Math 244-Final Exam   Name:_____

December 15, 2010   **Student #: _____**
Instructor: Dhar
Must show all work to receive full credit!!!

I pledge that I have not violated the NJIT code of honor_____

1. A poker hand consists of five cards. If all the cards are form the same suit but are not in consecutive order, we say that the hand is a flush. For instance, if we have five clubs that are not in consecutive order (such as 2, 4, 5, 10, K), then we have a flush. What is the probability of a flush but not a straight flush? Compute to five decimal places accuracy. (11 points)
    (Example 2.14, page 43)

2. Students in a college program have two opportunities to pass an exam required for graduation. The probability that a student passes the exam the first time is 0.88. For those who fail the first time, the probability of passing the test the second time is 0.70.
    a. Find the probability that a randomly student from the above program passes the exam.
    b. If the student passes the exam, what is the probability that she or he passed on the first try?   (12 points) (3.19, page 68)

3. Of the people who enter a blood bank to donate blood, 1 in 3 has type O^+ blood and 1 in 20 has O^- blood. For the next four people entering the blood bank, let X denote the number with O^+ blood, and let T denote the number of people with type O blood (either O^+ or O^-).
    a. Find the probability distribution of X.
    b. Find the probability distribution of T.   (10 points) (4.9, page 101)

4. A large lot of tires contain 4% defective. Four tires are to be chosen from the lot and place on a car.
    a. Find the probability that three defectives are found before four good ones. Compute to 5 decimal places accuracy.
    b. Find the expected value and the variance of the total number of selections that must be made to get the four good tires.   (10 points) (4.9, page 101)

5. When fishing off the shores of Florida, a spotted sea trout must be between 14 and 24 inches long before it can be kept; otherwise, it must be returned to the waters. In a region of the Gulf of Mexico the lengths of spotted sea trout that are caught are normally distributed with a mean of 20 inches and a standard deviation of 3.5 inches. Assume that each spotted sea trout is randomly caught by a fisherman.
    a. What is the probability that a fisherman catches a potted sea trout within the legal limits?   (5 points)
    b. The fisherman caught a large spotted sea trout. He wants to know whether the length is in the top 10% of the lengths of spotted sea trout in that region. Find the length x for which only 10% of the spotted sea trout in this region will be larger than x.   (9 points)
    c. What is the probability that the fisherman will catch two trout outside the legal limits before catching his first legal spotted sea trout (between 14 and 24 inches)? (5 points)

(5.94, page 252)

6. Find the moment generating function for the random variable X with probability

density function $f(x) = \begin{cases} x/2, & 0 < x < 2, \\ 0, & elsewhere. \end{cases}$ (10 points) (5.139, page 275)

7. The number of calls coming into a hotel's reservation center averages four per minute.
   a. Find the probability that at least two calls arrive in a given 1-minute period.
   b. Find the probability that no calls arrive in a give 2-minute period. Compute to four decimal places accuracy.
   (8 points) (4.90, page 160)

8. Let X denote the amount of gasoline stocked in a bulk tank at the beginning of a
    week and Y denote the amount sold during the week.

Let $f(x,y) = \begin{cases} 3x, 0 \le y \le x \le 1, \\ 0, \quad \text{elsewhere.} \end{cases}$

    a. Compute the marginal density of X. (3 points)
    b. Compute the probability P(2Y<X). (5 points)
    c. Is X independent of Y? Why or why not?   (4 points)
    d. Compute the conditional density of Y given X = 0.5. Show work. (5 points)
    e. Find the conditional mean of Y given X = 0.5. Show work. (3 points)

(Please see 6.40, p. 327, 6.26, p.313, 6.45a, p. 333)

END