**RC Circuits**

![RC Circuit Diagram]

\[ V_C = \frac{Q}{C} \rightarrow Q_{cap} = CE \]

First we close the switch to charge the capacitor. The charging process is governed by the loop eq.

\[ E - iR - \frac{Q}{C} = 0 \]

But \( i = \frac{dQ}{dt} \)

\[ R \frac{dQ}{dx} + \frac{Q}{C} = E \]

With the initial condition that at \( x = 0 \) \( q = 0 \)

So the solution is

\[ q = CE (1 - e^{-x/RC}) \]

\( t = RC \)

\[ x = 2 \quad (1 - \frac{1}{e^2}) = 1 - \frac{1}{e^2} = 63\% \quad q = 0.63CE \]

\[ x = 4 \quad (1 - \frac{1}{e^4}) = 1 - \frac{1}{e^4} = 88\% \quad q = 0.88CE \]

**Voltage across the capacitor during charging is**

\[ V_C = \frac{Q}{C} = CE (1 - e^{-x/RC}) \]
\[ i = \frac{dq}{dt} = \frac{V}{R} e^{-t/RC} \]

\[ \text{Units of the time constant} \]

\[ RC = \left[ \frac{\text{Volts}}{\text{Amp}} \right] \left[ \frac{\text{Coul}}{\text{Volt}} \right] = \frac{\text{Coul}}{\text{Coul/Sec}} \Rightarrow \text{Sec} \]

**Discharging a Capacitor**

**Loop equation yields**

\[ R \frac{dq}{dt} + \frac{q}{C} = 0 \]

**Yielding a solution**

\[ q = q_0 e^{-t/RC} \text{ where at } t=0 \quad q=q_0 \]

\[ i = \frac{dq}{dt} = -(\frac{q_0}{RC}) e^{-t/RC} \]

**Example 1: A capacitor leaks charge**

\[ V_0 = \frac{V_0}{4} \text{ in Sec. What is the equivalent resistance between the plates?} \]

\[ V = V_0 e^{-t/2} \quad t = RC \]
\[
\ln V = \ln V_0 + \frac{-x}{2}
\]

\[
-\ln V = -\ln V_0 + \frac{x}{2}
\]

\[
\ln V_0 - \ln V = \frac{x}{2}
\]

\[
\ln \left( \frac{V_0}{V} \right) = \frac{x}{2} = \frac{x}{RC}
\]

\[
R = \frac{1}{C \ln \left( \frac{V_0}{V} \right)} = \frac{2 \times 10^{-6} \ln (1/1)}{2 \times 10^{-6} (1.386)} = \frac{2}{1.386}
\]

\[
= 0.722 \times 10^{-6} \approx 0.722 \text{ M\Omega}
\]

**Example II**

**How many time constants must elapse for an initially uncharged capacitor in an RC circuit to be charged to a 99.9% value**

\[
q_f = CE \left(1 - e^{-x/2}\right)
\]

\[
q_f = 0.99 = 1 - e^{-x/2}
\]

\[
e^{-x/2} = 1 - 0.99 = 0.01
\]

\[
-\frac{x}{2} = \ln (0.01) = -4.605
\]

\[
x = 4.605 \text{ (time constants)}
\]

**Example IV**

\[
C = \frac{q}{V}
\]

Initially S is open for a long time. It is then closed for a long time. By how much does \( q \) on the capacitor change? (When S is closed)
\[ Q_{\text{initial}} = CV = 10 \times 10^{-6} \times 3 = 3 \times 10^{-5} \text{C} \]

After a long time the current \( i \) in the right hand loop is zero so \( V = \varepsilon_{\text{battery}} = 3 \)

Now when \( S \) is closed for a long time for the outer loop
\[ +3 - 0.4i' - 0.2i - 1 = 0 \]
\[ i' = \frac{2}{0.6} = 3.33 \text{A} \]

Now we find the voltage across the capacitor
\[ V_A + 3 - 0.4(3.33) = V_B \]
\[ V_B - V_A = 3 - 0.4(3.33) = 1.67 \text{ Volts} \]

\[ Q_{\text{final}} = C(V_B - V_A) = 10 \times 10^{-6} \times 1.67 = 1.67 \times 10^{-5} \]

Across the capacitor
\[ Q_f - Q_i = 1.67 \times 10^{-5} - 3 \times 10^{-5} = -1.33 \times 10^{-5} \text{C} \]

The charge has decreased by \( 1.33 \times 10^{-5} \text{C} \). 

\[ \text{VIII-4} \]
EXAMPLE V

\[ U_{\text{cap}} = \frac{1}{2} \text{ Joule} \]

\[ C = 1 \mu F \]

\[ R = 10^6 \Omega \]

\[ S \]

a) Find the initial charge on the capacitor

\[ U_{\text{cap}} = \frac{1}{2} = \frac{q_i^2}{2C} \]

\[ q_i = \sqrt{\frac{1}{2} (2 \times 10^{-6})} = 1 \times 10^{-3} \text{ C} \]

b) What is the current thru the resistor

\[ i = \frac{dq}{dt} = \frac{q_i}{2} (e^{-t/2}) \]

\[ L = RC = 10^6 (1 \times 10^{-6}) = 1 \]

\[ i = \frac{1 \times 10^{-3}}{1} e^{-0/2} = 1 \times 10^{-3} \text{ A} \]

c) Determine \( V_c(t) \) and \( V_R(t) \)

\[ V_c = \frac{q_i}{C} e^{-t/2} = \frac{1 \times 10^{-3}}{1 \times 10^{-6}} e^{-t/1.0} = 1 \times 10^3 e^{-t} \]

\[ V_R = \frac{q_i R}{C} = \frac{1 \times 10^{-3} (10^6)}{1} e^{-t/(10^6)} = 1 \times 10^3 e^{-t} \]