Formulas: $${}_nP_r = n(n-1)\ldots(n-r+1) = \frac{n!}{(n-r)!}.$$

$\binom{n}{n_1,\ n_2,\ \cdots\ n_r} = \frac{n!}{n_1!n_2!\cdots n_r!}$, where $n = n_1 + n_2 + \cdots + n_r$. $\binom{n}{r} = \frac{n!}{(n-r)!r!}$.

$P(A_1 \cup A_2 \cup A_3 \cup \cdots) = P(A_1) + P(A_2) + P(A_3) + \cdots$, where A_i's are mutually exclusive events.

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$.

$P(A') + P(A) = 1$. $P(B|A) = \frac{P(A \cap B)}{P(A)}$.

$$P(A_1 \cap A_2 \cap \cdots \cap A_k) = P(A_1)P(A_2|A_1)P(A_3|A_1 \cap A_2)\cdots P(A_k|A_1 \cap A_2 \cap \cdots \cap A_{k-1}).$$

If the events $B_1, B_2, \cdots, B_k$ constitute a partition of the sample space S,

$$P(A) = \sum_{i=1}^{k} P(B_i \cap A) = \sum_{i=1}^{k} P(B_i)P(A/B_i) \;\&\; P(B_r/A) = \frac{P(B_r \cap A)}{\sum_{i=1}^{k} P(B_i \cap A)} = \frac{P(B_r)P(A/B_r)}{\sum_{i=1}^{k} P(B_i)P(A/B_i)}.$$

$P(X = x) = f(x)$, $\sum_x f(x) = 1$. $F(x) = P(X \le x) = \sum_{t \le x} f(t)$.

$\int_{-\infty}^{\infty} f(x)dx = 1$. $P(a < X < b) = \int_a^b f(x)dx$. $F(x) = P(X \le x) = \int_{-\infty}^{x} f(t)dt$. $\frac{dF(x)}{dx} = f(x)$.

$\sum_x \sum_y f(x,y) = 1$. $P(X = x, Y = y) = f(x,y)$. $P[(X,Y) \in A] = \sum\sum_A f(x,y)$.

$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)dxdy = 1$. $P[(X,Y) \in A] = \iint_A f(x,y)dxdy$.

Marginal Distributions: $g(x) = \sum_y f(x,y)$ and $h(y) = \sum_x f(x,y)$,

$g(x) = \int_{-\infty}^{\infty} f(x,y)\,dy$ and $h(y) = \int_{-\infty}^{\infty} f(x,y)\,dx$. $f(x|y) = \frac{f(x,y)}{h(y)}$, provided $h(y) > 0$,

$\mu = E(X) = \sum_x xf(x)$, $\mu = E(X) = \int_{-\infty}^{\infty} xf(x)\,dx$. $\mu_{g(X)} = E[g(X)] = \int_{-\infty}^{\infty} g(x)f(x)\,dx$.

$\mu_{g(X,Y)} = E[g(X,Y)] = \sum_y\sum_x g(x,y)f(x,y)$, $\mu_{g(X,Y)} = E[g(X,Y)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x,y)f(x,y)\,dxdy$.

$\sigma^2 = E[(X-\mu)^2] = \sum_x (x-\mu)^2 f(x)$, $\sigma^2 = E[(X-\mu)^2] = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx$. $V(X) = E(X^2) - \mu^2$.

$\sigma^2_{g(X)} = E\{[g(X) - \mu_{g(X)}]^2\} = \sum_x [g(x) - \mu_{g(X)}]^2 f(x)$, $\sigma^2_{g(X)} = E\{[g(X) - \mu_{g(X)}]^2\}$

$= \int_{-\infty}^{\infty} [g(x) - \mu_{g(X)}]^2 f(x)\,dx$.

$$\sigma_{XY} = cov(X,Y) = E[(X-\mu_X)(Y-\mu_Y)] = \sum_y\sum_x (x-\mu_X)(y-\mu_Y) f(x,y),$$

$$\sigma_{XY} = cov(X,Y) = E[(X-\mu_X)(Y-\mu_Y)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (x-\mu_X)(y-\mu_Y) f(x,y)\,dxdy$$

$cov(X,Y) = E[XY] - \mu_X\mu_Y$. $\rho_{XY} = \dfrac{cov(X,Y)}{\sqrt{V(X)V(Y)}}$. $\sigma^2_{aX+bY+c} = a^2\sigma^2_X + b^2\sigma^2_Y + 2ab\sigma_{XY}$.

$$V\left(\sum_{i=1}^{n} a_i Y_i\right) = \sum_{i=1}^{n} a_i^2 V(Y_i) + 2\sum_{i<j}\sum a_i a_j Cov(Y_i, Y_j).$$

$$\sigma^2_{a_1X_1+a_2X_2+\cdots+a_nX_n} = a_1^2\sigma^2_{X_1} + a_2^2\sigma^2_{X_2} + \cdots + a_n^2\sigma^2_{X_n}.$$

Chebyshev's Theorem $P(\mu - k\sigma < X < \mu + k\sigma) \ge 1 - \dfrac{1}{k^2}$.

Binomial $b(x;n,p) = \dbinom{n}{x} p^x (1-p)^{n-x}, x = 0, 1, \ldots, n$, $\mu = E(X) = np$, $\sigma^2 = V(X) = np(1-p)$

$M(t) = (pe^t + \{1-p\})^n$.

Hypergeometric $h(x;N,n,k) = \dfrac{\dbinom{k}{x}\dbinom{N-k}{n-x}}{\dbinom{N}{n}}$, $\max\{0, n-(N-k)\} \le x \le \min\{n,k\}$. $\mu =$

$E(X) = \dfrac{nk}{N}$, $V(X) = \left(\dfrac{N-n}{N-1}\right) n \left(\dfrac{k}{N}\right)\left(1 - \dfrac{k}{N}\right)$.

Negative Binomial $b^*(x;k,p) = \dbinom{x-1}{k-1} p^k (1-p)^{x-k}$, $x = k, k+1, k+2, \ldots$, $E[X] = \dfrac{k}{p}$,

$V(X) = \dfrac{k(1-p)}{p^2}$. $M(t) = \left[\dfrac{pe^t}{1-(1-p)e^t}\right]^k$, $t < \ln\dfrac{1}{(1-p)}$.

Geometric $g(x;p) = p(1-p)^{x-1}, x = 1, 2, \ldots$ $E[X] = \dfrac{1}{p}$, $V(X) = \dfrac{1-p}{p^2}$.

$M(t) = \dfrac{pe^t}{1-(1-p)e^t}$, $t < \ln\dfrac{1}{(1-p)}$.

Poisson $p(x;\lambda t) = \dfrac{e^{-\lambda t}(\lambda t)^x}{x!}$, $x = 0, 1, 2, \ldots$, $E[X] = V(X) = \lambda t$,

$M(y) = \exp\{\lambda t(e^y - 1)\}$.

C**ontinuous uniform** on (A, B) then its variance $V(X) = \dfrac{(B-A)^2}{12}$.

Normal density $n(x;\mu,\sigma) = \dfrac{1}{\sigma\sqrt{2\pi}} e^{-(x-\mu)^2/2\sigma^2}$, $M(t) = e^{\mu t + \frac{\sigma^2 t^2}{2}}$, $Z = \dfrac{X-\mu}{\sigma}$.

Gamma density $f(x;\alpha,\beta)=\begin{cases}\dfrac{1}{\beta^{\alpha}\Gamma(\alpha)}x^{\alpha-1}e^{-x/\beta}, if \quad x>0\\ 0, \quad if \quad x\le 0\end{cases}$, $E[X]=\alpha\beta,\ Var(X)=\alpha\beta^2.$

$M(t)=\dfrac{1}{(1-\beta t)^{\alpha}},\quad t<\dfrac{1}{\beta}.\ \Gamma(\alpha)=\int_0^{\infty}x^{\alpha-1}e^{-x}dx,\ \Gamma(\alpha)=(\alpha-1)\Gamma(\alpha-1),\Gamma(n)=(n-1)!.$

Exponential density $f(x;\beta)=\begin{cases}\dfrac{1}{\beta}e^{-x/\beta}, if \quad x\ge 0\\ 0, \quad if \quad x<0\end{cases}$, $E[X]=\beta, Var(X)=\beta^2,$

$M(t)=\dfrac{1}{1-\beta t},\quad t<\dfrac{1}{\beta}.$

Beta density

$$f(x)=\begin{cases}\dfrac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}x^{\alpha-1}(1-x)^{\beta-1}, 0<x<1\\ 0, \quad elsewhere,\end{cases} \quad E(X)=\frac{\alpha}{\alpha+\beta}, V(X)=\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}.$$

Suppose X is a continuous random variable with probability density $f(x)$. Let $Y=u(X)$ define a one-to-one correspondence between the value of X and Y so that the equation $y=u(x)$ can be uniquely solved for x, $x=w(y)$. Then Y has a density function given by $g(y)\ \ f[w(y)]|J|$, where $J=w'(y)$.

Joint distribution of functions of random variables given joint density of X_1 and X_2 to be $f(x_{1,}x_2)$ need joint density of $Y_1=u_1(X_1,X_2)$ *and* $Y_2=u_2(X_1,X_2)$ define a one-to-one transformation between the points (x_1,x_2) and (y_1,y_2) so that the equations can be uniquely solved for x_1, *and* x_2 ,in terms of y_1 and y_2 say $x_1=w_1(y_1,y_2)$, $x_2=w_2(y_1,y_2)$. Then,

$$g(y_1,y_2)=f\left(w_1(y_1,y_2),w_2(y_1,y_2)\right)|J|, \quad \text{where } J=\begin{vmatrix}\dfrac{\partial x_1}{\partial y_1} & \dfrac{\partial x_1}{\partial y_2}\\ \dfrac{\partial x_2}{\partial y_1} & \dfrac{\partial x_2}{\partial y_2}\end{vmatrix}.$$

$$\mu_r'=E(X^r),\ M_X(t)=E\left(e^{tX}\right),\ \frac{d^r}{dt^r}M_X(t)\Big|_{t=0}=E\left(X^r\right)=\mu_r'.$$

Let $X_1,\dots,X_n$ be a random sample (independent and identically distributed) random variables with $E\left(X_i\right)=\mu$ and $V\left(X_i\right)=\sigma^2<\infty$, $\bar{X}$ the sample mean, the limiting form of the distribution of $Z=\dfrac{\bar{X}-\mu}{\sigma/\sqrt{n}}$, as $n\to\infty$, is the standard normal distribution $n(z;0,1)$.