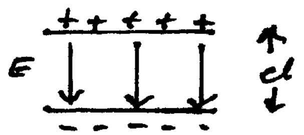
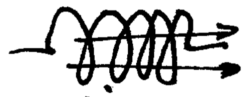
INDUCTANCE



$V = Ed$

$V = iR$

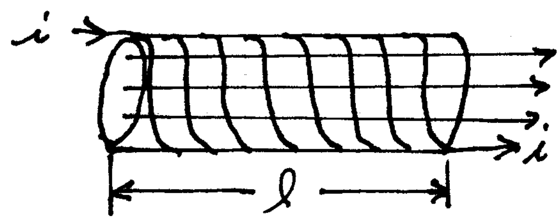
$\mathcal{E} = -\frac{\Delta\Phi}{\Delta t}$



$\frac{di}{dt}$

$$L = \frac{N\Phi}{i} \qquad [L] = \frac{[\Phi]}{[i]} = \frac{T\text{-}m^2}{Amp} = 1 \text{ HENRY}$$

SOLENOID



$$B = n i \mu_0$$

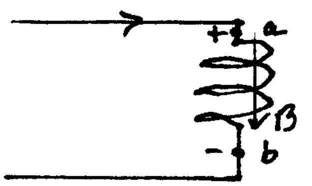
$$L = \frac{N\Phi}{i} = \frac{(nl)(BA)}{i} = \frac{nl(\overbrace{\mu_0 i n}^{B})A}{i} = \mu_0 n^2 l A$$

$$\frac{L}{l} = \mu_0 n^2 A \qquad \text{inductance / length } (H/m)$$

EXAMPLE I GIVEN A SOLENOID 20 CM LONG WITH A 5 CM RADIUS & 1000 TURNS FIND

$$L = \mu_0 n^2 l A = (1.26\times10^{-6})\underbrace{\left(\frac{1000}{0.2}\right)^2}_{25\times10^{6}}(0.2)\underbrace{(.05)^2\pi}_{0.00785} = 49 \text{ mH}$$

i increasing

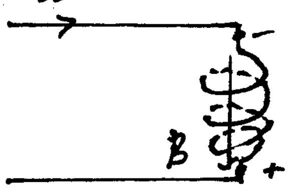


$V_a > V_b$

$\uparrow \mathcal{E}_L = -N\frac{\Delta\Phi}{\Delta t}$

VOLTAGE OPPOSES THE CURRENT INCREASE

i decreasing



$\downarrow \mathcal{E}_L = -N\frac{\Delta\Phi}{\Delta t}$

VOLTAGE OPPOSES THE CURRENT DECREASE

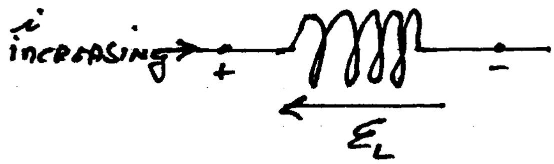
$$\mathcal{E}_L = -N\frac{d\phi}{dt} = -\frac{d}{dt}(N\phi) \qquad \text{BUT} \quad N\phi = Li$$

$$\mathcal{E}_L = -\frac{d}{dt}(Li) = -L\frac{di}{dt}$$

$$\boxed{\mathcal{E}_L = -L\frac{di}{dt}}$$

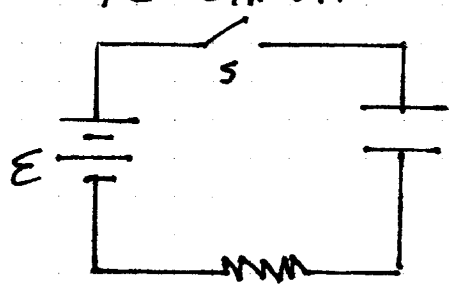
EXAMPLE II i GOES FROM $0 \to 1$ AMP IN 0.1SEC. FIND THE INDUCED VOLTAGE ACROSS A 50 mH INDUCTOR

$$\mathcal{E}_L = -L\frac{di}{dt} = -L\frac{\Delta i}{\Delta t} = -\underset{L}{(0.05)}\underset{\frac{\Delta i}{\Delta t}}{\left(\frac{1-0}{0.1}\right)} = -0.5 \text{ VOLTS}$$



RL CIRCUITS

Rc CIRCUIT



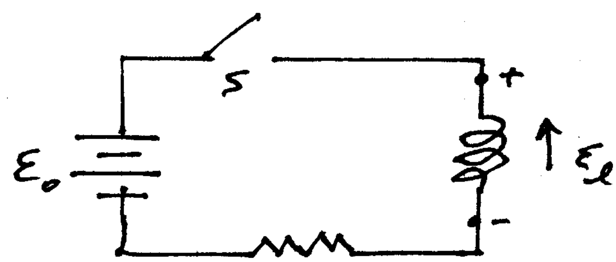
$$\mathcal{E}_0 - R\frac{dq}{dt} - \frac{q}{C} = 0$$

$$V_c = V_0(1 - e^{-t/\tau}) \qquad \tau = RC$$

$$q_c = q_0(1 - e^{-t/\tau})$$

$$i = \frac{\mathcal{E}}{R}e^{-t/\tau}$$

RL CIRCUIT



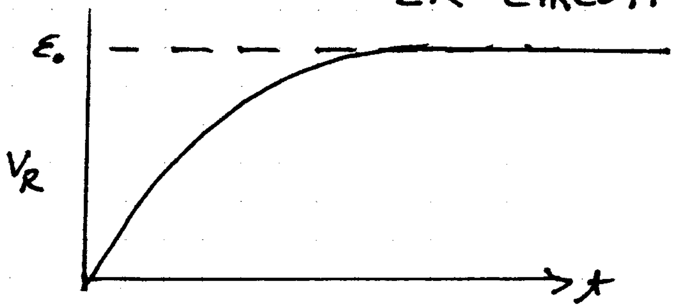
WHEN S IS CLOSED AT $t = 0$

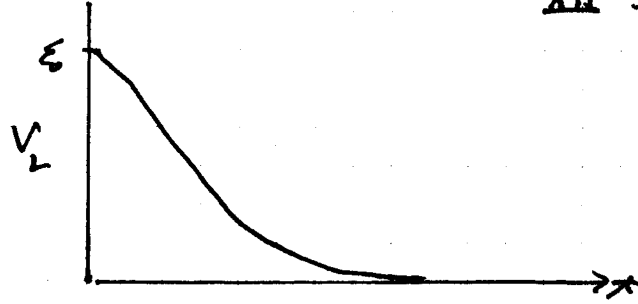
$$\mathcal{E}_0 - iR - L\frac{di}{dt} = 0$$

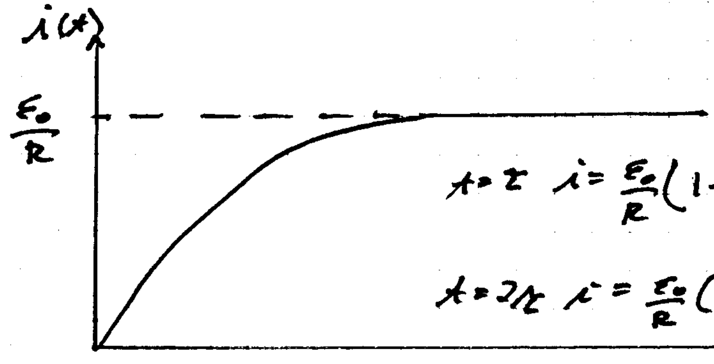
$$i = \frac{\mathcal{E}_0}{R}(1 - e^{-t/\tau}) \qquad \tau = \frac{L}{R}$$

$$V_L = \mathcal{E}_0 e^{-t/\tau}$$

LR CIRCUIT



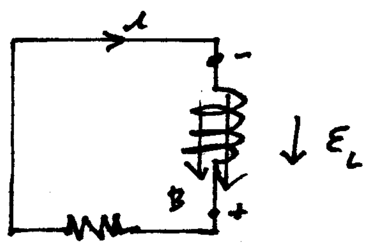




$t = \tau \quad i = \frac{\mathcal{E}_0}{R}\left(1 - \frac{1}{e}\right) = 63\% \ \frac{\mathcal{E}_0}{R}$

$t = 2\tau \quad i = \frac{\mathcal{E}_0}{R}\left(1 - \frac{1}{e^2}\right) = 86\% \ \mathcal{E}_0/R$

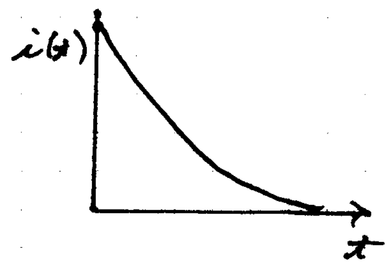
NEXT HAVING REACHED EQUILIB THE BATTERY IS NOW REMOVED FROM THE CIRCUIT



$$iR + L\frac{di}{dt} = 0$$

$$i = i_0 e^{-t/\tau}$$

$$i_0 = \frac{\mathcal{E}_0}{R}$$



$t = \tau \quad i = 37\% \ \mathcal{E}_0/R$

$t = 2\tau \quad i = 14\% \ \mathcal{E}_0/R$

ENERGY STORED IN AN INDUCTOR

FOR CAPACITORS $\quad U_C = \frac{q^2}{2C} = \frac{CV^2}{2}$ (JOULES)

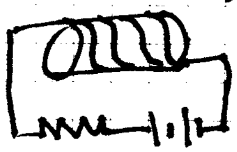
FOR INDUCTORS $\quad U_B = \frac{1}{2} L i^2$ (JOULES)

MAGNETIC ENERGY DENSITY

$$u_B = \frac{B^2}{2\mu_0} \qquad (\text{Joules}/m^3)$$

EXAMPE III

L = 6.3 μH



1200 Ω   14V

a) HOW MUCH TIME IS REQ'D FOR THE CURRENT THRU THE RESISTOR TO REACH 80% OF ITS FINAL VALUE

$$i = \frac{\mathcal{E}_0}{R}\left(1 - e^{-t/\tau_L}\right) \qquad \tau_L = \frac{L}{R} = \frac{6.3\times10^{-6}}{1.2\times10^{3}} = 5.25\times10^{-9}$$

$$\frac{\mathcal{E}_0}{R} = \frac{14}{1200} = 0.117 = i_i$$

$$\frac{i}{i_i} = .80 = 1 - e^{-t/\tau_L}$$

$$e^{-t/\tau} = 1 - 0.8 = 0.2$$

$$\ln\left(e^{-t/\tau_L}\right) = -\frac{t}{\tau_L} = \ln(0.2) = -1.609$$

$$t = -\tau_L(-1.609) = 8.4\times10^{-9}\ \text{SEC}$$

b) WHAT IS THE CURRENT THRU THE RESISTOR AT $t = \tau$

$$i = \frac{\mathcal{E}_0}{R}\left(1 - e^{-t/\tau_L}\right) = \frac{\mathcal{E}_0}{R}\left(1 - e^{-1}\right) = \frac{\mathcal{E}_0}{R}\underbrace{\left(1 - \frac{1}{2.718}\right)}$$

$$i = 0.632\left(\frac{\mathcal{E}_0}{R}\right)$$

$$= 0.632\left(\frac{14}{1200}\right) = 0.632\,(0.117) = 0.739\ \text{AMPS}$$