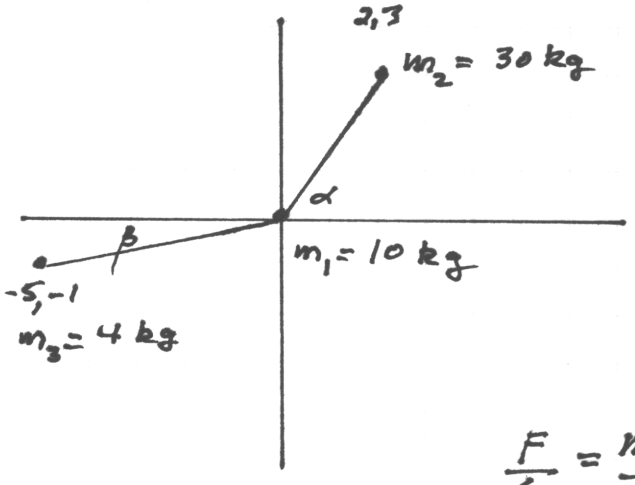
$$F = \frac{G m_e m}{(r_e + h)^2} \quad \& \quad W = mg \quad \text{SO THAT}$$

$$g = \frac{G m_e}{(r_e + h)^2}$$

h - ALTITUDE ABOVE the EARTH'S SURFACE

EXAMPLE	ALTITUDE km	ALTITUDE kilo-Feet	g m/s^2
EARTH'S SURFACE	0	0	9.83
MT. EVEREST	8.8	29	9.80
HIGHEST MANNED BALOON	36.6	121	9.71
SPACE SHUTTLE ORBIT	400	1,320	8.70
COMMUNICATION SATELLITE	37,700	1,244	0.225

EXAMPLE I - FIND THE RESULTANT FORCE ON MASS m_1 AS A RESULT OF m_2 & m_3



$r_{12} = \sqrt{2^2+3^2} = \sqrt{13} = 3.6$

$r_{13} = \sqrt{1^2+5^2} = \sqrt{26} = 5.1$

$\tan\alpha = \frac{3}{2} \quad \alpha = 56.3^\circ$

$\tan\beta = \frac{-1}{-5} \quad \beta = 11.3^\circ$

$$\frac{F}{G} = \frac{m_1 m_2}{r^2}$$

$$\frac{F_{12x}}{G} = \frac{10(30)}{(\sqrt{13})^2}\underbrace{\cos 56.3}_{.555} = 12.7$$

$$\frac{F_{12y}}{G} = \frac{10(30)}{13}\underbrace{\sin 56.3}_{.832} = 19.2$$

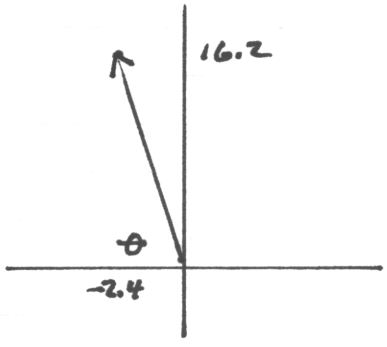
$$\frac{F_{13x}}{G} = -\frac{10(40)}{(\sqrt{26})^2}\underbrace{\cos 11.3}_{.981} = -15.1$$

$$\frac{F_{13y}}{G} = -\frac{10(40)}{26}\underbrace{\sin 11.3}_{.196} = -3.0$$

$\frac{\Sigma F_x}{G} = 12.7 - 15.1 = -2.4$

$\frac{\Sigma F_y}{G} = 19.2 - 3.0 = 16.2$

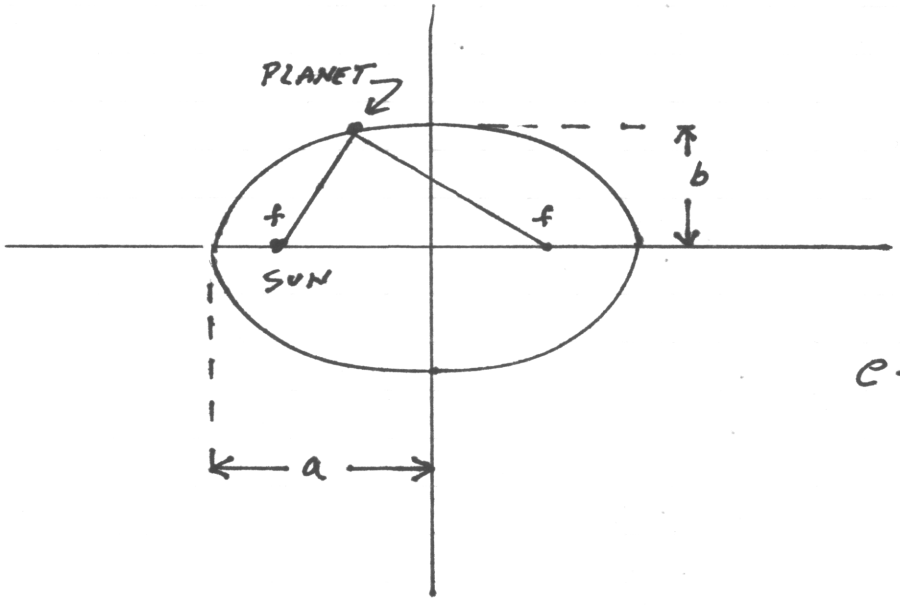
RESULTANT $R = G\sqrt{(-2.4)^2 + (16.2)^2} = 16.4\,G = 1.09 \times 10^{-9}$ NEWTONS



$\tan\theta = \frac{16.2}{-2.4}$ $\theta = 81.6^\circ$

KEPLER'S LAWS

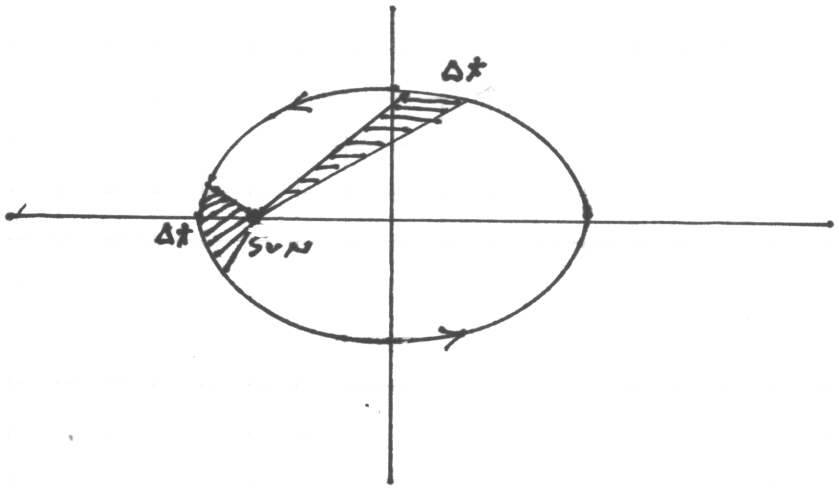
I LAW OF ORBITS - ALL PLANETS MOVE in ELLIPTICAL ORBITS WITH THE SUN AS A FOCUS



$e = \frac{b}{a}$

II LAW OF AREAS - A LINE THAT CONNECTS A PLANET TO THE SUN SWEEPS OUT = AREAS in = TIMES

$\frac{dA}{dt} = \frac{L}{2m} =$ CONSTANT



III LAW OF PERIODS - THE SQUARE OF THE PERIOD OF ANY PLANET IS PROPORTIONAL TO THE CUBE OF THE SEMI MAJOR AXIS (a) OF THE ORBIT

$$T^2 = \left(\frac{4\pi^2}{GM}\right) a^3$$

M IS THE MASS OF THE CENTRAL BODY AROUND WHICH THE MASS m IS ORBITING

FURTHER WE ASSUME $a = r$ i.e. WE ASSUME A CIRCULAR ORBIT SO THAT

$$\boxed{T^2 = \left(\frac{4\pi^2}{GM}\right) r^3}$$

r in meters
T in years

EXAMPLE II $r_{\substack{\text{MARS}\\\text{TO}\\\text{SUN}}} = 1.52\, r_{\substack{\text{EARTH}\\\text{TO}\\\text{SUN}}}$

HOW MANY YEARS ARE REQ'd FOR MARS TO MAKE 1 REVOLUTION AROUND THE SUN

$$T^2 = \left(\frac{4\pi^2}{GM}\right) r^3$$

$$T_{EARTH}^2 = \left(\frac{4\pi^2}{G M_{SUN}}\right) r_e^3 \qquad T_{MARS}^2 = \left(\frac{4\pi^2}{G M_{SUN}}\right) r_{MARS}^3$$

$$\left(\frac{T_{MARS}}{T_{EARTH}}\right)^2 = \frac{(4\pi^2/G M_{SUN})}{(4\pi^2/G M_{SUN})} \left(\frac{r_{MARS}}{r_{EARTH}}\right)^3$$

$$\frac{T_{MARS}}{T_{EARTH}} = \left(\frac{r_{MARS}}{r_{EARTH}}\right)^{3/2} = 1.52^{3/2} = 1.87 \text{ YEARS}$$

THE APPENDIX GIVES $T = 1.88$ YEARS

EXAMPLE III MARTIAN SATELLITE PHOBOS TRAVELS IN AN APPROXIMATE CIRCULAR ORBIT WITH $r = 9.4 \times 10^6$ m & A PERIOD OF 7HR 39min. CALCULATE THE MASS OF MARS

$$T_{PHOBOS}^2 = \left(4\pi^2/G M_{MARS}\right) r_{PHOBOS}^3$$

$$M_{MARS} = \frac{4\pi^2 r^3}{G T^2}$$

$$M_{MARS} = \frac{4\pi^2 (9.4 \times 10^6)^3}{(6.67 \times 10^{-11})(2.754 \times 10^4)^2} = 6.5 \times 10^{23} \text{ kg}$$

ACTUAL MASS IS 6.39×10^{23} kg