Fast, High-Order, Well-Conditioned Algorithms for the Solution of Three-Dimensional Acoustic and Electromagnetic Scattering Problems

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Abstract

We present a novel computational methodology based on Nyström discretizations to produce fast and very accurate solutions of acoustic and electromagnetic problems in small numbers of Krylov-subspace iterative solvers. At the heart of our approach are integral equation formulations that exhibit excellent spectral properties. In the case of scattering from perfectly conducting structures, and just as the classical Combined Field Integral Equation (CFIE), our well-conditioned equations result from representations of the scattered fields as a combination of surface magnetic- and electric-dipole distributions that involve use of certain types of regularizing operators. We call the resulting equations Regularized Combined Field Integral Equations (CFIE-R). A variety of numerical results demonstrate that, for a given accuracy, the new equations can give rise to significant reductions in computational costs over those resulting from previous approaches, especially in the high-frequency regime.

1 Introduction

The novel integral equations that we developed for the solution of scattering problems for the Maxwell equations [2] result from representations of the scattered fields as combinations of magnetic and electric-dipole distributions on the surface of the scatterer, *in which the electric dipole integral operators is composed with a regularizing operator* [2]: using the free-space Green's function $G_k(\mathbf{x}-\mathbf{y}) = e^{ik|\mathbf{x}-\mathbf{y}|}/4\pi|\mathbf{x}-\mathbf{y}|$, our representation formula reads

$$\mathbf{E}^s(\mathbf{z}) = \text{curl} \int_\Gamma G_k(\mathbf{z}-\mathbf{y})\mathbf{a}(\mathbf{y})d\sigma(\mathbf{y}) + i\xi \times$$
$$\text{curl curl} \int_\Gamma G_k(\mathbf{z}-\mathbf{y})(\mathbf{n}(\mathbf{y}) \times (\mathcal{R}_{el}\mathbf{a})(\mathbf{y}))d\sigma(\mathbf{y})$$

where Γ denotes the (closed) surface of the three-dimensional scatterer, with the magnetic field being retrieved from the Maxwell equations. The classical theory of traces of boundary layers and the PEC boundary conditions ($\mathbf{n} \times \mathbf{E} = 0$ on Γ) lead to the (indirect) CFIE-R formulation

$$\frac{\mathbf{a}}{2} + \mathcal{K}\mathbf{a} + \xi k \mathcal{T}(\mathbf{n} \times (\mathcal{R}_{el}\mathbf{a})) = -\mathbf{n} \times \mathbf{E}^i \qquad (1)$$

where $\mathbf{E}^i$ is the incident electric field and the magnetic and electric boundary integral operators are denoted by $\mathcal{K}$ and $\mathcal{T}$ respectively. We have proved in [2] that the choice of regularizing operators of the type $\mathcal{R}_{el} = \mathbf{S}_K,\ K = ik_1,\ k_1 \geq 0$, where $\mathbf{S}_K$ is a vector single layer operator associated with wavenumber K, leads to uniquely solvable CFIE-R equations in $H_{\text{div}}^{-\frac{1}{2}}(\Gamma)$.

2 Numerical Results

Used in conjunction with the high-order Nyström methods we developed for these operators (by generalizing previously available methodologies based on partitions of unity and changes of variables [1]), our scattering solvers can produce highly accurate solutions in small overall computational times [2]. We present in Figure 1 the numbers of iterations for the sequence of 320 wavenumbers $k = 0.1, 0.2, \cdots, 32$ and corresponding discretizations (9 points/wavelength) that deliver far-field errors of order 10^{-4}. The run-times/iteration are 223 sec for CFIE and 328 sec for CFIE-R on a 2.66 GHz CPU, 2Gb RAM desktop machine, C++ implementation for 37632 unknowns.

References

[1] O. Bruno and L. Kunyansky, *A fast, high-order algorithm for the solution of surface scattering problems: basic implementation, tests, and applications*, J. Comput. Physics, **169** (2001), pp. 80–110.

[2] O. Bruno, T. Elling, and C. Turc, *Electromagnetic integral equations requiring small numbers of Krylov-subspace iterations*, in press, J Comput. Physics, 2009.

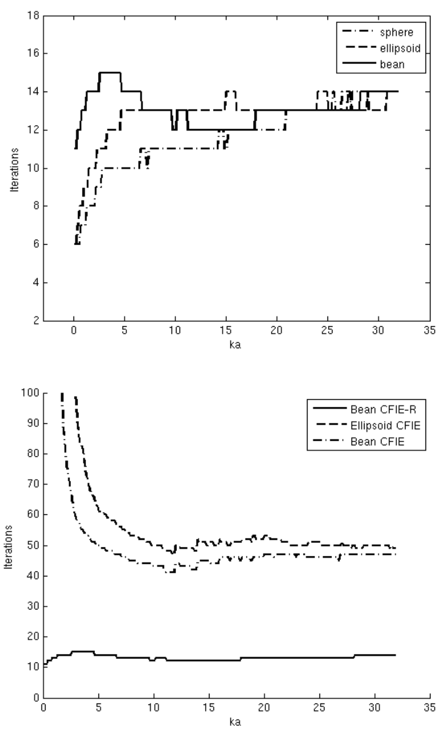

Figure 1: Numbers of iterations to 10^{-4} GMRES residuals for the CFIE-R formulations with $\xi = 1$ and $K = ik/2$ (top) and CFIE vs CFIE-R formulations (bottom), for three configurations: unit sphere, elongated ellipsoid and bean-shaped geometry with plane-wave incident fields; $k = 0.1, 0.2, \cdots, 32$