Efficient Solution of Three-Dimensional Problems of Acoustic and Electromagnetic Scattering by Open Surfaces

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Abstract

We present a computational methodology (a novel Nyström approach based on use of a non-overlapping-patch technique and Chebyshev discretizations) for efficient solution of problems of acoustic and electromagnetic scattering by open surfaces. Our integral equation formulations (**1**) Incorporate, as *ansatz*, the singular nature of open-surface integral-equation solutions, and (**2**) For the Electric Field Integral Equation (EFIE), use analytical regularizers that effectively reduce the number of iterations required by iterative linear-algebra solution based on Krylov-subspace iterative solvers.

1 Introduction

We consider two scattering problems by infinitely thin open surfaces Γ in $\mathbb{R}^3$, namely, (a) sound-soft acoustic scattering and (b) electromagnetic scattering with PEC boundary conditions, including cases in which Γ contains geometric singularities (corners). Our contribution generalizes the methods introduced in [1], [3] for acoustic scattering by open surfaces with smooth boundaries. In case (a) the scattered field can be represented in the form $u^s(\mathbf{z}) = \int_\Gamma G_k(\mathbf{z}-\mathbf{y})\varphi(\mathbf{y})ds(\mathbf{y})$ using the free-space Green's function $G_k(\mathbf{z}-\mathbf{y}) = e^{ik|\mathbf{z}-\mathbf{y}|}/4\pi|\mathbf{z}-\mathbf{y}|$ for $z \in \mathbb{R}^3 \setminus \Gamma$. This representation leads to a uniquely solvable integral equation of the first kind $S\varphi = -u^{inc}$ on Γ [6]. In the case when the boundary of Γ is a smooth curve, the density φ can be shown to be of the form $\varphi = \varphi^{reg}/\omega$ where $\omega(\mathbf{y}) \sim \sqrt{d(\mathbf{y})}$ and φ^{reg} is a smooth function [2]; here $d(\mathbf{y})$ denotes the distance from $\mathbf{y}$ to the open edge. In our numerical approach, we solve the weighted integral equation

$$S_\omega\varphi^{reg} = -u^{inc},\ (S_\omega\psi)(\mathbf{x}) = \int_\Gamma G_k(\mathbf{x}-\mathbf{y})\frac{\psi(\mathbf{y})}{\omega}ds(\mathbf{y}) \tag{1}$$

in the space of smooth functions. (As shown in [1], [3], integral equations of the second kind, for both Dirichlet and Neumann problems of scattering by open surfaces, can be obtained through regularization of weighted first kind equations, such as (1), by means of composition with a certain weighted integral operators. However, as indicated in that reference, use of the resulting second kind formulations for open surfaces, which is greatly beneficial for the Neumann case, does not provide significant advantages for the Dirichlet problem.)

In the PEC electromagnetic case the Electric Field Integral Equation (EFIE) is posed in terms of the unknown electrical current $\mathbf{J}$. The component of $\mathbf{J}$ along the open edge blows up as $1/\omega$, while the normal component behaves asymptotically as ω; the divergence of $\mathbf{J}$, in turn, blows up as $1/\omega$; see [5]. Previous related work [4] for the PEC case does not include use of the important singular edge behavior; in our context we incorporate the edge singularities by means of the ansatz $\mathbf{J} = W\mathbf{J}^{reg}$ where the singular weight is defined in parameter space (u,v) as $W = \begin{pmatrix} 1/\omega & -\theta(u,v)\omega \\ 0 & \omega \end{pmatrix}$ with $\theta(u,v) = \mathbf{r}_u\cdot\mathbf{r}_v/\mathbf{r}_u\cdot\mathbf{r}_u$. Here $\mathbf{r} = \mathbf{r}(u,v)$ denotes a parametrization of the surface Γ. With these notations, our weighted EFIE is given by $\mathcal{T}_W\mathbf{J}^{reg} = -\mathbf{n}\times\mathbf{E}^{inc}$, where

$$\mathcal{T}_W\mathbf{J}^{reg} = ikn\times\int_\Gamma G_k(|x-y|)W\mathbf{J}^{reg}(y)ds(y) + \frac{i}{k}n\times\nabla\int_\Gamma G_k(|x-y|)\mathrm{div}_\Gamma(W\mathbf{J}^{reg})ds(y).$$

The spectral properties of the operator $\mathcal{T}_W$ are extremely unfavorable for Krylov subspace iterative solvers. We address this issue by using a left regularizer $\mathcal{T}_\omega$ defined as

$$\mathcal{T}_\omega\mathbf{a} = ikn\times\int_\Gamma G_k(|x-y|)W\mathbf{a}(y)d\sigma(y) + \frac{i}{k}n\times\nabla\int_\Gamma G_k(|x-y|)\omega\ \mathrm{div}(\mathbf{a})d\sigma(y)$$

and solve the *regularized* equation EFIE-R:

$$\mathcal{T}_\omega\mathcal{T}_W\mathbf{J}^{reg} = -\mathcal{T}_\omega(\mathbf{n}\times\mathbf{E}^{inc}). \tag{2}$$

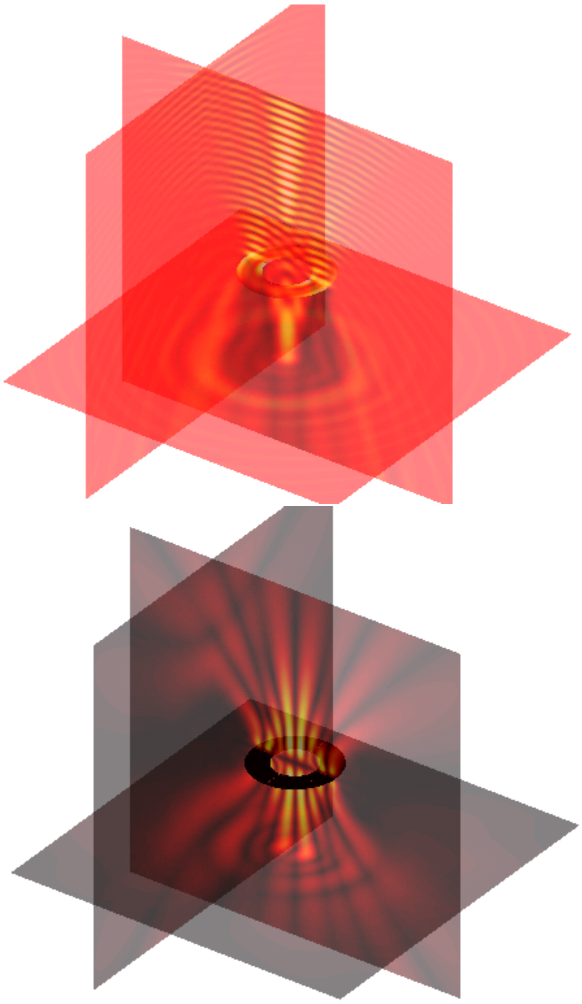

Figure 1: Electromagnetic fields scattered by an annular antenna element of diameter 8λ—x component (top) and z component (bottom); $8 \times 16 \times 16$ unknowns, 13 iterations, total computing time 17m45sec, GMRES residual 10^{-3}, solutions accurate with 3 digits in the far field.

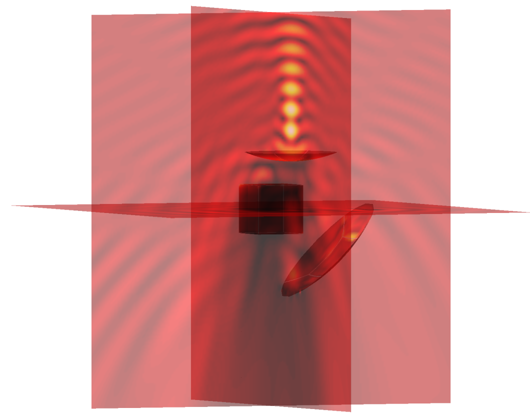

Figure 2: Scalar fields scattered by a configuration consisting of two parabolic antennas (infinitely thin open surfaces) and a closed cube.

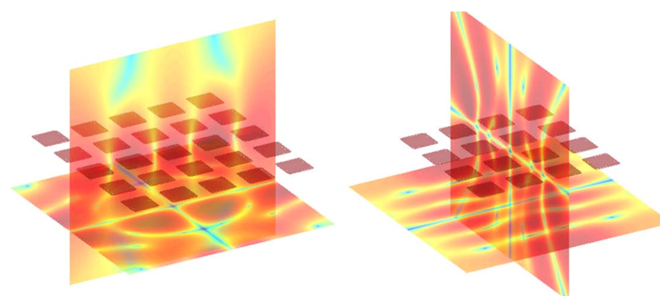

Figure 3: Electric fields diffracted by reflectarray antennas consisting of 5×5 (left) and 4×4 (right) infinitely thin square elements.

In the numerical evaluation of the composition $\mathcal{T}_\omega \mathcal{T}_W$ (which is performed in a sequential manner) it is critical to take into account the fact that, by design, the composition of the last term of $\mathcal{T}_W$ with the last term of $\mathcal{T}_\omega$, both of which are highly singular, actually vanishes.

2 Numerical Results

For the numerical solution of equations (1) and (2) we use a Nyström discretization based on integration patches that do not overlap. The integration surface Γ is tiled by polygonal regions that can be discretized with high-order accuracy by means of cosine transforms and Chebyshev approximations. We use (u, v) patch parametrizations for which the open edges always correspond to $v = \pm 1$, and we thus use $d = 1 - v^2$ together with underlying Chebyshev discretizations in the (u, v) space. In this manner the Jacobian associated to this parametrization annihilates the singularity in the weight matrix (since in this framework $W = W^{reg}/\sin v$ where W^{reg} is a smooth matrix function of u and v). Our high-order integration algorithm consists of two main stages corresponding to the treatment of well-separated interactions and adjacent/singular and near-singular interactions. For each integration patch, we treat the interactions of the first type by means of Clenshaw-Curtis-type integrations for all observation points sufficiently far away from the integration patch. The use of Chebyshev polynomials as spectrally accurate approximations of smooth densities in any given patch makes possible to resort to *finite-difference* approximations for the derivatives of the densities via

2D Chebyshev interpolators. On the other hand, the second stage of our algorithm consists of (a) Singular integration, based on polar coordinate transformations, which are used around each observation point in a given patch using floating partitions of unity; and (b) near singular integration: for smooth portions of the scattering surface and for observation points close to but *outside* the integration patch, we perform polar integration centered at the observation point.

In Figures 1 and 2 we present results obtained by means of the various algorithms described above.

Acknowledgments

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