

PHYSICS I FORMULAS

Physics 106:

$360^\circ = 2\pi$ radians = 1 revolution. $s = r\theta$ $v_t = r\omega$ $a_t = r\alpha$ $a_c = a_r = v_t^2/r = \omega^2 r$ $a_{tot}^2 = a_r^2 + a_t^2$
 $\omega = \omega_0 + \alpha t$ $\theta_f - \theta_o = \omega_o t + \frac{1}{2}\alpha t^2$ $\omega_f^2 - \omega_o^2 = 2\alpha(\theta_f - \theta_o)$ $\theta - \theta_o = \frac{1}{2}(\omega + \omega_o)t$ $K_{rot} = \frac{1}{2}I\omega^2$ $I = \sum m_i r_i^2$
 $I_{point} = mr^2$ $I_{hoop} = MR^2$ $I_{disk} = \frac{1}{2}MR^2$ $I_{sphere} = \frac{2}{5}MR^2$ $I_{shell} = \frac{2}{3}MR^2$ $I_{rod (center)} = \frac{1}{12}ML^2$ $I_{rod (end)} = \frac{1}{3}ML^2$

$\tau = \text{force} \times \text{moment arm} = F \times r \sin(\phi)$ $\tau_{net} = \Sigma \tau = I \alpha$ $F_{net} = \Sigma F = m a$ $t = r \times F$ $I_p = I_{cm} + Mh^2$

$W_{tot} = \Delta K = K_f - K_i$ $W = \tau_{net} \Delta \theta$ $K = K_{rot} + K_{cm}$ $E_{mech} = K + U$ $P_{average} = \Delta W / \Delta t$

$P_{instantaneous} = \tau \omega$ (for τ constant) $\Delta E_{mech} = 0$ (isolated system) $v_{com} = \omega r$ (rolling, no slipping)

$L = r \times p$ $p = mv$ $L = S I_i$ $\tau_{net} = dL/dt$ $L = I\omega$ $I_{point mass} = m \times r^2 \sin^2(\phi)$

For isolated systems: $\tau_{net} = 0$ L is constant $\Delta L = 0$ $L_o = S I_o \omega_o = L_f = S I_f \omega_f$

Equilibrium: $\Sigma \text{ forces} = 0$ and $\Sigma \text{ torques} = 0$, If net force on a system is zero, then the net torque is the same for any chosen rotation axis. COG definition: point about which torques due to gravity alone add to zero.

$F = G \frac{m_1 \cdot m_2}{R^2}$; $G = 6.67 \times 10^{-11} [N \cdot m^2/kg^2]$; $F_{net} = m \frac{v^2}{R}$; $a_g = G \frac{m}{R^2}$; $E_{mech} = K + U_g$ $K = \frac{1}{2}mv^2$;

$U_g = -G \frac{m_1 \cdot m_2}{R}$; $v_{escape} = \sqrt{\frac{2GM}{R}}$; $T^2 = \frac{4\pi^2}{GM} R^3$ ($T^2/R^3 = \text{Const}$ for all satellites of a given planet.

Angular momentum and mechanical energy are conserved for masses moving under gravitational forces.

$E_{mech} < 0 \rightarrow$ Bound, elliptical orbit; $E_{mech} > 0 \rightarrow$ Free particle, hyperbolic orbit; $E_{mech} = 0 \rightarrow$ Escape

threshold. For circular orbits: $F_{centri} = mv^2/r = F_{grav} = GmM/r^2$, $v_{orb} = \sqrt{GM/r}$, $E_{orb} = \frac{1}{2}U_{orb} = -\frac{1}{2}K_{orb}$

Earth: $M_E = 5.98 \times 10^{24}$ kg, $R_E = 6.37 \times 10^6$ m, orbital radius about Sun = 1.5×10^{11} m.

Mars: $M_m = 6.4 \times 10^{23}$ kg, $R_m = 3.395 \times 10^6$ m

Moon: $M_{moon} = 7.36 \times 10^{22}$ kg, $R_{moon} = 1.74 \times 10^6$ m, orbital radius about earth = 3.82×10^8 m

Oscillators in SHM: $\omega = \text{angular frequency [rad/s]} = 2\pi f = 2\pi/T$. Period $T = 2\pi/\omega$

$x(t) = x_m \cos(\omega t + \phi)$ $v(t) = v_m \sin(\omega t + \phi)$ with $v_m = -\omega x_m$ $a(t) = a_m \cos(\omega t + \phi)$ with $a_m = -\omega^2 x_m$

Oscillator equation: $a(t) = d^2x(t)/dt^2 = -\omega^2 x(t)$

Energy: $E_{osc} = \frac{1}{2}mv(t)^2 + \frac{1}{2}kx(t)^2$ if no damping, then $dE_{osc}/dt = 0$ and E_{osc} is constant

Spring osc: $F = -kx$ $\omega = \sqrt{k/m}$ Torsion pendulum.: $\tau = -\kappa\theta$ $\omega = \sqrt{\kappa/I}$

Pendulums: Simple $\omega = \sqrt{g/L}$ Physical $\omega = \sqrt{mgh/I}$, $h = \text{dist. to COM from pivot}$, $I = \text{rot. inertia}$

Physics 105:

$W = mg$ $g = 9.8 \text{ m/s}^2$ $1 \text{ m} = 100 \text{ cm} = 1000 \text{ mm}$, $1 \text{ kg} = 1000 \text{ g}$

$v = v_o + at$ $x - x_o = v_o t + \frac{1}{2}at^2$ $v^2 - v_o^2 = 2a(x - x_o)$ $x - x_o = \frac{1}{2}(v + v_o)t$

$F_{net} = ma$ $\Sigma F = ma = dP/dt$ $F_{st,max} = \mu_s N$ $F_k = \mu_k N$ incline: $W_{mgx} = mg \sin[\theta]$ $W_{mgy} = mg \cos[\theta]$

$F_r = ma_r = mv^2/r$ $a_r = v^2/r$ $f = 1/T$ $T = (2\pi r/v)$ Impulse: $F_{avr} \Delta t = mv_f - mv_i$

Momentum is conserved if net Impulse = 0. Then $(\Sigma mv)_{initial} = (\Sigma mv)_{final}$

Work: $W = F \cdot d \cos(\theta)$, $W_{grav} = -mg(y - y_o)$, $W_{spring} = -\frac{1}{2}k(x^2 - x_o^2)$, $W_{frict} = -F_k d$, $W_{tot} = K_f - K_i$

$U_g = mg(y - y_o)$, spring: $F = -kx$, $U_s = \frac{1}{2}kx^2$, $KE = \frac{1}{2}mv^2$

$W_{nc} = K_f - K_i + U_{gf} - U_{gi} + U_{sf} - U_{si}$ or $K_i + U_{gi} + U_{si} + W_{nc} = K_f + U_{gf} + U_{sf}$

Mass center: $X_{com} = S(x_i m_i) / \Sigma m_i$, similarly for Y_{com} , Z_{com} : $(Y_{com} = S(y_i m_i) / \Sigma m_i)$ and $Z_{com} = S(z_i m_i) / \Sigma m_i$

Vectors:

Components: $a_x = a \cos(\theta)$ $a_y = a \sin(\theta)$ $a = a_x i + a_y j$ $|a| = \sqrt{a_x^2 + a_y^2}$ $\theta = \tan^{-1}(a_y/a_x)$

Addition: $a + b = c$ implies $c_x = a_x + b_x$ $c_y = a_y + b_y$

Dot product: $a \cdot b = ab \cos(\phi) = a_x b_x + a_y b_y + a_z b_z$ unit vectors: $i \cdot i = j \cdot j = k \cdot k = 1$; $i \cdot j = i \cdot k = j \cdot k = 0$

Cross product: $|a \times b| = ab \sin(\phi)$; $c = |a \times b| = (a_x b_z - a_z b_x) i + (a_z b_x - a_x b_z) j + (a_x b_y - a_y b_x) k$

$a \times b = -b \times a$, $a \times a = 0$ always; $c = a \times b$ is perpendicular to a - b plane; if $a \parallel b$ then $|a \times b| = 0$

$i \times i = j \times j = k \times k = 0$, $i \times j = k$ $j \times k = i$ $k \times i = j$

Physics 106 Final Exam Sample 05 – Physics 106 (answers on last page)

Signature _____

Name (Print): _____

4 Digit ID: _____ Section: _____

A

Instructions:

- There are 30 multiple choice questions on the test. There is no penalty for guessing, so you should attempt to answer all of them. You will need to do calculations on the question papers for most of the questions and may need to use the backs of the pages for extra room. If you need more scrap paper, ask the proctor.
- Your final answers should be put on the Scantron sheet using #2 pencil.
- Be sure your name and section number are on both the Scantron form and the exam booklet.
- Be sure you are in the right room for your section.
- You may bring and use your own 8.5 x 11 formula sheet (both sides). A default formula sheet is also provided (see the final page of this booklet). Make sure to bring your own calculator; sharing of calculators is not permitted.
- As you know, NJIT has a zero tolerance policy for ethics code violations during and also after an exam. Students are not to communicate with each other once the test has started. All cell phones, pagers, or similar electronic devices should be turned off. If you have questions or need something call your proctor or instructor.

1. (FE) The dot product of two vectors $\mathbf{A} = 3\mathbf{i} - 2\mathbf{j}$ and $\mathbf{B} = 5\mathbf{i} - 6\mathbf{j}$ is
 A) 3 B) 27 C) $-3\mathbf{i} - 9\mathbf{j}$ D) $7\mathbf{i} + 3\mathbf{j}$ E) $2\mathbf{i} - 4\mathbf{j}$

$$\mathbf{A} \cdot \mathbf{B} = 3 \cdot 5 + 2 \cdot 6 = 15 + 12 = 27$$

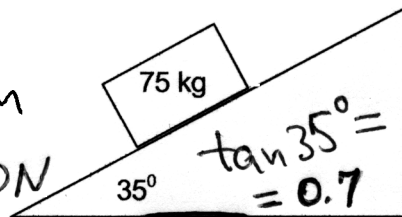
2. (FE) The block in the sketch is initially stationary until it is released. Then it may or may not slide down the ramp. Which answer is closest to the **actual** friction force (either static or kinetic) between the 75 kg block and the ramp? The coefficient of static friction is 0.20 and the coefficient of kinetic (sliding) friction is 0.15.

- A) 180 N.
 B) 90 N.
 C) 80 N.
 D) 120 N.
 E) 60 N

$$f = N \cdot \mu = mg \cdot \cos \theta \cdot \mu$$

$$f_s = 75 \cdot 9.8 \cdot 0.2 \cdot \cos 35^\circ [N] = 120 N$$

$$f_k = 75 \cdot 9.8 \cdot 0.15 \cdot \cos 35^\circ [N] = 90 N$$



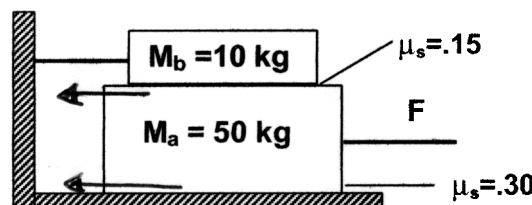
3. (FE) The horizontal force F is gradually increased until the 50-kg block begins moving to the right. The 10-kg block cannot move because of the cord attaching it to the wall at left. For what force F does the lower block start to move?

- A) 14.7 N.
 B) 176.4 N.
 C) 132.3 N.
 D) 147 N.
 E) 191 N.

$$F = f_{s1} + f_{s2}$$

$$f_{s1} = M_b \cdot g \cdot \mu_{s1}$$

$$f_{s2} = (M_a + M_b) \cdot g \cdot \mu_{s2}$$



$$F = g \cdot [M_b \cdot \mu_{s1} + (M_b + M_a) \mu_{s2}] =$$

4. (FE) An archer's bow shoots arrows at a speed of 49 m/s. If he shoots on level ground and fires an arrow at 45 degrees above the horizontal (for maximum range), how far will the arrow travel horizontally before it hits the ground. Neglect air resistance.

- A) 104.5 m
 B) 245 m
 C) 347 m
 D) 490 m
 E) 980 m

$$R_{max} = \frac{2V_0^2}{g} \sin \theta \cdot \cos \theta$$

$$R_{max} = \frac{2 \cdot 49^2}{9.8} \cdot \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} = 245 m$$

5. A 70-kg skier starts from rest and slides down a 12-m long hill making an angle of 20° with the horizontal. The kinetic friction coefficient between the skis and the hill is 0.2. What is his speed at the bottom of the hill?

(A) 6.0 m/s
(B) 7.0 m/s
(C) 8.5 m/s
(D) 9.7 m/s
(E) 10.0 m/s

$$\Delta U = mgh = mg \cdot L \cdot \sin 20^\circ$$

$$W_{k.f.} = \underbrace{mg \cos 20^\circ}_N \cdot \mu_k \cdot L$$

$$\frac{1}{2} m v^2 = mgL \sin 20^\circ - mg \cos 20^\circ \cdot \mu_k \cdot L ; v = \sqrt{2 \cdot g \cdot L (\sin 20^\circ - \mu_k \cos 20^\circ)}$$

6. (FE) The spring constant k of the spring shown in the figure is 5.0 newtons per millimeter (1000 millimeters = 1 meter). When it is uncompressed, the left end of the spring is opposite zero on the length scale. The work done on the spring in compressing it from $x = 30$ millimeters to $x = 60$ millimeters is:

A) 2.25 J
B) 22.5 J
C) 6.75 J
D) 9.0 J
E) 0.9 J

$$k = 5000 \text{ N/m}$$

$$W = \frac{1}{2} k (x_f^2 - x_i^2) = \frac{1}{2} \cdot 5000 \cdot (0.06^2 - 0.03^2) = \frac{1}{2} \cdot 5000 \cdot 0.009 = 6.75 \text{ J}$$

7. A 2.0 kg ball starts from rest at point A and rolls down a frictionless track with a circular loop-the-loop on it. The loop diameter $d = 0.6$ m as shown in figure. Point A is 1.0 m above the bottom of the track. The speed of the ball at point B is closest to:

A) 2.8 m/s
B) 3.7 m/s
C) 4.4 m/s
D) 9.8 m/s
E) 7.8 m/s

$$mgy = mgd + \frac{1}{2} m v^2$$

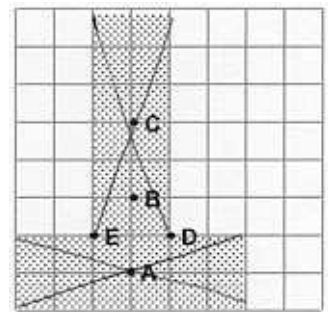
$$v^2 = 2g(y - d)$$

$$v = \sqrt{2g(y - d)} = \sqrt{2 \cdot 9.8 \frac{\text{m}}{\text{s}^2} (1\text{m} - 0.6\text{m})} = 2.8 \text{ m/s}$$

8. (FE) A T-shaped piece, represented by the shaded area on the figure, is cut from a metal plate of uniform thickness. The point that corresponds to the center of mass of the T-shaped piece is

A) A
B) B
C) C
D) D
E) E

com is between A and C



9. (FE) An 15 gram bullet with a horizontal speed of 900 meters per second collides with and becomes embedded in an 200 gram block of wood that is initially at rest on a horizontal frictionless surface. The speed of the block with the bullet in it after the collision is about:

A) 900 m/s
B) 34 m/s
C) 680 m/s
D) 63 m/s
E) 68 m/s

$$P_i = P_f ; P_i = m \cdot v_i$$

$$P_f = (m + M) \cdot v_f$$

$$v_f = \frac{m}{m + M} \cdot v_i = \frac{15}{215} \cdot 900 \text{ m/s} = 63 \text{ m/s}$$

10. A soccer player is about to give a penalty kick to a 0.5 kg ball which is at rest at the penalty mark. He needs to give the ball an initial velocity of at least 24 m/s to have a chance of scoring a goal. He can kick with an average force of 600 N. Using the impulse-momentum theorem, find how long the player's foot must be in contact with the ball during the kick.

- A) 0.2 s
- B) 0.04 s
- C) 0.02 s
- D) 0.01 s
- E) 0.1 s

$$\Delta p = mv_i - \emptyset = mv_i$$

$$\Delta p = F \cdot \Delta t; \quad \Delta t = \frac{mv_i}{F} = \frac{0.5 \text{ kg} \cdot 24 \text{ m/s}}{600 \text{ N}} = 0.02 \text{ s}$$

11. A disk starts from rest with angular speed $\omega_i = 0$ and rotates with constant angular acceleration α . If it takes 100 rev to reach angular speed ω , then how many (total) revolutions are required to reach angular speed 6ω ?

- A) 600 rev
- B) 750 rev
- C) 1200 rev
- D) 1800 rev
- E) 3600 rev

$$2\alpha \cdot \theta = \omega_f^2 - \omega_i^2; \quad \omega_i = \emptyset$$

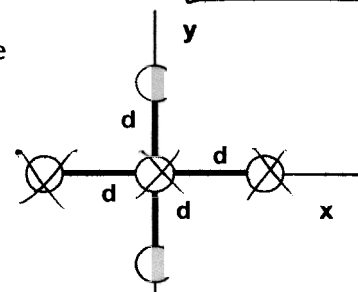
$$2\alpha \cdot \theta = \omega_f^2$$

$$\left. \begin{aligned} 2\alpha \cdot 100 \text{ rev} &= \omega^2 \\ 2\alpha \cdot X \text{ rev} &= (6\omega)^2 \end{aligned} \right\} X = 100 \cdot 36 = 3,600 \text{ rev}$$

12. Five equal 3.0-kg point masses are arranged in the x-y plane as shown. They are connected by massless rods so that they form a rigid body. The distance d is 0.5 m. The rotational inertia in $\text{kg} \cdot \text{m}^2$ of the array about the **x axis** is:

- A) 0.5
- B) 1.0
- C) 1.5
- D) 2.0
- E) 3.0

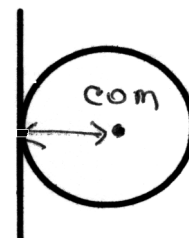
$$I = 2 \cdot md^2 = 2 \cdot 3 \text{ kg} \cdot (0.5 \text{ m})^2 = 1.5 \text{ kg} \cdot \text{m}^2$$



13. A 5 kg sphere is glued to a massless stick that is tangent to it and then spun about the axis formed by the stick. What is the sphere's rotational inertia I about this axis, if its radius is $1/2$ m? The rotational inertia of a sphere about its **mass center** $I_{cm} = \frac{2}{5} mR^2$.

- A) $\text{kg} \cdot \text{m}^2$
- B) $0.5 \text{ kg} \cdot \text{m}^2$
- C) $1.5 \text{ kg} \cdot \text{m}^2$
- D) $1.75 \text{ kg} \cdot \text{m}^2$
- E) $1.25 \text{ kg} \cdot \text{m}^2$

$$I = \frac{2}{5} mR^2 + mR^2 = \frac{7}{5} mR^2 = 1.4 \cdot 5 \cdot 0.5^2 = 1.75 \text{ kg} \cdot \text{m}^2$$



14. A Ferris wheel with rotational inertia $5.0 \times 10^5 \text{ kg} \cdot \text{m}^2$ has to accelerate from rest to an angular velocity of 0.5 rad/s in 20 sec. The minimum torque that its motor must provide to cause this acceleration is

- A) 7500 Nm
- B) 10,000 Nm
- C) 12,500 Nm
- D) 15,000 Nm
- E) 5000 Nm

$$I \cdot \alpha = \tau; \quad \alpha = \omega_f / \Delta t$$

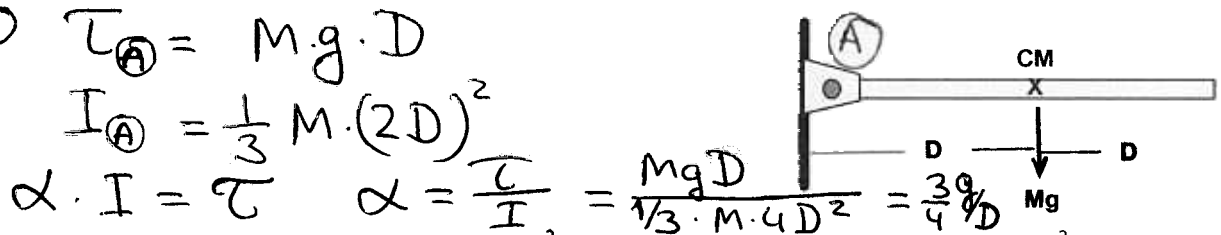
$$\tau = I \cdot \omega_f / \Delta t = \frac{5 \cdot 10^5 \text{ kg} \cdot \text{m}^2 \cdot 0.5 \text{ rad/s}}{20 \text{ s}} = 12,500 \text{ N} \cdot \text{m}$$

15. To increase the rotational inertia of a solid wheel about its axis without changing its radius or mass:

- A) drill holes near the rim and put the material near the axis
- B) drill holes near the axis and put the material near the rim
- C) drill holes at points on a circle near the rim and put the material at points between the holes
- D) drill holes at points on a circle near the axis and put the material at points between the holes
- E) do none of the above: the rotational inertia cannot be changed without changing the mass

16. (FE) A long, uniform, thin rod of mass M and length $2D$ is pivoted at one end. The free end is held so that the rod is horizontal and then the rod is released. If g is the acceleration of gravity, find the magnitude of the angular acceleration the instant the rod is released. You may need to know that $I_{cm} = M(2D)^2/12$.

- A) $(3/4)g/D$
 B) g/D
 C) $gD/3$
 D) $(3/2)g/D$
 E) gD^2



$$\tau_A = M \cdot g \cdot D$$

$$I_A = \frac{1}{3} M \cdot (2D)^2$$

$$\alpha \cdot I = \tau \quad \alpha = \frac{\tau}{I} = \frac{MgD}{\frac{1}{3} M \cdot 4D^2} = \frac{3g}{4D}$$

17. A bicycle wheel whose rotational inertia is $I = 0.5 \text{ kg} \cdot \text{m}^2$ is given a constant acceleration of 0.5 rad/s^2 by the net torque acting on it. If the wheel starts from rest, the work done on it during the first 8.0 s is closest to:

- A) 1 J
 B) 4 J
 C) 16 J
 D) 64 J
 E) 256 J

$$\tau = I \cdot \alpha \quad \Delta\theta = \alpha \cdot \frac{\Delta t^2}{2}$$

$$W = \tau \cdot \Delta\theta = I \cdot \alpha \cdot \alpha \cdot \frac{\Delta t^2}{2} = I \frac{\alpha^2 \Delta t^2}{2} = 0.5 \text{ kg} \cdot \text{m}^2 \cdot (0.5 \text{ rad/s}^2)^2 \cdot (8 \text{ s})^2 / 2 = 4 \text{ J}$$

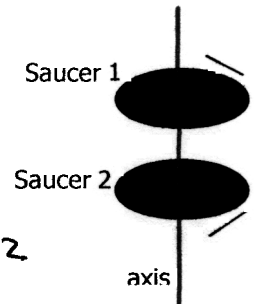
18. At a certain time, a 2.0 kg object is located at $\mathbf{r} = 3.0\mathbf{i} - 4.0\mathbf{k}$ meters and its velocity is $\mathbf{v} = 60\mathbf{i} - 80\mathbf{k} \text{ m/s}$. What is the angular momentum vector \mathbf{L} for the object about the origin?

- A) $960\mathbf{j} \text{ N} \cdot \text{m}$
 B) $480\mathbf{j} \text{ N} \cdot \text{m}$
 C) $0 \text{ N} \cdot \text{m}$
 D) $-480\mathbf{j} \text{ N} \cdot \text{m}$
 E) $-960\mathbf{j} \text{ N} \cdot \text{m}$

$$\mathbf{L} = m \cdot [\mathbf{r} \times \mathbf{v}] = 2 \text{ kg} \cdot [(3\mathbf{i} - 4\mathbf{k}) \times (60\mathbf{i} - 80\mathbf{k})] = 2 \text{ kg} \cdot \frac{\text{m}^2}{\text{s}} \cdot \mathbf{j} (3 \cdot 80 - 4 \cdot 60) = \mathbf{0} \quad \mathbf{r} \parallel \mathbf{v}$$

19. A pair of identical flying saucers need to hook together along their common rotation axis in order to transfer crew members to the mother ship, as shown in the sketch. Saucer one is rotating freely counterclockwise at 30 revolutions per minute (rpm). The second identical disc is initially rotating freely at 15 revolutions per minute in the opposite, clockwise direction. The two are suddenly coupled together along the common rotation axis. The final rotation rate in revolutions per minute is:

- A) 15 rpm counterclockwise
 B) 15 rpm clockwise
 C) 7.5 rpm clockwise
 D) 7.5 rpm counterclockwise
 E) 0 rpm



$$\omega_1 = +30 \text{ rpm} \quad I_1 = I_2$$

$$\omega_2 = -15 \text{ rpm}$$

$$L_i = L_f \quad L_i = I_1 \omega_1 + I_2 \omega_2 \quad L_f = (I_1 + I_2) \omega_f$$

$$\omega_f = (\omega_1 + \omega_2) / 2 = 7.5 \text{ rpm}$$

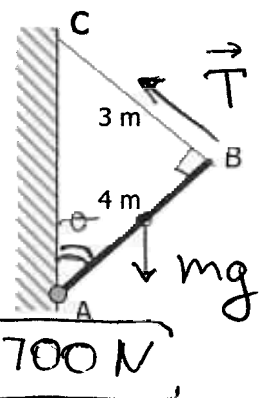
20. A 4.0 m long, uniform beam is pinned at one end and supported at the other end by a 3.0 m long cable attached to a vertical wall as shown. Angle ABC is a right angle. The mass of the beam is 5000 kg . The tension in the cable is closest to:

- A) 1725 N
 B) 1470 N
 C) 2450 N
 D) 2940 N
 E) 3920 N

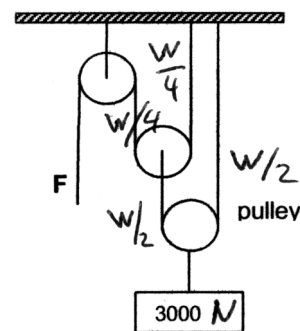
$$\sum \tau_A = 0$$

$$T \cdot L_{AB} - mg \cdot \frac{L_{AB}}{2} \cdot \sin\theta = 0$$

$$= \frac{mg}{2} \cdot \sin\theta = \frac{mg}{2} \cdot \frac{3}{5} = 14,700 \text{ N}$$



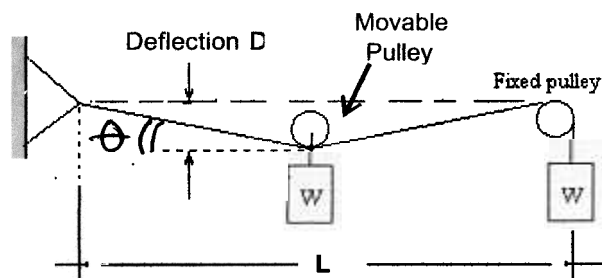
21. (FE) For the system of pulleys shown in the sketch, what is the smallest force F that can hold the 3000 N. load in place or lift it very slowly at constant speed. All three of the pulleys are massless and frictionless.



- A) 750 N.
B) 1500 N.
C) 250 N.
D) 500 N.
E) 375 N.

$$F = W/4 = \frac{3000 \text{ N}}{4} = 750 \text{ N}$$

22. (FE) Assume that the system shown in the figure is in equilibrium and that the pulleys have negligible friction. The two blocks have equal weights. Draw the free body diagram for the movable pulley and find the distance D that measures the deflection below the horizontal. D is most nearly:



- A) L
B) $0.289 L$
C) $0.500 L$
D) $0.707 L$
E) $0.866 L$

$$2D/L = \tan \theta$$

$$D = \frac{L}{2} \cdot \tan \theta$$

$$T = W \text{ (for the right weight)}; 2T \sin \theta = W; \sin \theta = \frac{1}{2}$$

$$D = L \cdot \tan 30^\circ/2 = 0.289 L$$

23. What is the approximate orbital speed for the Earth's Moon, assuming it is in a circular orbit whose radius is about

$$3.82 \times 10^5 \text{ km?}$$

- A) 8900 m/s
B) 3100 m/s
C) 6300 m/s
D) 1000 m/s
E) 2230 m/s

$$V = \sqrt{G \cdot \frac{M_{\text{Earth}}}{R}} = \sqrt{6.67 \cdot 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2} \cdot \frac{6 \cdot 10^{24} \text{ kg}}{3.82 \cdot 10^8 \text{ m}}} = 1022 \text{ m/s}$$

The escape velocity at the surface of the earth is approximately 11.2 km/s. What is the escape velocity on the surface of a very dense planet whose radius is one half that of earth and whose mass is 100 times that of earth?

- A) 160 km/s
B) 112 km/s
C) 2240 km/s
D) 320 km/s
E) 224 km/s

$$V_{\text{esc}} = \sqrt{2gR} = \sqrt{2GM/R}$$

$$V_{\text{esc}} = \sqrt{2G \frac{100 \cdot M}{0.5R}} = \sqrt{2G \frac{M}{R}} \cdot \sqrt{200} = 11.2 \cdot \sqrt{200} \frac{\text{km}}{\text{s}} = 158 \text{ km/s}$$

A "year" on the planet Mercury is 88 days long - about 0.241 earth years. What is the approximate ratio of the radius of Mercury's orbit around the Sun to that of the Earth's orbit?

- A) 0.491
B) 0.387
C) 0.622
D) 0.118
E) 2.584

$$\frac{T^2}{R^3} = \text{Const}$$

$$\frac{T_M^2}{R_M^3} = \frac{T_E^2}{R_E^3} \Rightarrow \frac{R_M}{R_E} = \sqrt[3]{\frac{T_M^2}{T_E^2}} = \left(\frac{T_M}{T_E}\right)^{2/3} = 0.387$$

26. In simple harmonic motion, the maximum restoring force and the maximum acceleration are directly proportional to the:

- A) amplitude X_m B) frequency f C) velocity $v(t)$ D) displacement $x(t)$ E) mass

27. A simple pendulum consists of a 3-kg mass (bob) attached to a 2.0-m string. The pendulum executes simple harmonic motion (at an unknown altitude) according to $\theta(t) = (\pi/6)\cos(4\pi \cdot t)$, where θ is the displacement in radians and t is in seconds. How much time does it take for the pendulum bob to move from one point of zero angular velocity to the **next** such point?

- A) 0.10 s
- B) 0.25 s
- C) 0.30 s
- D) 0.45 s
- E) 0.50 s

$$\theta(t) = \theta_m \cdot \cos(\omega \cdot t + \phi)$$

$$\theta_m = \pi/6 \quad \omega = 4\pi \text{ s}^{-1}$$

$$t = \frac{T}{2}; \quad T = 2\pi/\omega \Rightarrow t = \frac{\pi}{\omega} = \frac{\pi}{4\pi} = \boxed{0.25 \text{ s}}$$

28. A 0.25-kg block is attached to a spring whose spring constant is 5000 N/m. The block is pulled out so that the spring is stretched by 6 cm and then released so that the block oscillates. The maximum speed of the block is:

- A) 2.7 m/s
- B) 4.3 m/s
- C) 8.5 m/s
- D) 17.0 m/s
- E) 26.7 m/s

$$v_{\max} = x_{\max} \cdot \sqrt{\frac{k}{m}} = 0.06 \text{ m} \cdot \sqrt{\frac{5000 \text{ N/m}}{0.25 \text{ kg}}}$$

$$= \boxed{8.5 \text{ m/s}}$$

29. In simple harmonic motion, the displacement is zero when the :

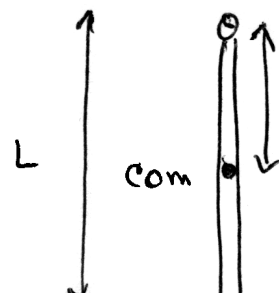
- A) kinetic energy is zero
- B) velocity is maximum
- C) velocity is zero
- D) acceleration is a maximum
- E) oscillations have died away

$$E = \frac{1}{2} kx^2 + \frac{1}{2} mv^2$$

$$\left\{ \begin{array}{ll} x = 0 & v = v_{\max} \\ x = x_{\max} & v = 0 \end{array} \right\}$$

30. A thin rod with length $L = 2.0 \text{ m}$ is suspended at one end and started oscillating as a physical pendulum. Its rotational inertia is $(1/3) mL^2$, where L is the total length of the rod. What would be the **length of a simple pendulum** having the **same period T** oscillation?

- A) 2.0 m
- B) 2.67 m
- C) 1.33 m
- D) 0.75 m
- E) 1.5 m



$$T_{\text{SHO}} = 2\pi \sqrt{\frac{L}{g}}$$

$$T_{\text{PhPend}} = 2\pi \sqrt{\frac{I}{mgh}}$$

$$I = \frac{1}{3} mL^2$$

$$2\pi \sqrt{\frac{L_{\text{SHO}}}{g}} = 2\pi \sqrt{\frac{I}{mgL/2}}$$

$$\frac{L_{\text{SHO}}}{g} = \frac{1}{3} \frac{mL^2}{mgL} \cdot 2 = \frac{2}{3} L = \boxed{1.33 \text{ m}}$$