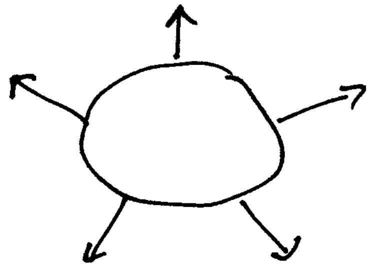
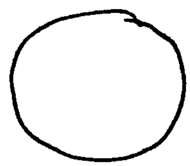
GAUSS'S LAW

GAUSS'S LAW RELATES THE NET CHARGE WITHIN A CLOSED SURFACE TO THE $\vec{E}$ CROSSING THE SURFACE



$\vec{E}$ CROSSING THE SURFACE $\Longrightarrow$ CHARGE WITHIN THE SURFACE

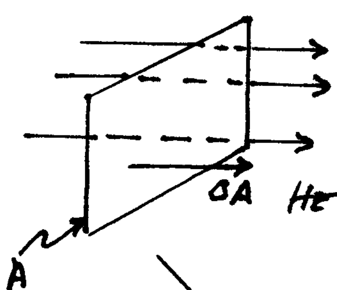


DOES THIS IMPLY NO CHARGE WITHIN THE SURFACE } ?

FLUX (TO FLOW) $\Longrightarrow$ VOLUME FLOW OF SOMETHING / TIME

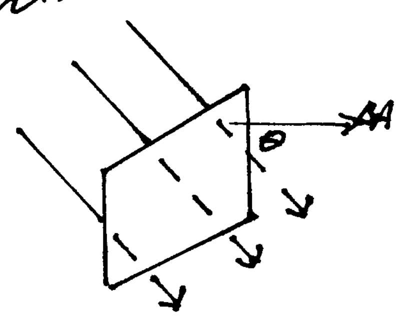
$$\Phi = \Delta A\, v \qquad [m^3/s]$$

v IS $\parallel$ TO THE VECTOR ΔA



$$\Phi = \Delta A\, v$$

HERE $v \perp A$ & $\vec{A}$ IS $\perp$ TO THE SURFACE



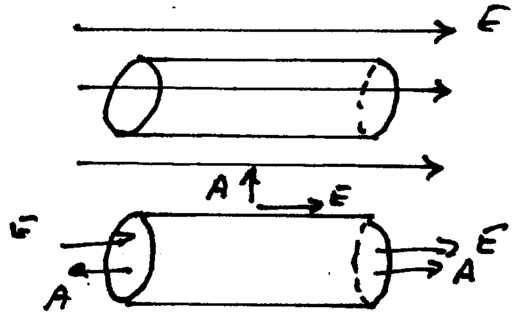
$$\Phi = \Delta A\, v \cos\theta = \vec{v} \cdot \vec{\Delta A}$$

IN THE CASE WHERE $v \rightarrow E$

$$\Phi = \sum \vec{E} \cdot \Delta\vec{A} = \int \vec{E} \cdot d\vec{A}$$

THE ELECTRIC FLUX PASSING THROUGH A GAUSSIAN SURFACE $\propto$ # OF $\vec{E}$ LINES PASSING THROUGH THE SURFACE

EXAMPLE I - FIND THE NET FLUX THROUGH THE CYLINDER SHOWN BELOW



$$\Phi = \int \vec{E} \cdot d\vec{A} = \int E\,dA\cos\theta$$

$A \uparrow \xrightarrow{E}$ $\quad EA\cos\theta = 0$, $\theta = 90°$

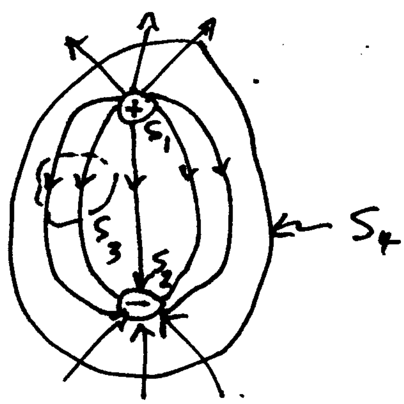
$\rightrightarrows$ $\quad EA\cos\theta = EA$, $\theta = 0$

$\overset{E}{\rightarrow}\underset{A}{\leftarrow}$ $\quad EA\cos\theta = -EA$, $\theta = 180°$

$$\Phi = -EA + 0 + EA = 0$$

GAUSS'S LAW

$$\boxed{\epsilon_0 \Phi = q_{\substack{\text{NET} \\ \text{ENCLOSED}}}}$$



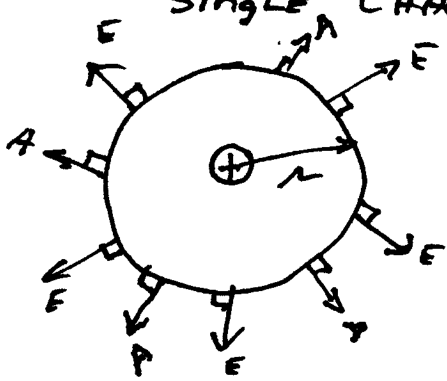
$\int_{S_1} \vec{E} \cdot d\vec{A} > 0 \qquad q > 0$

$\int_{S_2} \vec{E} \cdot d\vec{A} < 0 \qquad q < 0$

$\int_{S_3} \vec{E} \cdot d\vec{A} = 0 \qquad q = 0$

$\int_{S_4} \vec{E} \cdot d\vec{A} = 0 \qquad q = 0$

EXAMPLE II APPLY GAUSS'S LAW TO A SINGLE CHARGE q ENCLOSED WITHIN A SPHERE



$$\epsilon_0 \int E \cdot dA = q$$

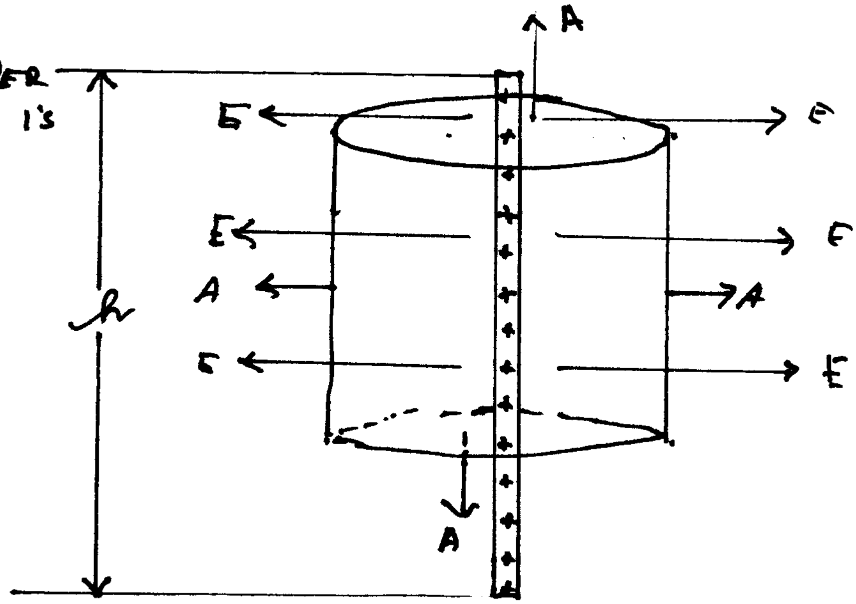
BUT E IS ALWAYS $\parallel$ TO dA

$$\epsilon_0 \int \vec{E} \cdot d\vec{A} = \epsilon_0 \int E\,dA \underbrace{\cos 0}_{1} = \epsilon_0 E (4\pi r^2) = q$$

$$E = \frac{q}{4\pi\epsilon_0 r^2}$$

EXAMPLE III FIND E AT A DISTANCE r FROM THE AXIS OF THE ROD

THE CHARGE PER UNIT LENGTH IS λ [C/m]



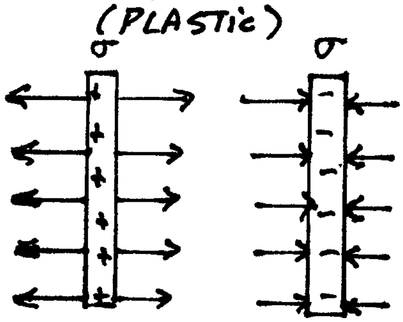
$$\underset{\text{TOP CAP}}{\int \vec{E}\cdot d\vec{A}} + \underset{\text{CYLINDER}}{\int \vec{E}\cdot d\vec{A}} + \underset{\text{BOTTOM CAP}}{\int \vec{E}\cdot d\vec{A}} = \frac{q}{\epsilon_0}$$

$E \perp A$	$E \parallel A$	$E \perp A$
$\theta = 90°$ $\cos 90 = 0$	$\theta = 0$ $\cos 0 = 1$	$\theta = 90$ $\cos 90 = 0$
$\int_{\text{Top}} \vec{E}\cdot d\vec{A} = 0$	$\int \vec{E}\cdot d\vec{A} = EA$	$\int_{\text{BOT}} \vec{E}\cdot d\vec{A} = 0$

$$E(2\pi r h) = q/\epsilon_0 = \frac{\lambda h}{\epsilon_0}$$

$$\boxed{E = \frac{\lambda}{2\pi\epsilon_0 r}}$$

NON CONDUCTING PLATES (PLASTIC)

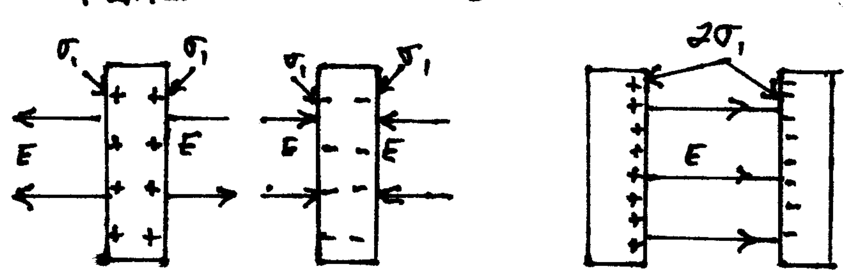


$E = \frac{\sigma}{2\epsilon_0}$ $\qquad E = +\frac{\sigma}{2\epsilon_0}$

σ - CHARGE DENSITY

CONDUCTING PLATES

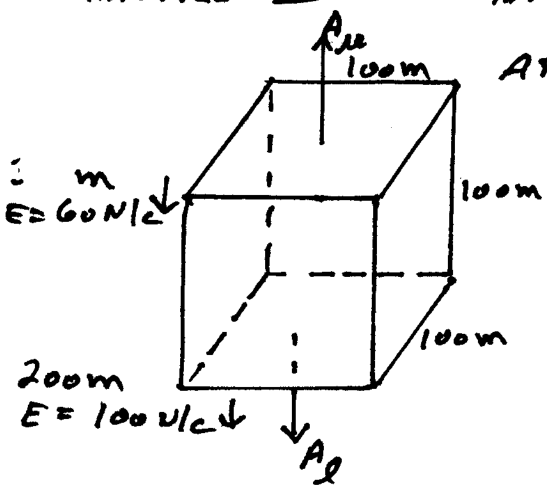
$$\sigma_{\text{TWO PLATES}} = \frac{2\sigma_1}{\epsilon_0} = \frac{\sigma}{\epsilon_0}$$



EXAMPLE IV

AT 300m $E = 60\ N/c \downarrow$

AT 200m $E = 100\ N/c \downarrow$



FIND THE NET CHARGE IN A 100m CUBE WITH HORIZONTAL FACES AT 200m & 300m

$$\epsilon_0 \Phi = E_u \Delta A_u \underset{\theta=180}{\cos\theta} + E_L \Delta A_l \underset{\theta=0}{\cos\theta} + 4\underbrace{E_{SIDE}\, \Delta A_{SIDE} \cos\theta}_{E \perp \Delta A_{SIDE} \quad \theta = 90 \quad \cos 90 = 0}$$

$\Delta A = (100)^2 = 10^4\ m^2$

$$= E_u \Delta A_u (-1) + E_L \Delta A_l (+1)$$

$$= \Delta A (E_L - E_U)$$

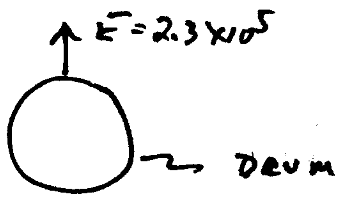
$$= (100)^2 (100 - 60) = 4 \times 10^5$$

$$q = \epsilon_0 \Phi = 8.85 \times 10^{-12} (4 \times 10^5) = 35.4 \times 10^{-7} = 3.54\ \mu C$$

$$\boxed{q = 3.54\ \mu C}$$

EXAMPLE V

$\vec{E}$ JUST ABOVE THE SURFACE OF A CHARGED DRUM IS $2.3 \times 10^5\ N/c$. WHAT IS THE SURFACE CHG. DENSITY σ OF THE DRUM



FOR A CONDUCTOR

$$E = \frac{\sigma}{\epsilon_0}$$

FOR A NON CONDUCTOR

$$E = \frac{\sigma}{2\epsilon_0}$$

$$\sigma = E\epsilon_0 = 2.3 \times 10^5 (8.85 \times 10^{-12}) = 2.04\ \mu \text{ COULOMBS}$$