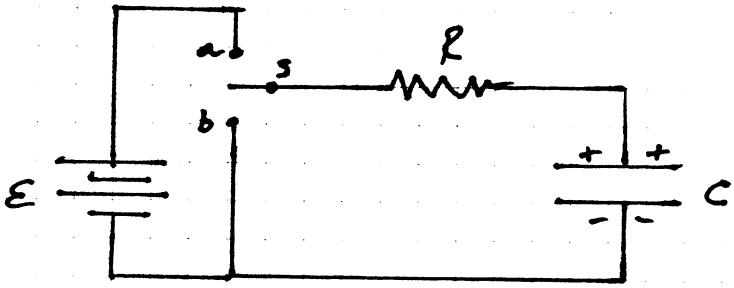
RC CIRCUITS



$V_c = \frac{q}{C} \Rightarrow q_{cap} = \mathcal{E}C$

FIRST WE CLOSE THE SWITCH TO a TO CHARGE THE CAPACITOR. THE <u>CHARGING PROCESS</u> IS GOVERNED BY THE LOOP EQ.

$$\mathcal{E} - iR - \frac{q}{C} = 0 \qquad \text{BUT } i = \frac{dq}{dt}$$

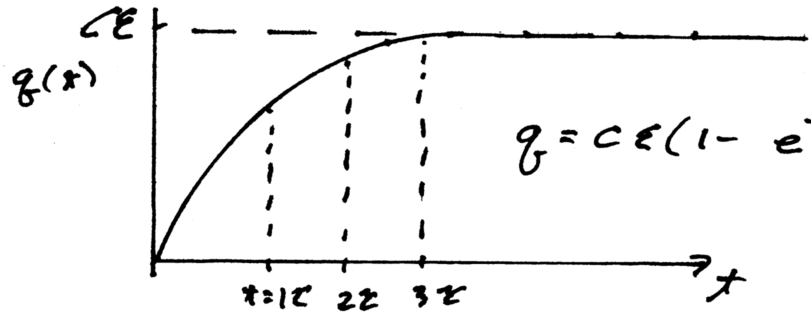
$$R\frac{dq}{dt} + \frac{q}{C} = \mathcal{E}$$

WITH THE INITIAL CONDITION THAT AT $t=0$ $q=0$

SO THE SOLUTION IS

$$q = C\mathcal{E}\left(1 - e^{-t/RC}\right) \qquad \text{At } t=0 \quad q=0$$

$$\text{At } t=\infty \quad q = C\mathcal{E}$$



$q = C\mathcal{E}(1 - e^{-t/\tau})$

$\tau = RC$

$t=\tau \quad (1 - \frac{1}{e}) = 1 - .367 = 63\% \qquad q = .63C\mathcal{E}$

$t=2\tau \quad (1 - \frac{1}{e^2}) = 1 - \frac{1}{7.38} = 86\% \qquad q = .86C\mathcal{E}$

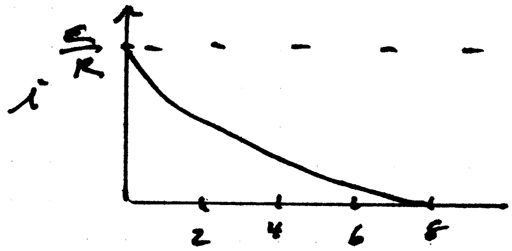
$t=3\tau \quad (1 - \frac{1}{e^3}) = 1 - \frac{1}{20} = 95\% \qquad q = .95C\mathcal{E}$

$t=4\tau \quad (1 - \frac{1}{e^4}) = 1 - \frac{1}{55} = 98\% \qquad q = .98C\mathcal{E}$

<u>VOLTAGE</u> ACROSS THE CAPACITOR <u>DURING</u> <u>CHARGING</u> IS

$$V_c = \frac{q}{C} = \mathcal{E}\left(1 - e^{-t/RC}\right)$$

$$i = \frac{dq}{dt} = \frac{E}{R} e^{-t/RC}$$



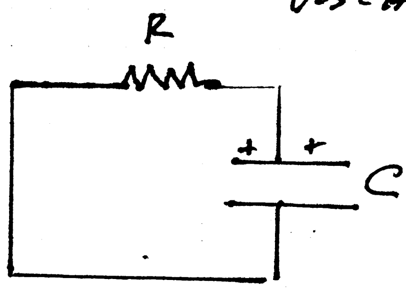
UNITS OF THE TIME CONSTANT

$$R = \frac{V}{I} \qquad C = \frac{q}{V}$$

$$RC \Rightarrow \left[\frac{\text{VOLTS}}{\text{AMP}}\right]\left[\frac{\text{COUL}}{\text{VOLT}}\right]$$

$$= \frac{\text{COUL}}{\text{COUL/SEC}} \Rightarrow \text{SEC}$$

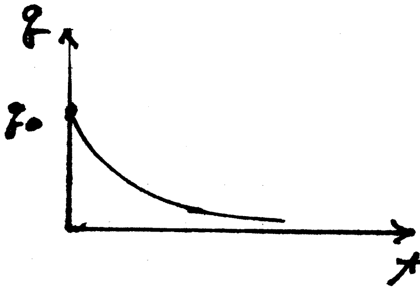
DISCHARGING A CAPACITOR



LOOP EQUATION YIELDS

$$R\frac{dq}{dt} + \frac{q}{C} = 0$$

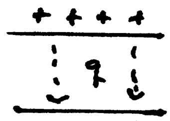
YIELDING A SOLUTION



$$q = q_0 e^{-t/RC}$$ WHERE AT $t=0$ $q=q_0$

$$i = \frac{dq}{dt} = -\left(\frac{q_0}{RC}\right) e^{-t/RC}$$

EXAMPLE I A CAPACITOR LEAKS CHARGE SUCH THAT $V_0 \Rightarrow V_0/4$ in SEC. WHAT is THE EQUIVALENT RESISTANCE BETWEEN THE PLATES



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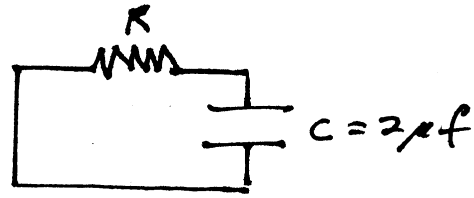


$$V = V_e e^{-t/\tau} \qquad \tau = RC$$

$$\ln V = \ln V_0 + -\frac{t}{\tau}$$

MULT THROUGH BY -1

$$-\ln V = -\ln V_0 + \frac{t}{\tau}$$

$$\ln V_0 - \ln V = t/\tau$$

$$\ln\left(\frac{V_0}{V}\right) = \frac{t}{\tau} = \frac{t}{RC}$$

$$R = \frac{t}{C \ln(V_0/V)} = \frac{2}{2\times10^{-6} \underbrace{\ln(4/1)}_{1.386}} = \frac{2}{2\times10^{-6}(1.386)}$$

$$= 0.722\times10^{+6} = 0.722\ M\Omega$$

EXAMPLE II

HOW MANY TIME CONSTANTS MUST ELAPSE FOR AN INITIALLY UNCHARGED CAPACITOR IN AN RC CIRCUIT TO BE CHARGED TO A 99% VALUE

$$q = CE(1 - e^{-t/\tau})$$

$$\frac{q}{CE} = 0.99 = 1 - e^{-t/\tau}$$

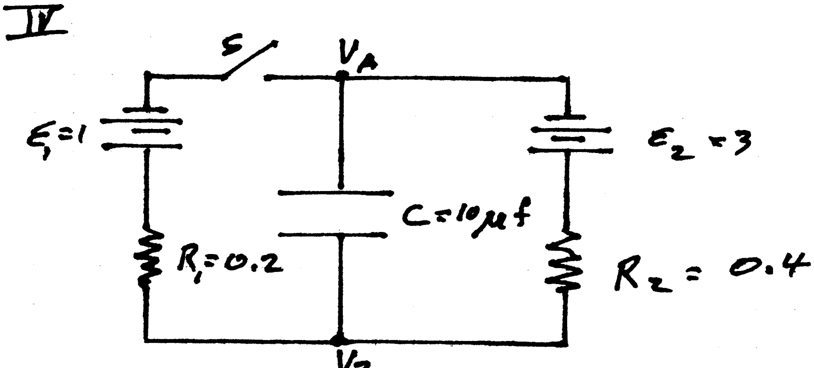
$$e^{-t/\tau} = 1 - .99 = 0.01$$

$$-\frac{t}{\tau} = \ln(.01) = -4.605$$

$$t = 4.605 \text{ (TIME CONSTANTS)}$$

EXAMPLE IV

$C = \frac{q}{V}$



INITIALLY S IS OPEN FOR A LONG TIME. IT IS THEN CLOSED FOR A LONG TIME. BY HOW MUCH DOES q ON THE CAPACITOR CHANGE? (WHEN S IS CLOSED)

$q_{initial} = CV = 10\times10^{-6}(3) = 3\times10^{-5}\,C$

AFTER A Long time THE CURRENT i in THE RIGHT HAND Loop is ZERO So $V_{Cap} = \mathcal{E}_{BATTERY} = 3$

NOW WHEN S is CLOSED FOR A LONG TIME

FOR THE OUTER Loop

$$+3 - 0.4i - 0.2i - 1 = 0$$

$$i = \frac{2}{0.6} = 3.33A$$

NOW WE FIND THE VOLTAGE ACROSS THE CAPACITOR

$$V_A + 3 - 0.4(3.33) = V_B$$

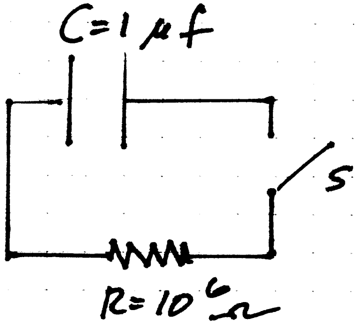
$$V_B - V_A = 3 - \underbrace{0.4(3.33)}_{1.33} = 1.67\text{ VOLTS}$$

q_{final} ACROSS THE CAPACITOR $= C(V_B - V_A) = 10\times10^{-6}(1.67) = 1.67\times10^{-5}$

$$q_f - q_i = 1.67\times10^{-5} - 3\times10^{-5} = -1.33\times10^{-5}\,C$$

THE CHARGE HAS DECREASED BY $1.33\times10^{-5}\,C$

EXAMPLE V



$U_{CAP} = \frac{1}{2}$ JOULE

a) find the initial charge on the capacitor

$$U_{CAP} = \frac{1}{2} = \frac{q^2}{2C} \qquad q_i = \sqrt{\tfrac{1}{2}(2)(1\times10^{-6})} = 1\times10^{-3}\,C$$

b) WHAT IS THE CURRENT THRU THE RESISTOR

$$i = -\frac{dq}{dt} = \frac{q_0}{\tau}\left(e^{-t/\tau}\right)$$

$$\tau = RC = 10^6(1\times10^{-6}) = 1$$

$$i = \frac{1\times10^{-3}}{1}e^{-0/\tau} = 1\times10^{-3}\,A$$

c) DETERMINE $V_C(t)$ & $V_R(t)$

$$V_C = \frac{q_0}{C}e^{-t/\tau} = \frac{1\times10^{-3}}{1\times10^{-6}}e^{-t/1.0} = 1\times10^{3}e^{-t}$$

$$V_R \; \frac{q_0 R}{\tau} = \frac{1\times10^{-3}(10^6)}{1}e^{-t/(1.0)} = 1\times10^{3}e^{-t}$$