

TABLE 2-1 Laplace Transform Pairs

	$f(t)$	$F(s)$
1	Unit impulse $\delta(t)$	1
2	Unit step $1(t)$	$\frac{1}{s}$
3	t	$\frac{1}{s^2}$
4	$\frac{t^{n-1}}{(n-1)!} \quad (n = 1, 2, 3, \dots)$	$\frac{1}{s^n}$
5	$t^n \quad (n = 1, 2, 3, \dots)$	$\frac{n!}{s^{n+1}}$
6	e^{-at}	$\frac{1}{s+a}$
7	te^{-at}	$\frac{1}{(s+a)^2}$
8	$\frac{1}{(n-1)!} t^{n-1} e^{-at} \quad (n = 1, 2, 3, \dots)$	$\frac{1}{(s+a)^n}$
9	$t^n e^{-at} \quad (n = 1, 2, 3, \dots)$	$\frac{n!}{(s+a)^{n+1}}$
10	$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$
11	$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$
12	$\sinh \omega t$	$\frac{\omega}{s^2 - \omega^2}$
13	$\cosh \omega t$	$\frac{s}{s^2 - \omega^2}$
14	$\frac{1}{a}(1 - e^{-at})$	$\frac{1}{s(s+a)}$
15	$\frac{1}{b-a}(e^{-at} - e^{-bt})$	$\frac{1}{(s+a)(s+b)}$
16	$\frac{1}{b-a}(be^{-bt} - ae^{-at})$	$\frac{s}{(s+a)(s+b)}$
17	$\frac{1}{ab} \left[1 + \frac{1}{a-b}(be^{-at} - ae^{-bt}) \right]$	$\frac{1}{s(s+a)(s+b)}$

(continued)

TABLE 2-1 (continued)

	$f(t)$	$F(s)$
18	$\frac{1}{a^2}(1 - e^{-at} - ate^{-at})$	$\frac{1}{s(s+a)^2}$
19	$\frac{1}{a^2}(at - 1 + e^{-at})$	$\frac{1}{s^2(s+a)}$
20	$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2 + \omega^2}$
21	$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2 + \omega^2}$
22	$\frac{\omega_n}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin \omega_n \sqrt{1-\zeta^2} t$	$\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$
23	$-\frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1-\zeta^2} t - \phi)$ $\phi = \tan^{-1} \frac{\sqrt{1-\zeta^2}}{\zeta}$	$\frac{s}{s^2 + 2\zeta\omega_n s + \omega_n^2}$
24	$1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta\omega_n t} \sin(\omega_n \sqrt{1-\zeta^2} t + \phi)$ $\phi = \tan^{-1} \frac{\sqrt{1-\zeta^2}}{\zeta}$	$\frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$
25	$1 - \cos \omega t$	$\frac{\omega^2}{s(s^2 + \omega^2)}$
26	$\omega t - \sin \omega t$	$\frac{\omega^3}{s^2(s^2 + \omega^2)}$
27	$\sin \omega t - \omega t \cos \omega t$	$\frac{2\omega^3}{(s^2 + \omega^2)^2}$
28	$\frac{1}{2\omega} t \sin \omega t$	$\frac{s}{(s^2 + \omega^2)^2}$
29	$t \cos \omega t$	$\frac{s^2 - \omega^2}{(s^2 + \omega^2)^2}$
30	$\frac{1}{\omega_2^2 - \omega_1^2} (\cos \omega_1 t - \cos \omega_2 t) \quad (\omega_1^2 \neq \omega_2^2)$	$\frac{s}{(s^2 + \omega_1^2)(s^2 + \omega_2^2)}$
31	$\frac{1}{2\omega} (\sin \omega t + \omega t \cos \omega t)$	$\frac{s^2}{(s^2 + \omega^2)^2}$

TABLE 2-2 Properties of Laplace Transforms

1	$\mathcal{L}[Af(t)] = AF(s)$
2	$\mathcal{L}[f_1(t) \pm f_2(t)] = F_1(s) \pm F_2(s)$
3	$\mathcal{L}_{\pm} \left[\frac{d}{dt} f(t) \right] = sF(s) - f(0\pm)$
4	$\mathcal{L}_{\pm} \left[\frac{d^2}{dt^2} f(t) \right] = s^2 F(s) - sf(0\pm) - \dot{f}(0\pm)$
5	$\mathcal{L}_{\pm} \left[\frac{d^n}{dt^n} f(t) \right] = s^n F(s) - \sum_{k=1}^n s^{n-k} f^{(k-1)}(0\pm)$ where $f^{(k-1)}(t) = \frac{d^{k-1}}{dt^{k-1}} f(t)$
6	$\mathcal{L}_{\pm} \left[\int f(t) dt \right] = \frac{F(s)}{s} + \frac{[\int f(t) dt]_{t=0\pm}}{s}$
7	$\mathcal{L}_{\pm} \left[\iint f(t) dt dt \right] = \frac{F(s)}{s^2} + \frac{[\int f(t) dt]_{t=0\pm}}{s^2} + \frac{[\iint f(t) dt dt]_{t=0\pm}}{s}$
8	$\mathcal{L}_{\pm} \left[\int \cdots \int f(t)(dt)^n \right] = \frac{F(s)}{s^n} + \sum_{k=1}^n \frac{1}{s^{n-k+1}} \left[\int \cdots \int f(t)(dt)^k \right]_{t=0\pm}$
9	$\mathcal{L} \left[\int_0^t f(t) dt \right] = \frac{F(s)}{s}$
10	$\int_0^{\infty} f(t) dt = \lim_{s \rightarrow 0} F(s) \quad \text{if } \int_0^{\infty} f(t) dt \text{ exists}$
11	$\mathcal{L}[e^{-at}f(t)] = F(s+a)$
12	$\mathcal{L}[f(t-\alpha)1(t-\alpha)] = e^{-as}F(s) \quad \alpha \geq 0$
13	$\mathcal{L}[tf(t)] = -\frac{dF(s)}{ds}$
14	$\mathcal{L}[t^2f(t)] = \frac{d^2}{ds^2} F(s)$
15	$\mathcal{L}[t^n f(t)] = (-1)^n \frac{d^n}{ds^n} F(s) \quad n = 1, 2, 3, \dots$
16	$\mathcal{L} \left[\frac{1}{t} f(t) \right] = \int_s^{\infty} F(s) ds \quad \text{if } \lim_{t \rightarrow 0} \frac{1}{t} f(t) \text{ exists}$
17	$\mathcal{L} \left[f\left(\frac{t}{a}\right) \right] = aF(as)$