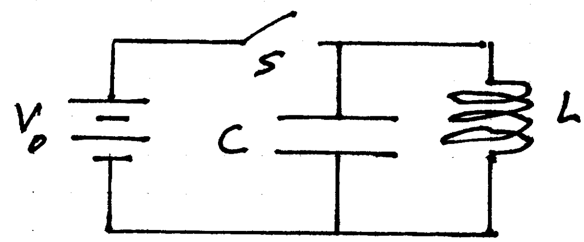
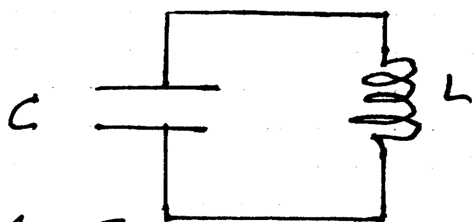
LC CIRCUIT



FIRST WE CHARGE THE CAPACITOR SO $q_0 = CV_0$ & THEN WE DISCONNECT THE BATTERY



$$V_C - \mathcal{E}_L = 0$$

$$\frac{q}{C} + L\frac{di}{dt} = 0 \qquad i = \frac{dq}{dt}$$

$$q = q_0 \cos\omega t$$

$$q_0 \cos\omega t - Lw^2 q_0 \cos\omega t = 0$$

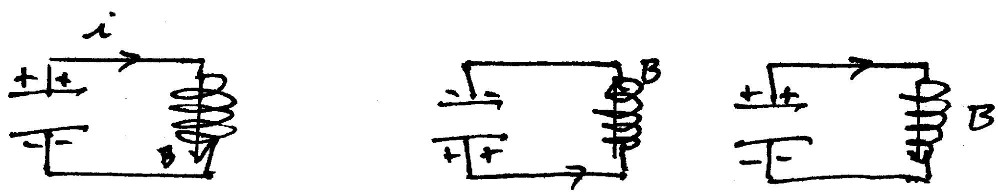
$$1 - LCw^2 = 0$$

$$\omega = \frac{1}{\sqrt{LC}} \qquad f = \frac{1}{2\pi\sqrt{LC}}$$

EXAMPLE I A $4\mu f$ CAPACITOR IS CHARGED TO 5V & THEN DISCHARGED INTO A 0.3H INDUCTOR

a) WHAT IS THE FREQUENCY OF OSCILLATION

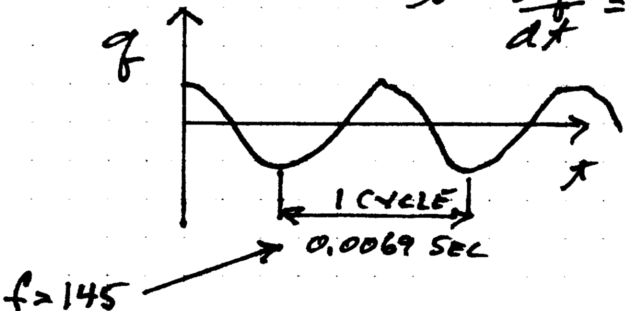
$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{6.28\sqrt{(.3)(4\times10^{-6})}} = 145\ \text{Hz}$$

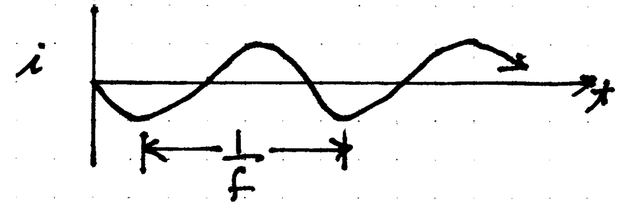


b) WHAT IS THE MAX CURRENT FLOWING IN THE CIRCU

$$q = q_0 \cos \omega t \qquad q_0 = CV_0 = 4\times10^{-6}(5) = 2\times10^{-5}$$

$$i = \frac{dq}{dt} = -q_0 \omega \sin \omega t$$





$$i_{MAX} = \underbrace{q_0}_{\text{AMPLITUDE}} \omega = q_0 2\pi f = 2\times10^{-5}(6.28)145 = 18.2 \text{ m}$$

c) AT WHAT TIME DOES THE MINIMUM CURRENT OCCUR

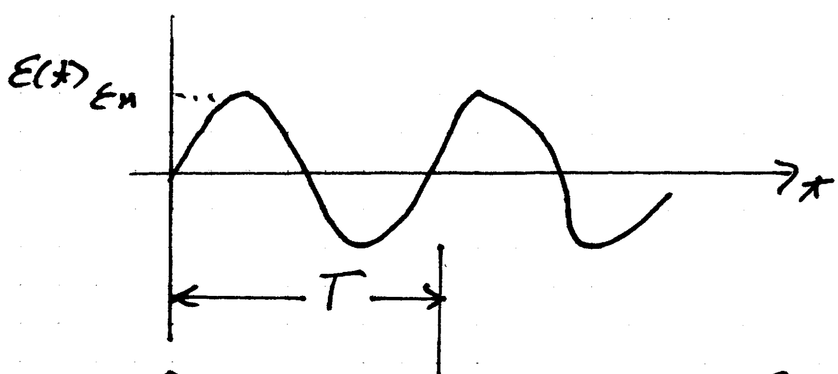
$$1 \text{ CYCLE} = \text{PERIOD } T = \frac{1}{f}$$

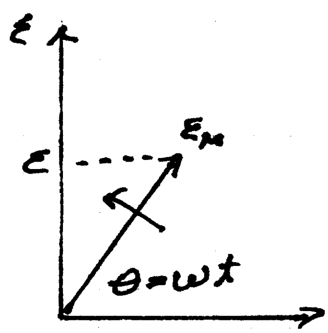
$$\text{TIME FOR } I_{MIN} = \frac{T}{4} = \frac{1}{4f} = \frac{1}{4(145)} = 1.72\times10^{-3} = 1.72 \text{ m SE}$$

AC CIRCUITS

APPLIED VOLTAGE VARIES WITH TIME

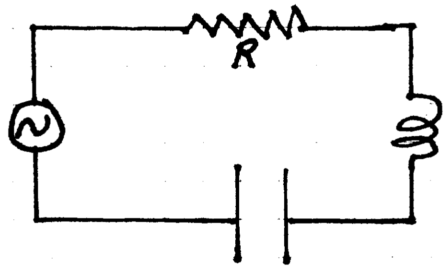
$$\mathcal{E}(t) = \mathcal{E}_{MAX} \sin \omega t$$





$\theta = \omega t = 2\pi$

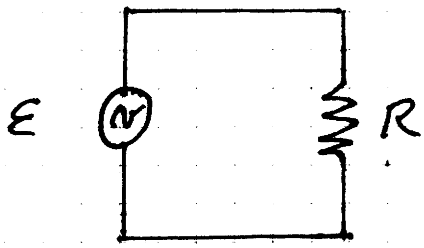
$$T = \frac{2\pi}{\omega} = \frac{1}{f}$$



GEN'L $\quad i(t) = I_{MAX} \sin(\omega t - \phi) \qquad \phi \neq \phi'$

$\qquad\qquad v(t) = V_{MAX} \sin(\omega t - \phi')$

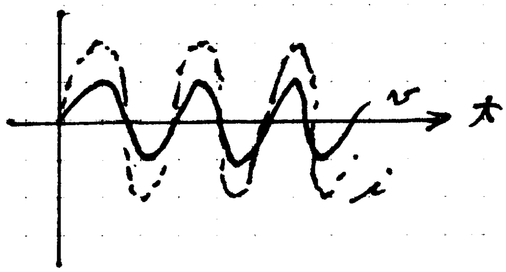
RESISTIVE LOAD



$V_R = IR$

$v_R = E_M \sin \omega t$

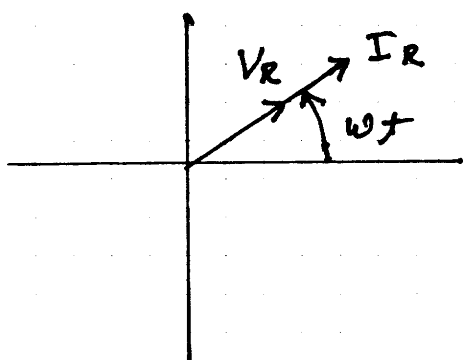
$i = \frac{V_R}{R} = \frac{E_M}{R} \sin \omega t$



R<1

v_R & i_R ARE in PHASE i.e. THE MAX + MIN VALUES OCCUR AT THE SAME TIME

PHASOR DIAGRAMS

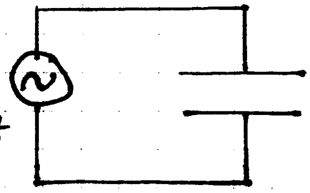


EXAMPLE II FOR $E_M = 10\,V$ ($f = 60$ HZ) WHICH IS APPLIED TO A $20\,\Omega$ RESISTIVE CIRCUIT find i AT 1/240 SEC

$i = I_{max} \sin \omega t = \frac{E_M}{R} \sin \omega t$

$\omega = 2\pi f = 6.28(60) = 377$ RAD/SEC

CAPACITIVE LOAD



$\mathcal{E} = \mathcal{E}_M \sin\omega t$

$$\mathcal{E} - v_c = 0$$

$$v_c = V_c \sin \omega t$$

$$q_c = C v_c = C V_c \sin \omega t$$

$$i_c = \frac{dq_c}{dt} = \omega C V_c \cos\omega t$$

BUT SINCE $\cos(\omega t) = \sin(\omega t + \pi/2)$

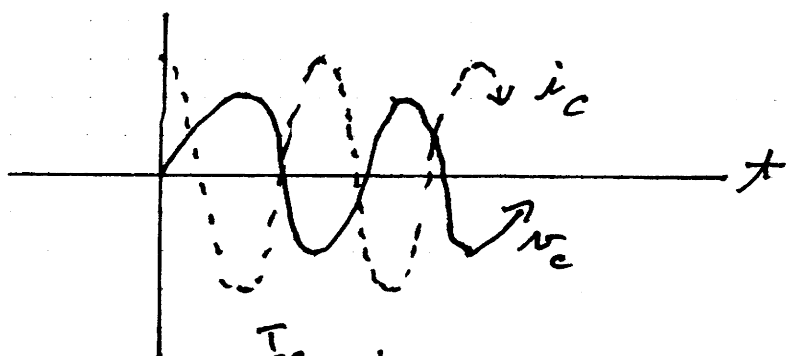
NOW DEFINE
CAPACITIVE REACTANCE $X_c = \frac{1}{\omega C}$ $[\Omega]$

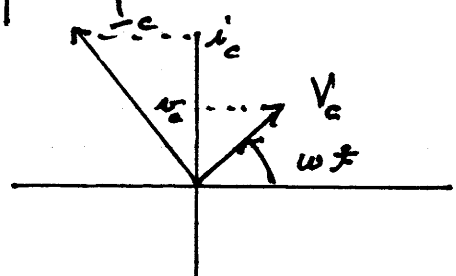
$$i_c = \frac{V_c}{X_c} \sin(\omega t + \pi/2) = I_c \sin(\omega t + \pi/2)$$

$$v_c = V_c \sin(\omega t)$$
$$i_c = I_c \sin(\omega t + \pi/2)$$

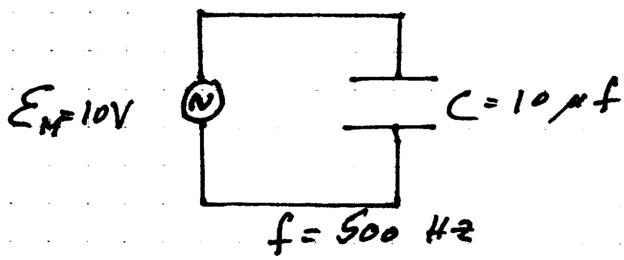
$\boxed{i_c \text{ LEADS } v_c}$

i.e. CURRENT REAC
A MAXIMUM VAL
BEFORE THE VOLT
(i.e. TIMEWISE)





EXAMPLE III



$\omega = 2\pi f$

a) FIND THE CAPACITIVE REACTANCE

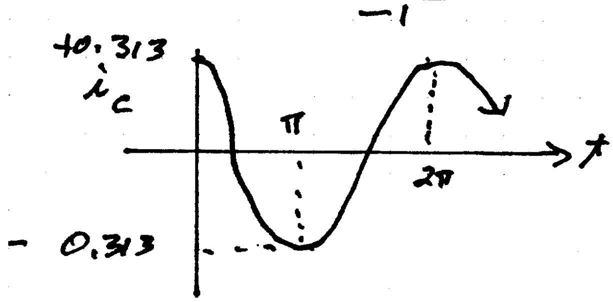
$$X_c = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{6.28(500)(10\times10^{-6})} = 3.2\times10^{-5+6} = 32\,\Omega$$

b) DETERMINE THE MAX CURRENT

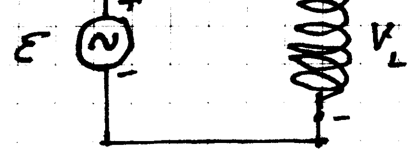
$$I_c = \frac{V_c}{X_c} = \frac{10}{32} = 0.313\,A$$

c) DETERMINE THE CURRENT AT $t = 1\,ms$

$$i_c = I_c \sin(\omega t + \pi/2)$$
$$= 0.313 \sin(2\pi f t + \pi/2)$$
$$= 0.313 \sin(\underbrace{2\pi(500)(1\times10^{-3})}_{(3.14\ \text{RADIANS} = 180^\circ} + \pi/2)$$
$$+ 90)$$
$$= 0.313\ \underbrace{\sin(270^\circ)}_{-1} = -0.313\ \text{Amps}$$



w/o THE PHASE ANGL



$$+\mathcal{E} - V_L = 0$$

$$v_L = V_L \sin \omega t$$

BUT $\quad v_L = -L\dfrac{di}{dt}$

$$\int di = -\int \frac{v_L}{L}\,dt = -\frac{1}{L}\int V_L \sin\omega t\, dt = -\frac{V_L}{L}\underbrace{\int \sin \omega t\, dt}_{\frac{1}{\omega}(-\cos\omega t)}$$

$$i_L = \frac{V_L}{L\omega}\underbrace{\left(-\cos\omega t\right)}_{= \sin(\omega t - \phi) = \sin(\omega t - \frac{\pi}{2})}$$

$$i_L = \frac{V_L}{L\omega}\sin(\omega t - \tfrac{\pi}{2})$$

ϕ IS THE ANGLE BETWEEN i & v

& $\phi = \frac{\pi}{2}$

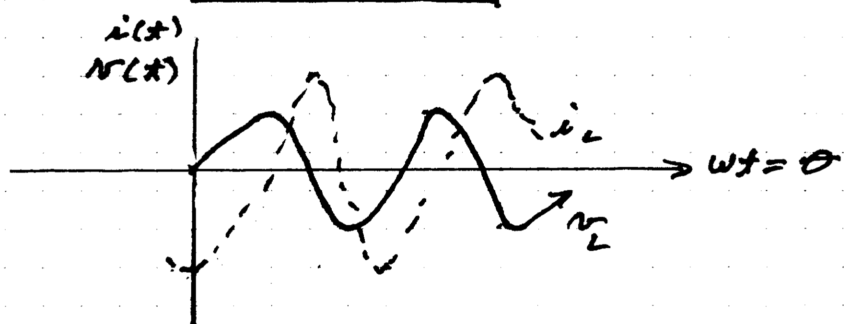
INDUCTIVE REACTANCE $X_L = \omega L$

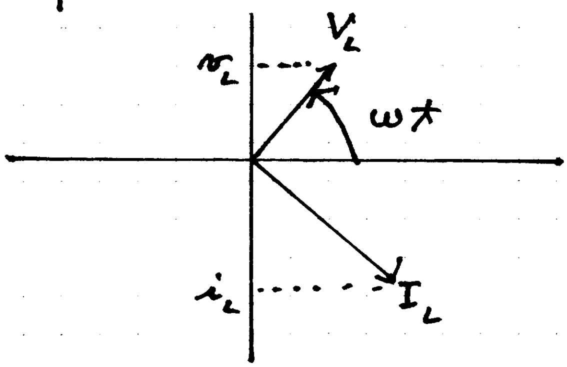
$$i_L = \frac{V_L}{X_L}\sin(\omega t - \tfrac{\pi}{2})$$

$$v_L = V_L \sin \omega t$$

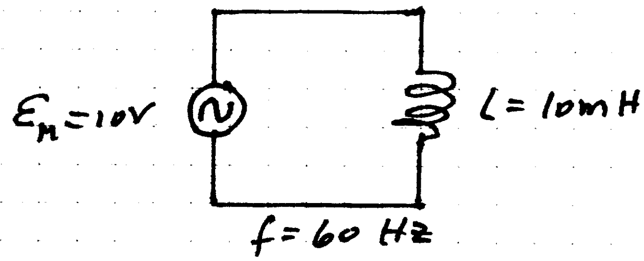
$$\boxed{i_L \text{ LAGS } V_L}$$

i.e. VOLTAGE REACHES MAXIMUM BEFORE THE CURRENT (i.e. TIMEWI





EXAMPLE IV



a) find $X_L = \omega L = 2\pi f L = 2\pi(60)(10\times10^{-3}) = 3.77\,\Omega$

b) CALCULATE I_L $\qquad I_L = \frac{\mathcal{E}_M}{X_L} = \frac{10}{3.77} = 2.65A$

c) FIND i FOR $t = 1/120$ SEC $\qquad \omega t = 2\pi f t = 6.28(60)(\frac{1}{120}) =$

$$i_L = \frac{V_L}{X_L}\sin(\omega t - \pi/2)$$

$$= \frac{10}{3.77}\sin\underbrace{(\pi - \pi/2)}_{\pi/2} = \frac{10}{3.77}\sin(\pi/2) = 2.65A$$

THE MAX VALUE