Lecture 10: Ch 16 Cost-Volume-Profit Analysis & Intro to Time-Value of Money

IE618 Eng Cost & Production Economics

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Chapter 16 Objectives

1. Determine the number of units and amount of sales revenue needed to break even and to earn a target profit.
2. Determine the number of units and sales revenue needed to earn an after-tax target profit.
3. Apply cost-volume-profit analysis in a multiple-product setting.
4. Prepare a profit-volume graph and a cost-volume-profit graph, and explain the meaning of each.
5. Explain the impact of risk, uncertainty, and changing variables on cost-volume-profit analysis.
6. Discuss the impact of non-unit cost drivers on cost-volume-profit analysis.

The Break Even Point and Target Profit in Units and Sales Revenue

Operating Income: income or profit before income taxes (includes only revenues and expenses from the firm's normal operations)

Net income: operating income minus income taxes

Objective 1

The Break Even Point and Target Profit in Units and Sales Revenue

How many units will yield the desired profit?

Operating income = Sales Revenues – Variable Expenses – Fixed expenses

Operating income = (Price × Number of units) – (Variable cost per unit × Number of units) – Total fixed costs

Note: All further CVP equations are derived from the contribution margin based income statement.

The Break Even Point and Target Profit in Units and Sales Revenue

Contribution Margin = Sales revenue minus total variable costs

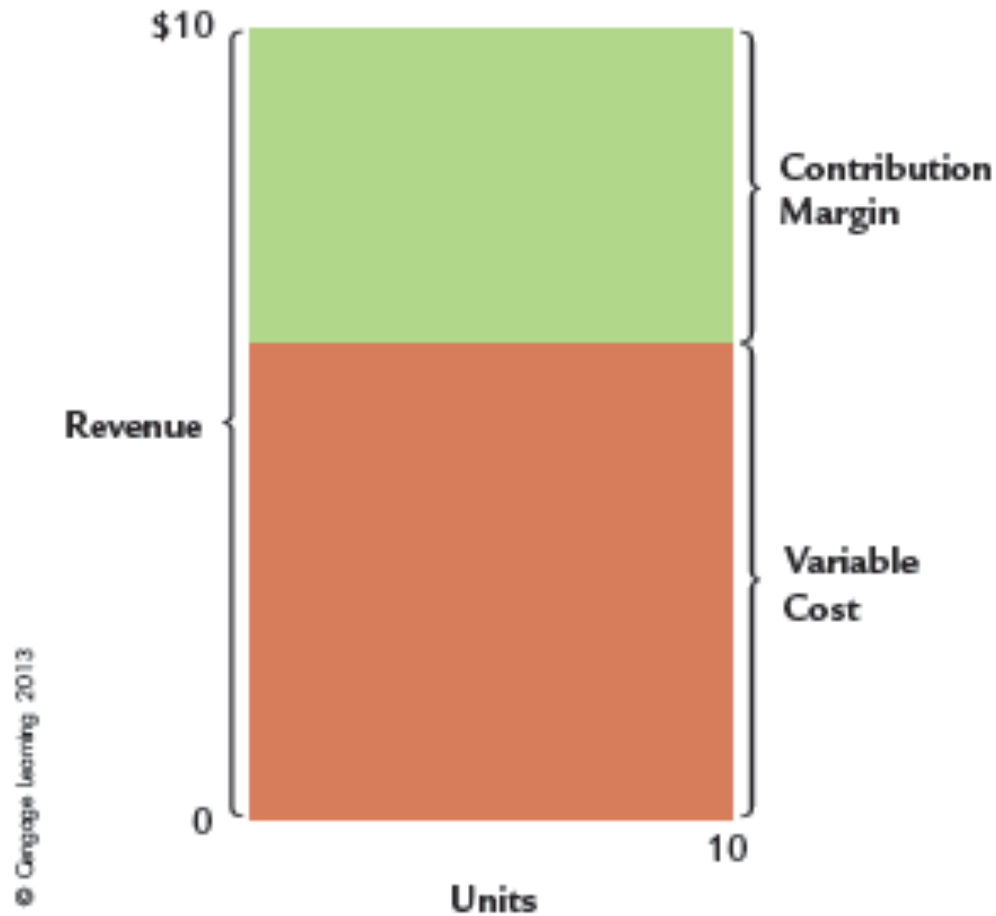
By substituting the unit contribution margin for price minus unit variable cost in the operating income equation:

Number of units = Fixed costs /Unit contribution margin

Objective 1

The Break Even Point and Target Profit in Units and Sales Revenue

Division of Revenue into Variable Cost and Contribution Margin



Objective 1

Break-Even Point in Sales Dollars

The following More-Power Company contribution margin income statement is shown for sales of 72,500 sanders.

Sales	$2,900,000
Less: Variable expenses	1,740,000
Contribution margin	$1,160,000
Less: Fixed expenses	800,000
Operating income	$ 360,000

To determine the break-even in sales dollars, the contribution margin ratio must be determined ($1,160,000 ÷ $2,900,000)

Contribution Margin Ratio = **40%**

Therefore, Variable Cost Ratio = **60%**

The Break Even Point and Target Profit in Units and Sales Revenue

Sales Revenue Approach

Operating income = Sales – Variable costs – Total fixed costs

Operating income = Sales – (Variable cost ratio × Sales) – Total fixed costs

Operating income = Sales (1- Variable cost ratio) – Total fixed costs

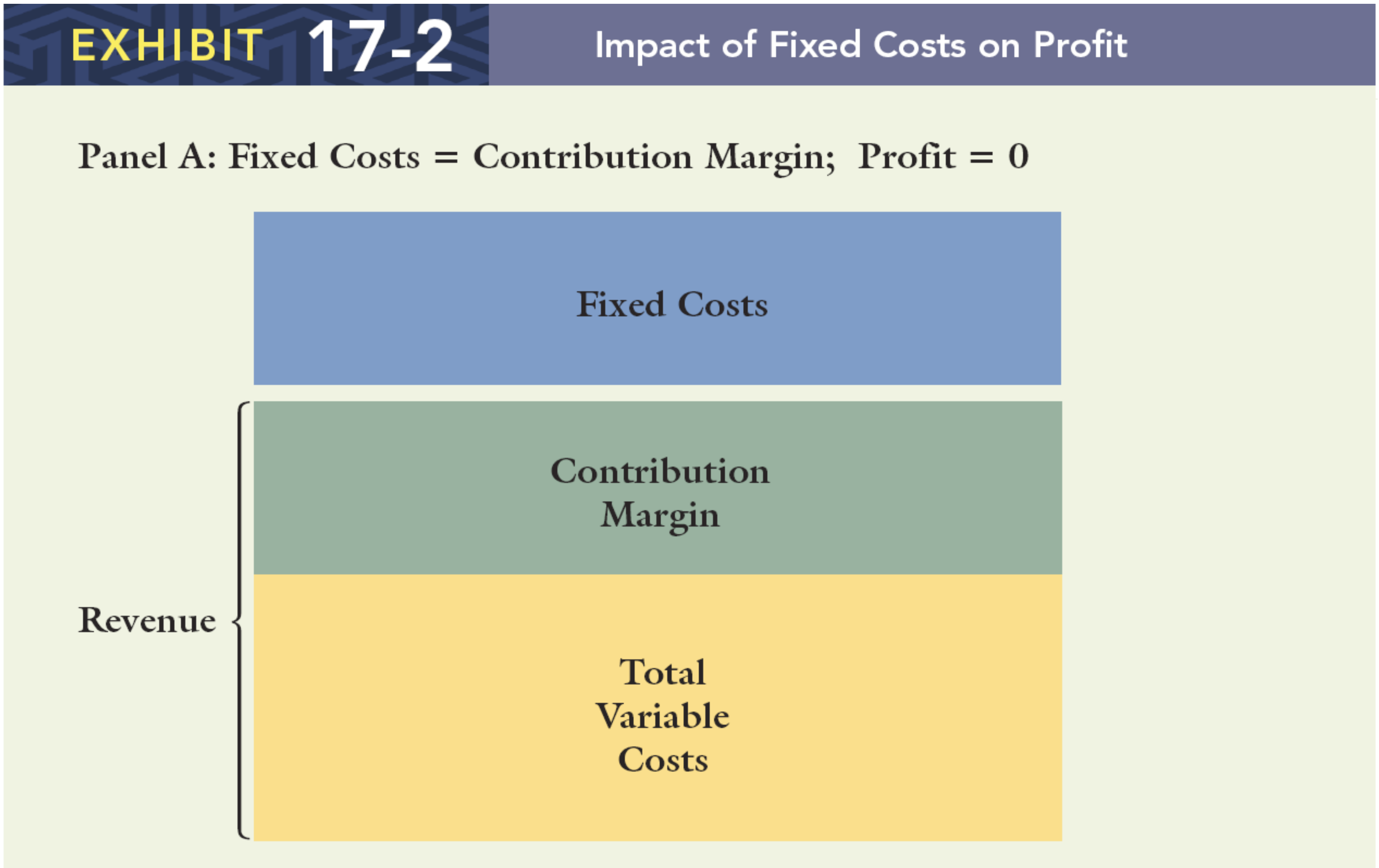
Operating income = Sales × Contribution margin ratio – Total fixed costs

Sales = (Total fixed costs + Operating income)/ Contribution margin ratio

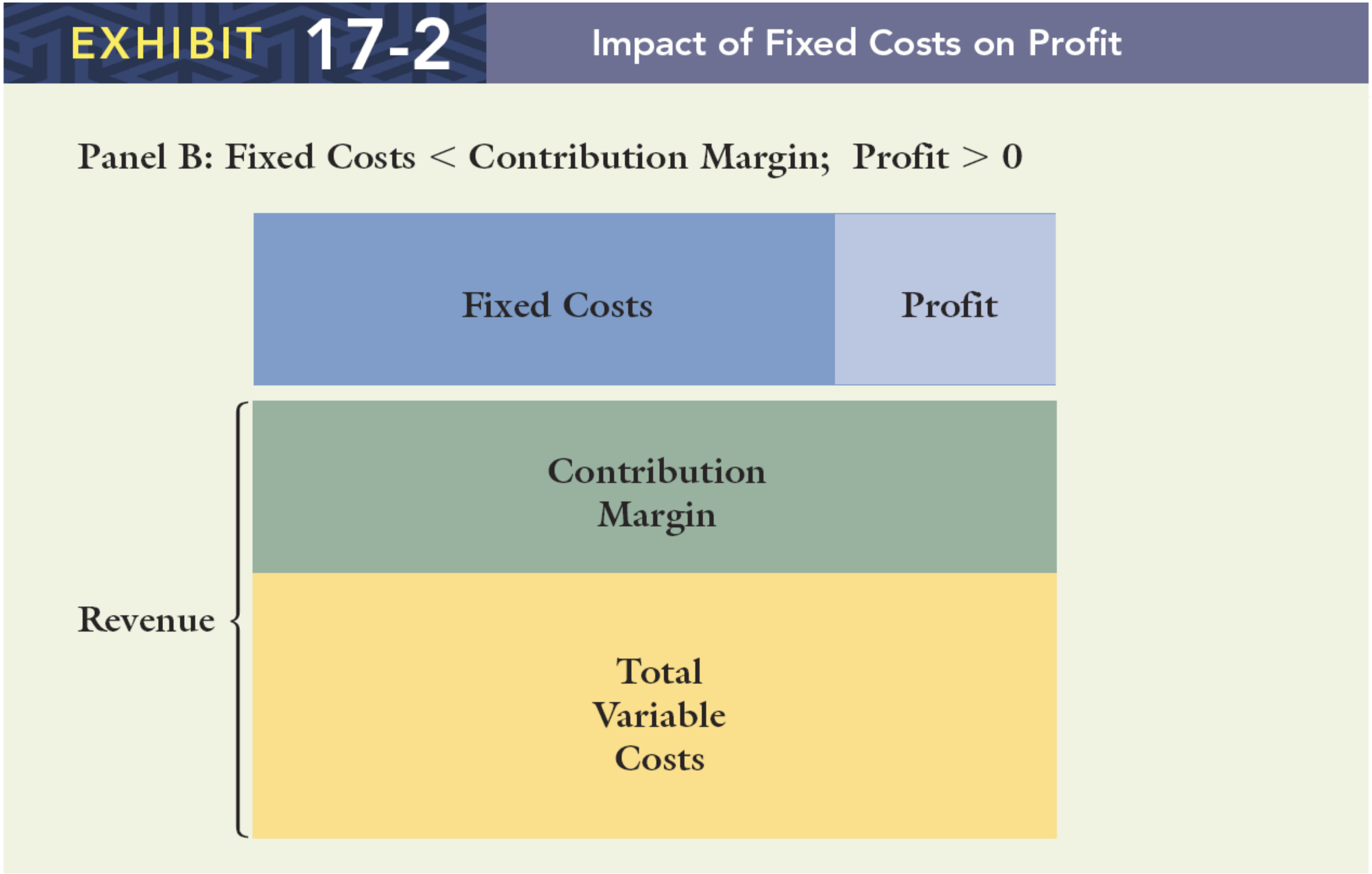
So, at break even:

Break-even sales = Total fixed costs/Contribution margin ratio

17.2 Break-Even Point in Sales Dollars



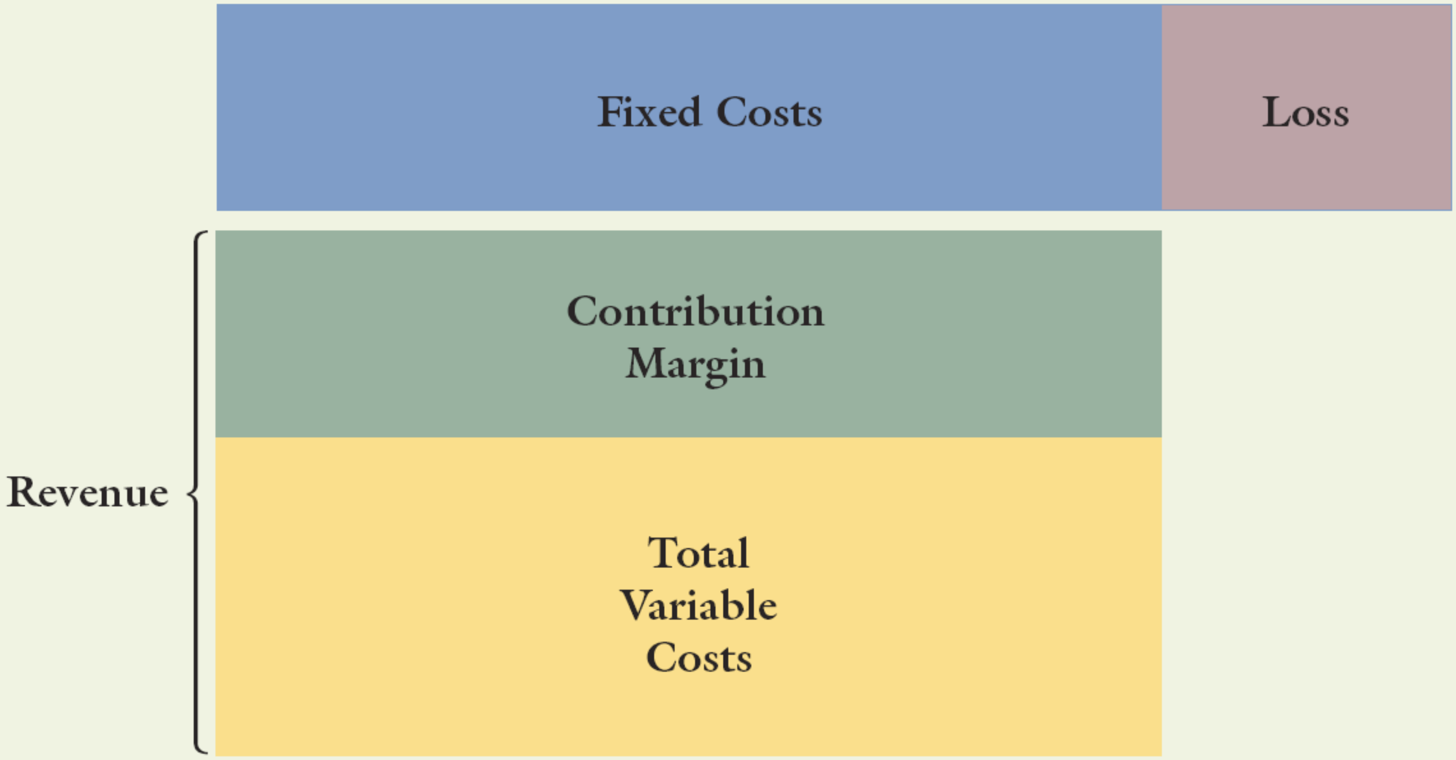
17.2 Break-Even Point in Sales Dollars



17.2 Break-Even Point in Sales Dollars

EXHIBIT 17-2 Impact of Fixed Costs on Profit

Panel C: Fixed Costs > Contribution Margin; Profit < 0



After Tax Profit Targets

- When calculating the break-even point, income taxes play no role because the taxes paid on zero income are zero
- After tax profit: computed by subtracting income taxes from the operating income

Operating Income = Net income /(1-tax rate)

Objective 2

Break-Even Point in Sales Dollars

Profit Targets

How much sales revenue must More-Power generate to earn a before-tax profit of $424,000?

Sales = (Fixed Cost + Profit)/Contribution Margin

Sales = ($800,000 + $424,000) ÷ 0.40

= $1,224,000 ÷ 0.40

= **$3,060,000**

Multiple Product Analysis

Direct fixed expenses: those fixed costs which can be traced to each segment and would be avoided if the segment did not exist

Common fixed expenses: fixed costs that are not traceable to the segments and that would remain even if one of the segments was eliminated

Sales mix: the relative combination of products being sold by a firm

Break-even sales = Fixed costs/Contribution margin ratio

Objective 3

Multiple Product Analysis

Blazin-Boards Company plans to sell 10,000 regular snowboards and 2,500 deluxe snowboards in the coming year. Product price and cost information includes:

	Regular Snowboard	Deluxe Snowboard
Price	$ 400	$ 600
Unit variable cost	240	300
Direct fixed cost	400,000	200,000

Common fixed selling and administrative expense totals $200,000.

Why:

The break-even point in units gives managers a starting point for increasing profitability. If the company is making a loss, the break-even point tells management just what needs to be done to stop losing money. Once the break-even point is passed, the company will earn a profit. By looking at break-even points for each product, managers can see whether one product is being "carried" by other products.

Multiple Product Analysis

Blazin-Boards plans on selling 10,000 regular snowboards and 2,500 deluxe snowboards. The sales mix is 4:1

	Regular Snowboards	Deluxe Snowboards	Total
Sales	$4,000,000	$1,500,000	$5,500,000
Less: Variable expenses	2,400,000	750,000	3,150,000
Contribution margin	$1,600,000	$ 750,000	$2,350,000
Less: Direct fixed expenses	400,000	200,000	600,000
Product margin	$ 1,200,000	$ 550,000	$1,750,000
Less: Common fixed exp.			200,000
Operating income			$ 1,550,000

Note: Regular snowboard unit price = $4,000,000/10,000 = $400
Variable unit cost=$2,400,000/10,000 = $ 240
Deluxe Snowboard unit price = $600
Variable unit cost = $300

Multiple Product Analysis

Break-Even Point in Units

Regular Snowboard break-even units

= Fixed costs ÷ (Price – Unit variable)

= $400,000 ÷ $160

= **2,500** units

Deluxe Snowboard break-even units

= Fixed costs ÷ (Price – Unit variable)

= $200,000 ÷ $300

= **667** units

Multiple Product Analysis

Sales Mix and CVP Analysis

Product	Price	Unit Variable Cost	Unit Contribution Margin	Sales Mix	Package Unit Contribution Margin
Regular Snowboard	$400	$240	$160	4	$640
Deluxe Snowboard	600	300	300	1	300
Package total					$940

Package break-even units

= Fixed costs ÷ Package contribution margin

= ($400,000+$200,000+$200,000) ÷ $940

= **851.064** units

Sales volume for break-even

Regular Snowboard: (851.064 x 4) = 3,404 units

Deluxe Snowboard: (851.064 x 1) = 851 units

Multiple Product Analysis

Sales Dollar Approach

Projected Income:

Sales	$5,500,000	
Less: Variable expenses	3,150,000	
Contribution margin	$2,350,000	0.4273
Less: Fixed expenses	800,000	
Operating income	$ 1,550,000	

Break-even sales = Fixed costs ÷ contribution margin ratio

= 800,000 ÷ 0.4273

= $1,872,220

Or from break-even units approach:

Break-even Sales = 3,404 reg units x $400 + 851 deluxe units x $600

= $1,872,200 (with round-off error)

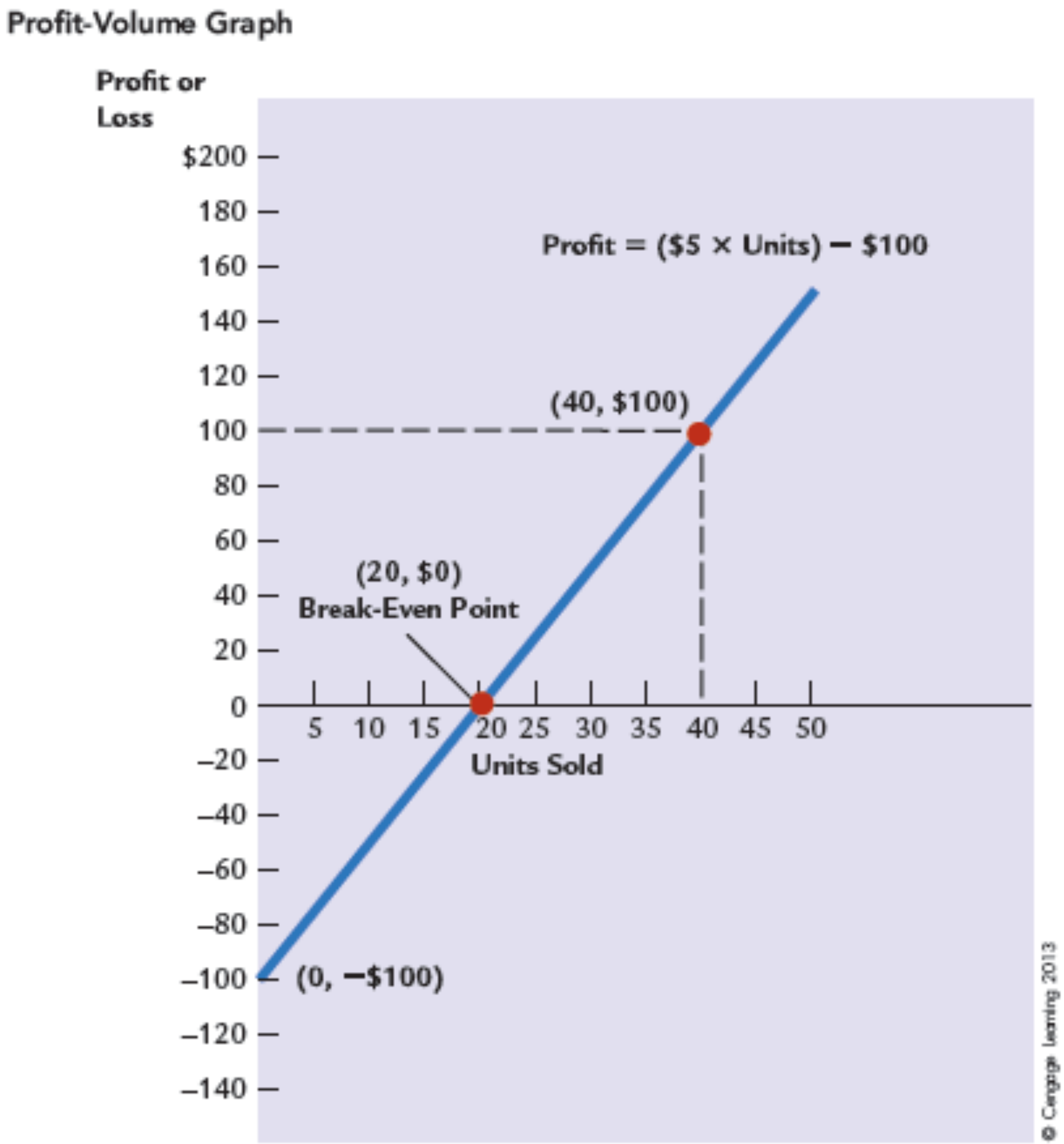
Graphical Representations of CVP Relationships

Profit-volume graph: portrays the relationship between profits and sales volume

- The graph of the operating income equation [Operating income = (Price × Units) – (Unit variable cost × Units) – Fixed Costs]
- Operating income is the dependent variable
- Number of units is the independent variable

Objective 4

Graphical Representations of CVP Relationships



Objective 4

Graphical Representations of CVP Relationships

The cost-volume-profit graph depicts the relationships among cost, volume, and profits

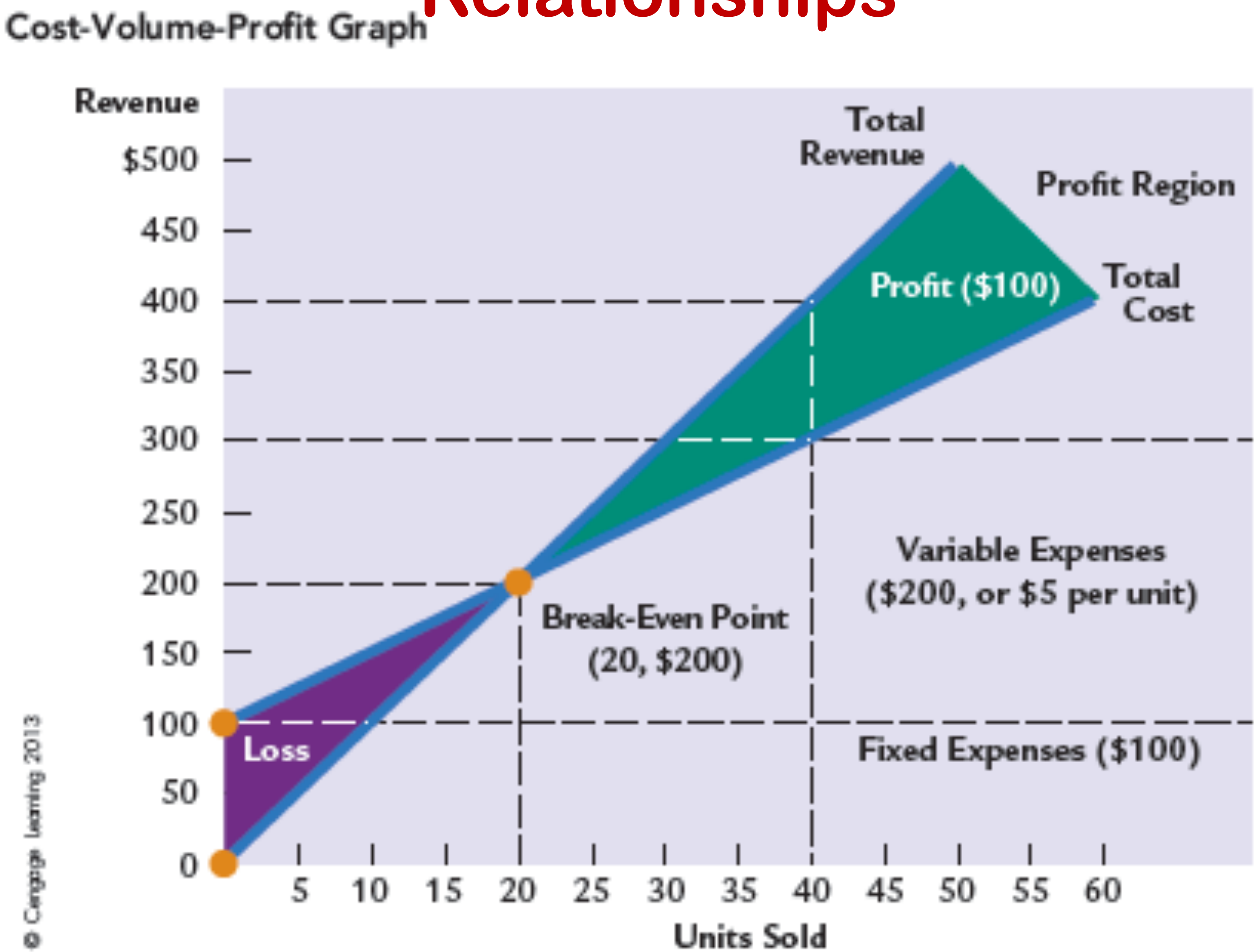
Necessary to graph two separate lines:

1) The total revenue line: revenue = price × units
2) The total cost line: (unit variable cost × units) + Fixed costs

The vertical axis is measured in dollars and the horizontal axis is measured in units sold

Objective 4

Graphical Representations of CVP Relationships



Objective 4

Graphical Representations of CVP Relationships

Assumptions of Cost-Volume-Profit Analysis

1) The analysis assumes a linear revenue function and a linear cost function.
2) The analysis assumes that price, total fixed costs, and unit variable costs can be accurately identified and remain constant over the relevant range.
3) The analysis assumes that what is produced is sold.
4) For multiple-product analysis, the sales mix is assumed to be known.
5) The selling price and costs are assumed to be known with certainty.

Objective 4

Changes in the CVP Variables

Consider three alternatives … which do you recommend?

	Total	Per Unit
Sales ($400 × 10,000 snowboards)	$4,000,000	$400
Total variable expense ($240 × 10,000)	2,400,000	240
Total contribution margin	$1,600,000	$160
Total fixed expense	1,200,000	
Operating income	$ 400,000	

Alternative 1: If advertising expenditures increase by $16,500, sales will increase from 10,000 units to 10,100 units.

Alternative 2: A price decrease from $400 per snowboard to $375 would increase sales from 10,000 units to 12,000 units.

Alternative 3: Decreasing price to $375 and increasing advertising expenditures by $16,500 will increase sales from 10,000 units to 15,000 units.

Changes in the CVP Variables

EXHIBIT 16.5

Summary of the Effects of Alternative 1

	Before the Increased Advertising	With the Increased Advertising
Units sold	10,000	10,100
Unit contribution margin	×$160	× $160
Total contribution margin	$1,600,000	$1,616,000
Less: Fixed expenses	1,200,000	1,216,500
Profit	$ 400,000	$ 399,500

	Difference in Profit
Change in sales volume	100
Unit contribution margin	×$160
Change in contribution margin	$16,000
Less: Increase in fixed expenses	16,500
Decrease in profit	$ (500)

Objective 5

Changes in the CVP Variables

EXHIBIT 16.6

Summary of the Effects of Alternative 2

New contribution margin ($135 × 12,000 units)	$1,620,000
Old contribution margin ($160 × 10,000 units)	1,600,000
Increased contribution margin	$ 20,000

Changes in the CVP Variables

EXHIBIT 16.7

Summary of the Effects of Alternative 3

	Before Changes	With the Increased Advertising and Decreased Price
Units sold	10,000	12,000
Unit contribution margin	×$160	×$135
Total contribution margin	$1,600,000	$1,620,000
Less: Fixed expenses	1,200,000	1,216,500
Profit	$ 400,000	$ 403,500

	Difference in Profit
Decrease in contribution margin on 10,000 units	$(250,000)
Increase contribution margin on 2,000 units	270,000
Change in contribution margin	$ 20,000
Less: Increase in fixed expenses	16,500
Increase in profit	$ 3,500

Objective 5

Changes in the CVP Variables

Margin of Safety: the units sold or expected to be sold or the revenue earned or expected to be earned above the break-even volume

If a firm's margin of safety is large given the expected sales for the coming year, the risk of suffering losses should sales take a downward turn is less than if the margin of safety is small.

CVP Analysis and Non-Unit Cost Drivers

Conventional CVP analysis assumes that all costs can be divided into variable and fixed costs.

An ABC system divides costs into unit and non unit based categories.

Objective 6

Changes in the CVP Variables

EXHIBIT 16.8

Differences Between Manual and Automated Systems

	Manual System	Automated System
Price	Same	Same
Variable costs	Relatively higher	Relatively lower
Fixed costs	Relatively lower	Relatively higher
Contribution margin	Relatively lower	Relatively higher
Break-even point	Relatively lower	Relatively higher
Margin of safety	Relatively higher	Relatively lower
Degree of operating leverage	Relatively lower	Relatively higher
Downside risk	Relatively lower	Relatively higher
Upside potential	Relatively lower	Relatively higher

CVP Analysis and Non-Unit Cost Drivers

The ABC Cost Equation

Total cost = Fixed costs + (Unit variable cost × Number of units) + (Setup cost × Number of setups) + (Engineering cost X Number of engineering hours)

Operating Income

Operating income = Total revenue – [Fixed costs + (Unit variable cost × Number of units) + (Setup cost × Number of setups) + (Engineering cost × Number of engineering hours)]

Objective 6

CVP Analysis and Non-Unit Cost Drivers

Break-Even in Units

Break-even units = [Fixed costs + (Setup cost × Number of setups) + (Engineering cost × Number of engineering hours)]/Price – Unit variable cost)

Differences Between ABC Break-Even and Conventional Break-Even

- The fixed costs differ
- The numerator of the ABC break-even equation has two nonunit-variable cost terms

Objective 6

CVP Analysis and Non-Unit Cost Drivers

Cost Driver	Unit Variable Cost	Level of Cost Driver
	Data About Variables	
Units sold	$ 10	—
Setups	1,000	20
Engineering hours	30	1,000

Other data:

Total fixed costs (conventional)	$100,000
Total fixed costs (ABC)	50,000
Unit selling price	20

The units that must be sold to earn a before-tax profit of $20,000 are computed as follows:

Units = (Targeted income + Fixed costs)/(Price − Unit variable cost)
= ($20,000 + $100,000)/($20 − $10)
= $120,000/$10
= 12,000

Using the ABC equation, the units that must be sold to earn an operating income of $20,000 are computed as follows:

Units = [Targeted income + Fixed costs + (Setup cost × Setups)
+ (Engineering rate × Engineering hours)]/(Price − Unit variable cost)
= ($20,000 + $50,000 + $20,000 + $30,000)/($20 − $10)
= $120,000/$10
= 12,000

Introduction to Time-Value of Money

Time Value of Money

- Capital refers to wealth in the form of money or property that can be used to produce more wealth.

- Engineering economic studies involve the commitment of capital for extended periods of time.

- A dollar today is worth more than a dollar one or more years from now

Capital Return

- Return of capital in the form of interest and profit is an essential ingredient of engineering economy studies.
- Interest and profit pay the providers of capital for forgoing its use during the time the capital is being used.
- Interest and profit are payments for the *risk* the investor takes in letting another use his or her capital.
- Any project or venture must provide a sufficient return to be financially attractive to the suppliers of money or property.

Compound Interest

- Compound interest reflects both the remaining principal and any accumulated interest.
- Compound interest is commonly used in personal and business financial transactions.
- For the following example, assume $1,000 is invested at 10% compound interest

Compound Interest Example

Period	(1) Amount owed at beginning of period	(2)=(1)x10% Interest amount for period	(3)=(1)+(2) Amount owed at end of period
1	$1,000	$100	$1,100
2	$1,100	$110	$1,210
3	$1,210	$121	$1,331

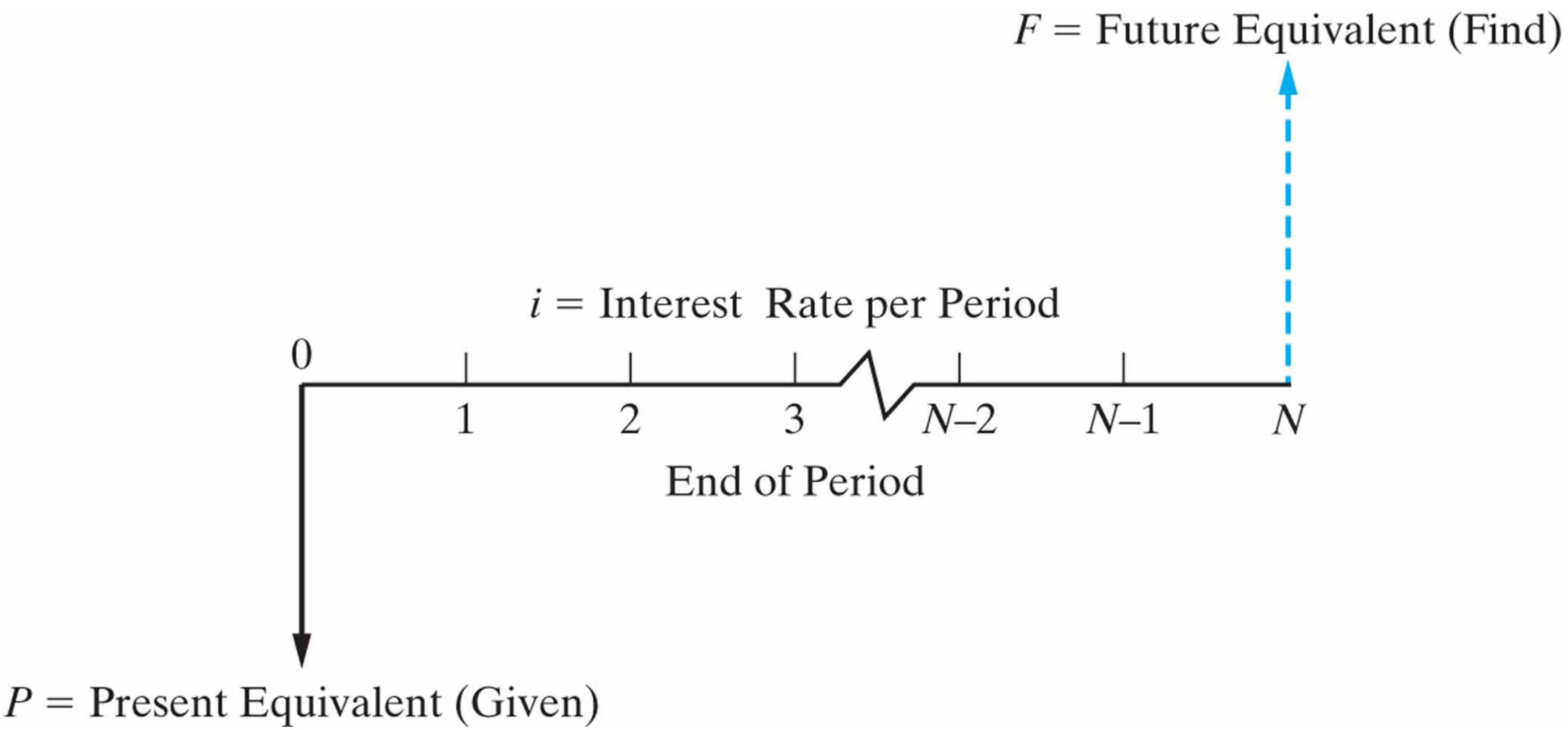
Economic Equivalence

- Allows us to compare alternatives on a common basis.
- Each alternative can be reduced to an *equivalent basis* dependent on
  - interest rate,
  - amount of money involved, and
  - timing of monetary receipts or expenses.
- Using these elements we can "move" cash flows so that we can compare them at particular points in time.

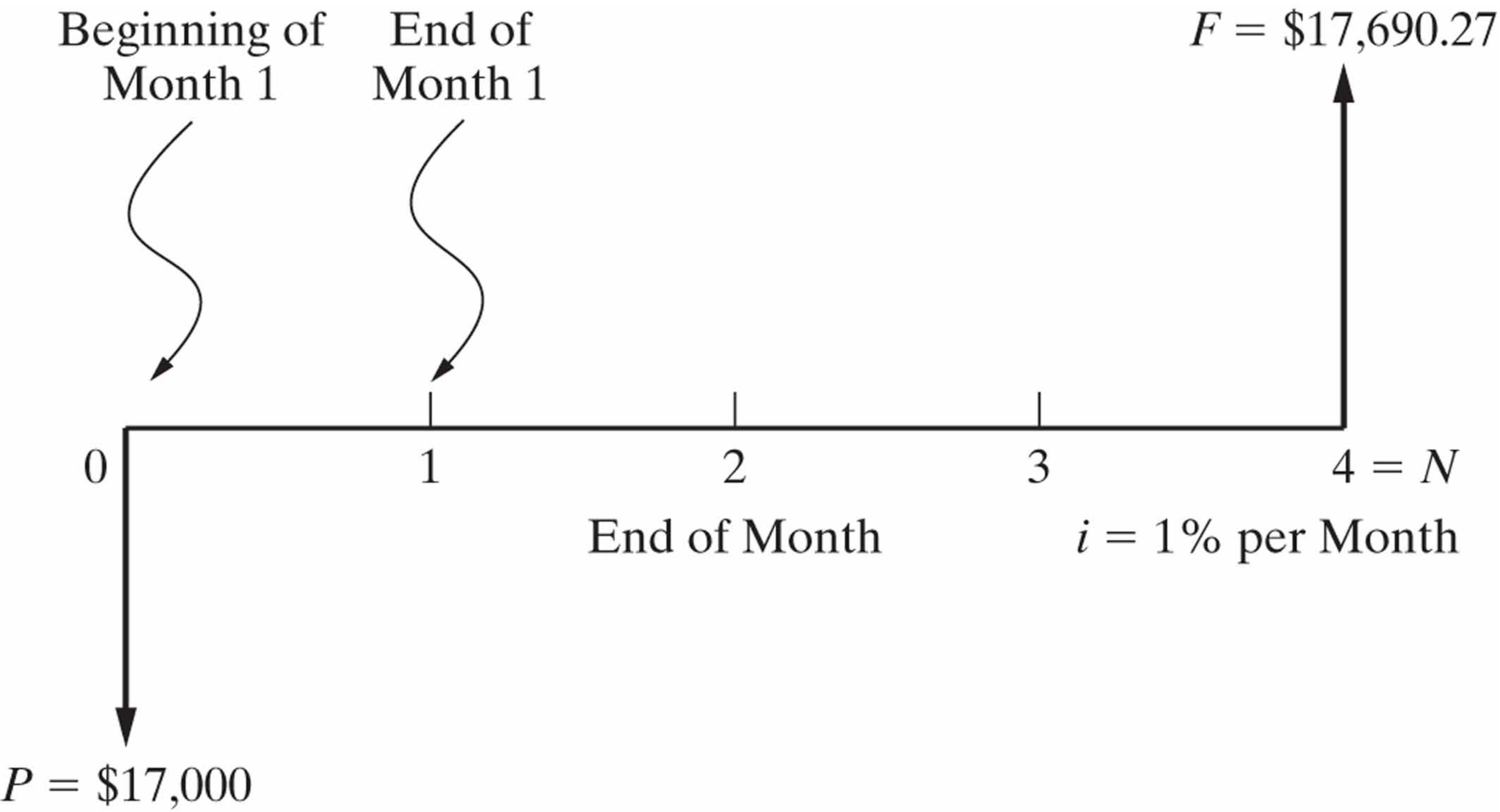
Notation

- i = effective interest rate per interest period
- N = number of compounding (interest) periods
- P = present sum of money; *equivalent* value of one or more cash flows at a reference point in time; the present
- F = future sum of money; *equivalent* value of one or more cash flows at a reference point in time; the future
- A = end-of-period cash flows in a uniform series continuing for a certain number of periods, starting at the end of the first period and continuing through the last

Cash-Flow Diagram



Cash-Flow Diagram



Cash-Flow Diagram

- Horizontal Line: time scale

- Arrows: cash flows
  - Downward arrows represent disbursements or negative cash flows (cash outflows)
  - Upward arrows represent receipts or positive cash flows (cash inflows)

- Cash flow depends on point of view
  - The direction of arrows will be reversed from a lender's point of view vs. the borrower's

Compound Interest Formula

Period	Future Value (F) at the end of the period
0	P
1	$P + Pi = P(1+i)$
2	$P(1+i) + P(1+i)i = P(1+i)^2$
3	$P(1+i)^2 + P(1+i)^2 i = P(1+i)^3$
4	$P(1+i)^3 + P(1+i)^3 i= P(1+i)^4$
.	
.	
N	$P(1+i)^N$

Compound Interest Formulas

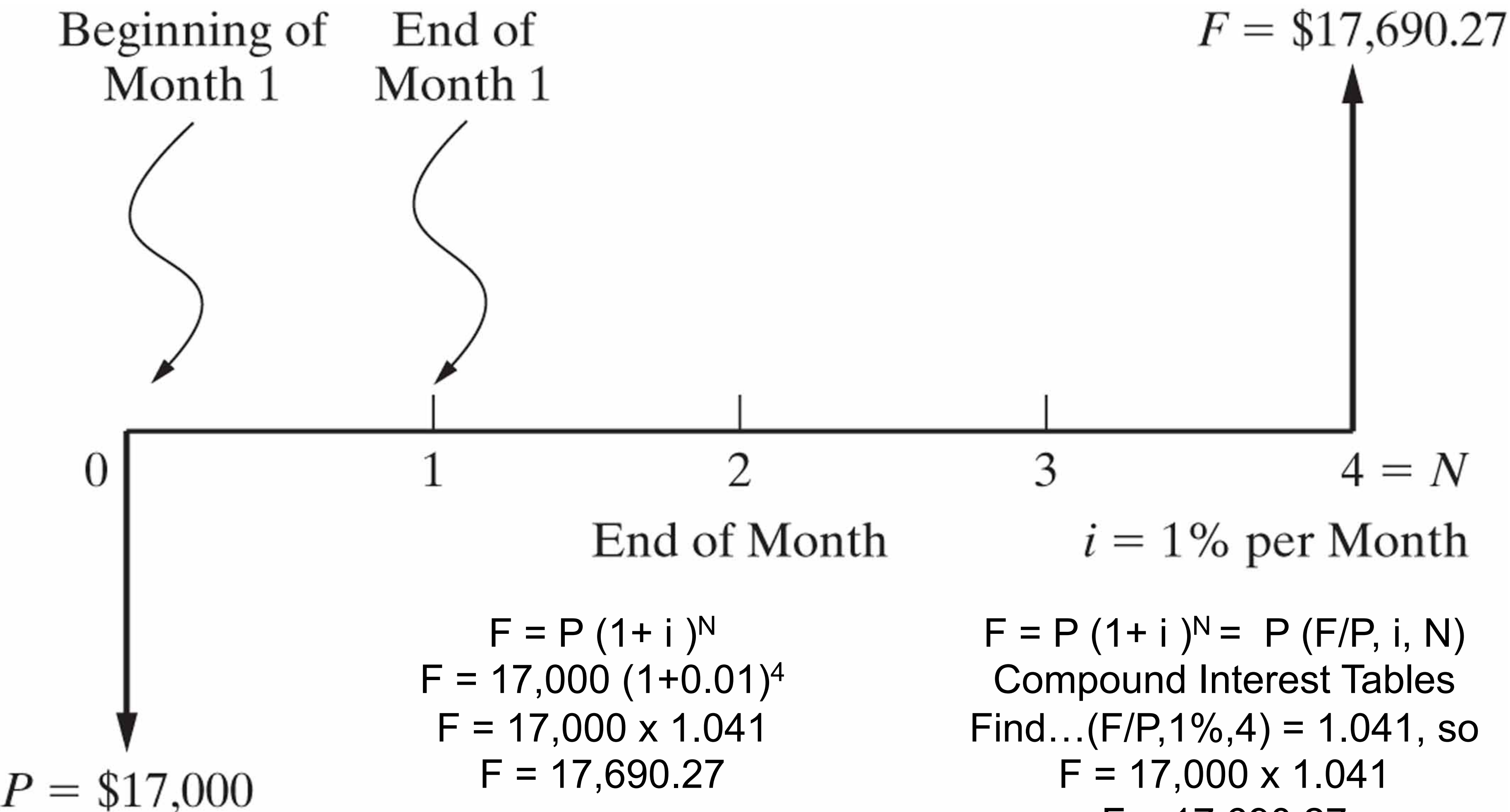
Using the standard notation, we find that a present amount, *P*, can grow into a future amount, *F*, in *N* time periods at interest rate *i* according to the formula below.

$$F = P(1+i)^{N}$$

In a similar way we can find *P* given *F* by

$$P = F(1+i)^{-N} = \frac{F}{(1+i)^{N}}$$

Cash-Flow Diagram



$F = P\,(1+i)^N$

$F = 17{,}000\,(1+0.01)^4$

$F = 17{,}000 \times 1.041$

$F = 17{,}690.27$

$F = P\,(1+i)^N = P\,(F/P, i, N)$

Compound Interest Tables

Find…$(F/P, 1\%, 4) = 1.041$, so

$F = 17{,}000 \times 1.041$

$F = 17{,}690.27$

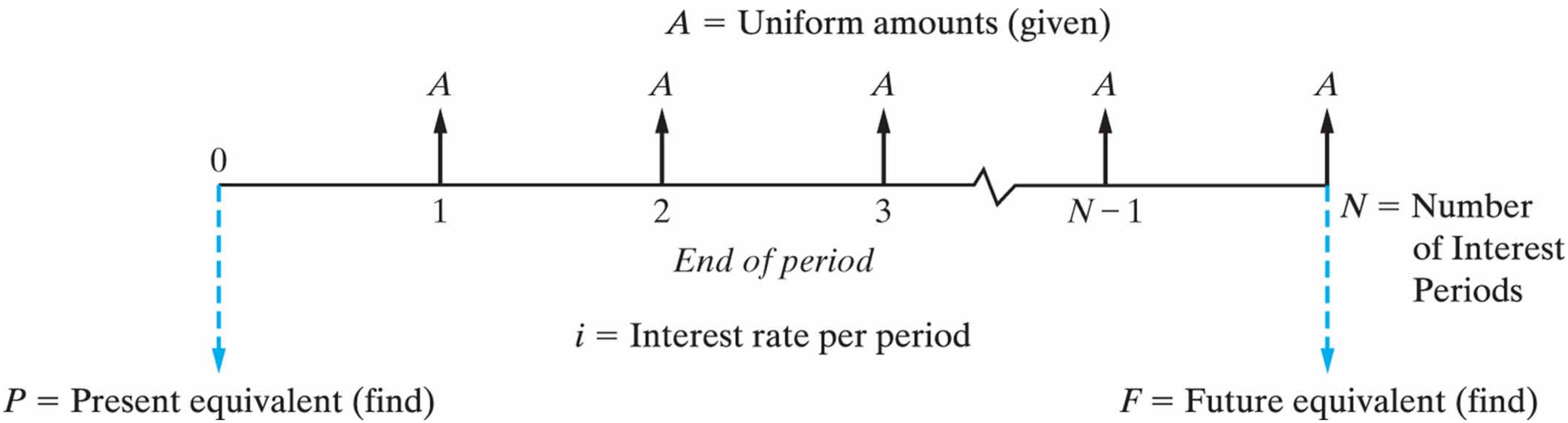
Finding Equivalent Values of Cash Flows- Six Scenarios

Given Information	Find
Present sum of money	Equivalent future value
Future sum of money	Equivalent present value
Uniform end-of-period series	Equivalent present value
Present sum of money	Equivalent uniform end-of-period series
Uniform end-of-period series	Equivalent future value
Future sum of money	Equivalent uniform end-of-period series

Basic Cash Flow Rules

- Rule 1. Cash flows cannot be added or subtracted unless they occur at the same point in time.
- Rule 2. To move a cash flow forward in time by one time unit, multiply the magnitude of the cash flow by (1 + i), where i is the interest rate that reflects the time value of money.
- Rule 3. To move a cash flow backward in time by one time unit, divide the magnitude of the cash flow by (1 + i).

Annuity Cash Flow



A Series of End-of-Period Cash Flows

- $F = A[(1+i)^{N-1} + (1+i)^{N-2} + (1+i)^{N-3} + \ldots$
- $+ (1+i)^2 + (1+i) + 1]$
- Using Geometric Progression formula:

$$F = A\left[\frac{(1+i)^N - 1}{i}\right] \qquad A = F\left[\frac{i}{(1+i)^N - 1}\right]$$

How much will you have in 40 years if you save $3,000 each year and your account earns 8% interest each year?

$$F = \$3,000(F/A, 8\%, 40) = \$3,000(259.0565) = \$777,170$$

Find P Given A

- Write a Present worth expression

$$P = A\left[\frac{1}{(1+i)^1} + \frac{1}{(1+i)^2} + \ldots + \frac{1}{(1+i)^{n-1}} + \frac{1}{(1+i)^n}\right] \quad [1]$$

$$\frac{P}{1+i} = A\left[\frac{1}{(1+i)^2} + \frac{1}{(1+i)^3} + \ldots + \frac{1}{(1+i)^n} + \frac{1}{(1+i)^{n+1}}\right] \quad [2]$$

Uniform Series Present Worth and Capital Recovery Factors

- Setting up the subtraction

$$\frac{P}{1+i} = A\left[\frac{1}{(1+i)^2} + \frac{1}{(1+i)^3} + .. + \frac{1}{(1+i)^n} + \frac{1}{(1+i)^{n+1}}\right] \quad [2]$$

$$-P = A\left[\frac{1}{(1+i)^1} + \frac{1}{(1+i)^2} + .. + \frac{1}{(1+i)^{n-1}} + \frac{1}{(1+i)^n}\right] \quad [1]$$

$$= \quad \frac{-i}{1+i}P = A\left[\frac{1}{(1+i)^{n+1}} - \frac{1}{(1+i)}\right] \quad [3]$$

Uniform Series Present Worth and Capital Recovery Factors

- Simplifying Eq. [3] further

$$\frac{-i}{1+i}P = A\left[\frac{1}{(1+i)^{n+1}} - \frac{1}{(1+i)}\right]$$

$$P = A\left[\frac{(1+i)^N - 1}{i(1+i)^N}\right] \text{ for } i \neq 0 \qquad A = P\left[\frac{i(1+i)^N}{(1+i)^N - 1}\right]$$

Example Find P Given A

How much would be needed today to provide an annual amount of $50,000 each year for 20 years, at 9% interest each year?

$$P = A\left[\frac{(1+i)^N - 1}{i(1+i)^N}\right] \text{ for } i \neq 0$$

$$P = \$50,000(P/A, 9\%, N) = \$50,000(9.1285) = \$456,427$$

Lotto Example

- If you win $5,000,000 in the lottery, how much will you be paid each year? How much money must the lottery commission have on hand at the time of the award? Assume interest = 3%/year.
- Given: Jackpot = $5,000,000, N = 19 years (1st payment immediate), and i = 3% year
- Solution: A = $5,000,000/20 payments = $250,000/ payment (This is the lottery's calculation of A)

  P = $250,000 + $250,000 (P/A,3%,19) (=14.324)

  P = $250,000 + $3,580,950 = $3,830,950

Finding A Given F Example

How much would you need to set aside each year for 25 years, at 10% interest, to have accumulated $1,000,000 at the end of the 25 years?

$$A = F\left[\frac{i}{(1+i)^N - 1}\right]$$

$$A = \$1,000,000(A/F, 10\%, 25) = \$1,000,000(0.0102) = \$10,200$$

Example Finding A Given P

If you had $500,000 today in an account earning 10% each year, how much could you withdraw each year for 25 years?

$$A = P\left[\frac{i(1+i)^N}{(1+i)^N - 1}\right]$$

$$A = \$500,000(A/P, 10\%, 25) = \$500,000(0.1102) = \$55,100$$

It can be challenging to solve for *N* or *i*.

- We may know *P*, *A*, and *i* and want to find *N*.
- We may know *P*, *A*, and *N* and want to find *i*.
- These problems present special challenges that are best handled on a spreadsheet.

Finding i - Example

Jill invested $1,000 each year for five years in a local company and sold her interest after five years for $8,000. What annual rate of return did Jill earn?

$$\$8,000 = \$1,000(F/A, i\%, 5)$$

So,

$$(F/A, i\%, 5) = \frac{\$8,000}{\$1,000} = 8 = \frac{(1+i)^5 - 1}{i}$$

Again, this can be solved using the interest tables and interpolation, but we generally resort to a computer solution.

Finding N - Example

Acme borrowed \$100,000 from a local bank, which charges them an interest rate of 7% per year. If Acme pays the bank \$8,000 per year, now many years will it take to pay off the loan?

$$\$100,000 = \$8,000(P/A, 7\%, N)$$

So,

$$(P/A, 7\%, N) = \frac{\$100,000}{\$8,000} = 12.5 = \frac{(1.07)^N - 1}{0.07(1.07)^N}$$

This can be solved by using the interest tables and interpolation, but we generally resort to a computer solution. Or Solve using logarithms gives N=30.7 yrs

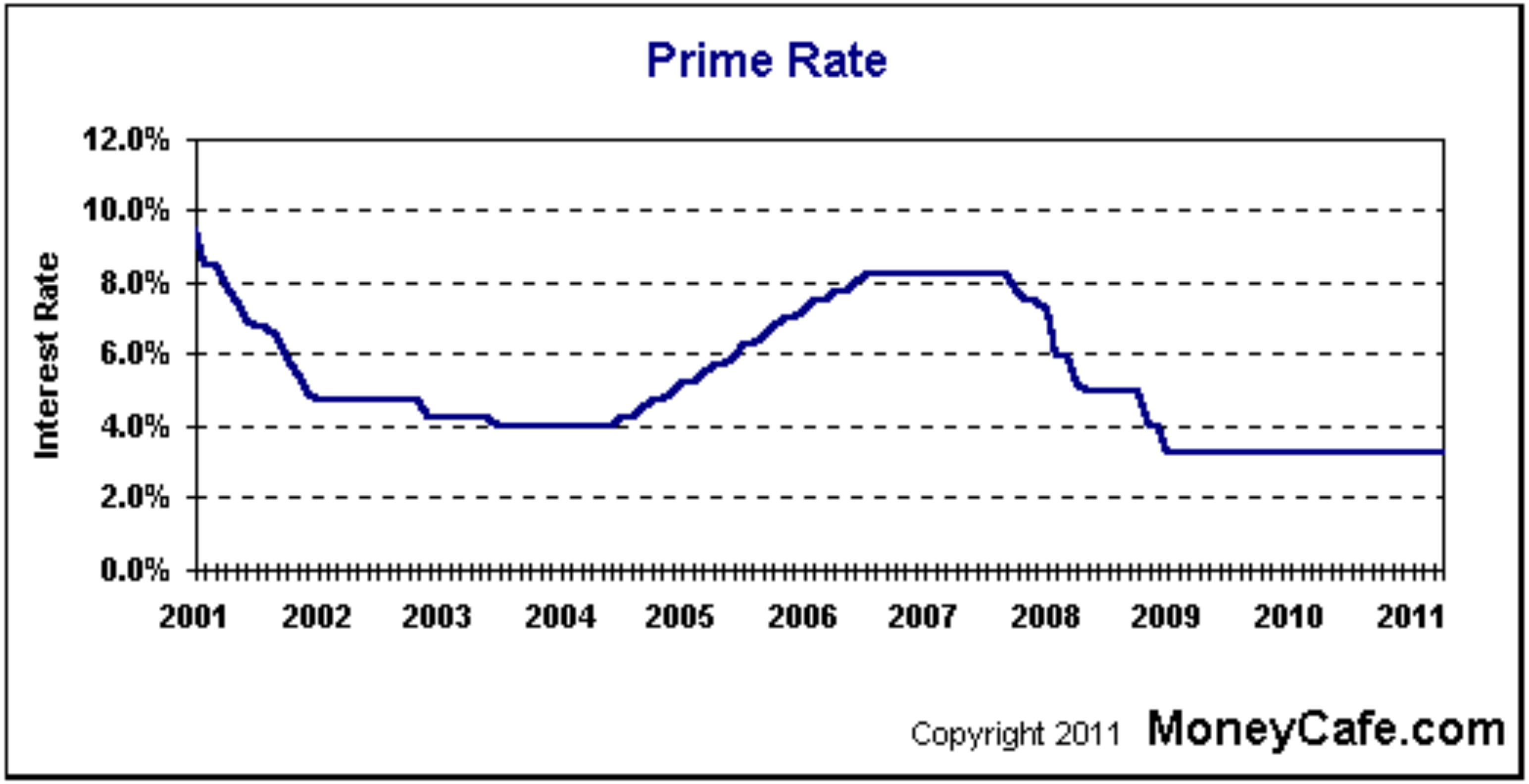
There are specific spreadsheet functions to find *N* and *i*.

The Excel function used to solve for *N* is

NPER(*rate*, *pmt*, *pv*), which will compute the number of payments of magnitude *pmt* required to pay off a present amount (*pv)* at a fixed interest rate (*rate*).

One Excel function used to solve for *i* is

RATE(*nper*, *pmt*, *pv*, *fv*), which returns a fixed interest rate for an annuity of *pmt* that lasts for *nper* periods to either its present value (*pv*) or future value (*fv*).



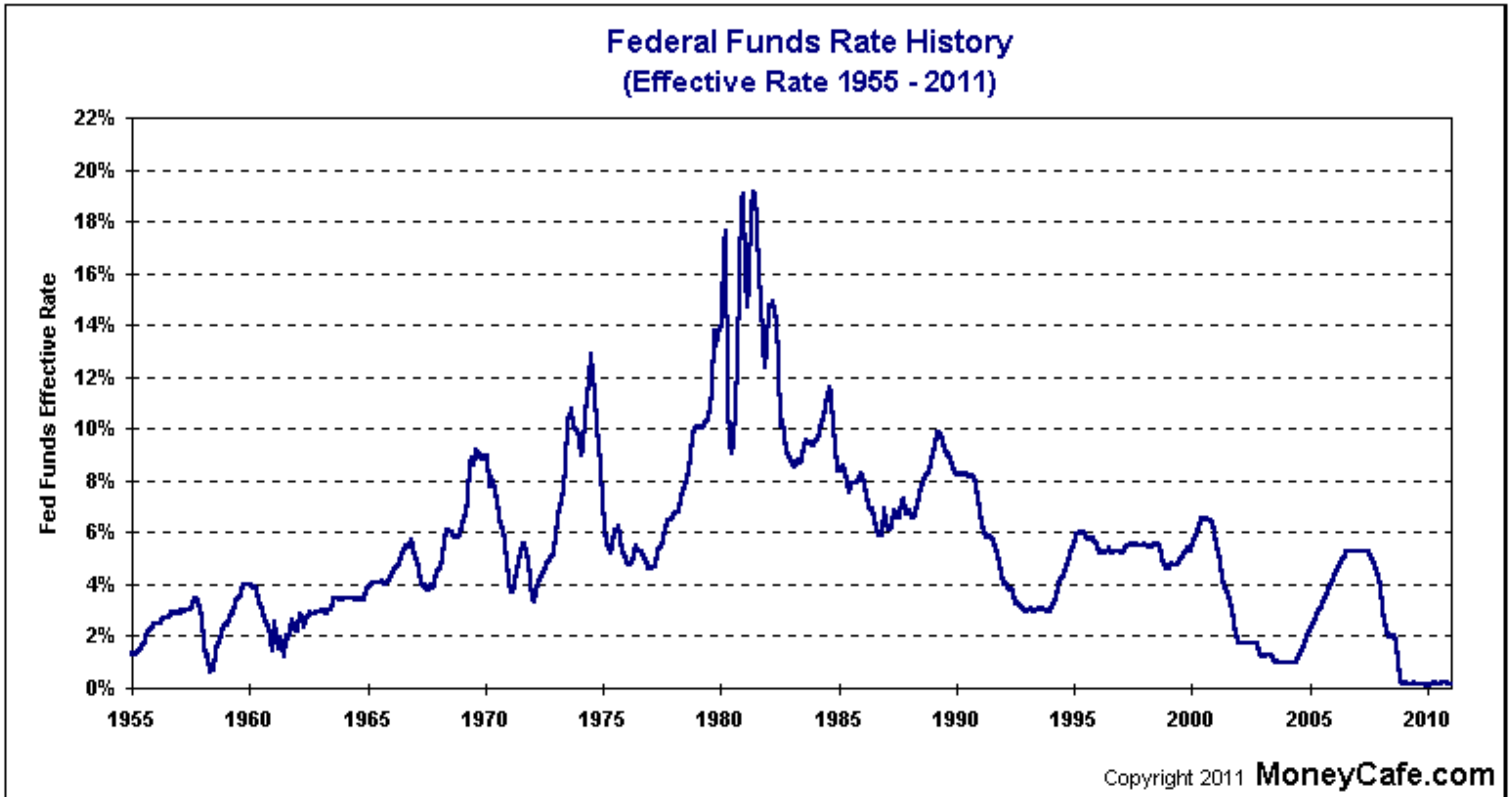
Historical Chart

Fed Funds Rate (Rate Fed Charges Banks)

Past Trend Present Value & Future Projection

Historical Graph

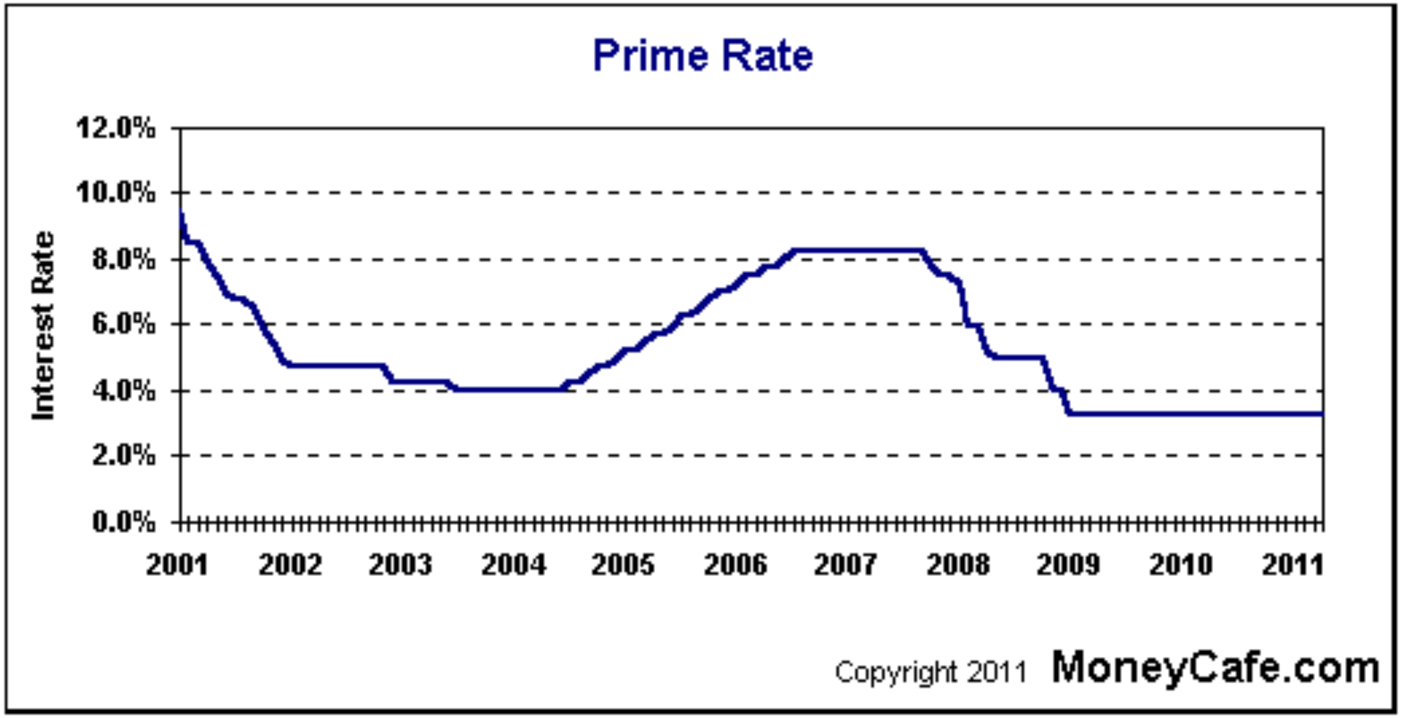
Below is a graph of the Fedl Funds Rate from 1955 to 2011.



Source: Federal Reserve Board

Prime Interest Rate (Rate Banks Charge Best Customers)

Past Trend Present Value



Historical Chart

Team Project

(5 pts)   Proposal Overview: Company Well Defined & Product Key Characteristics Identified

(10 pts) Customer Expectations and Alternative Solutions Clearly Considered

(30 pts) Cost Management Principles and Techniques Clear & Fully Utilized

(35 pts) Analysis with Assumptions Complete & Calculations Accurate

(20 pts) Costs & Benefits with Sustainability Impacts Identified & Investment Justified