1. Verify that the following function \( p(x) \) is a probability mass function for a discrete random variable \( X \) (5 pts):

\[
p(x) = \frac{3x + 2}{40}, \quad \text{for } x = 0, 1, 2, 3, 4
\]

Compute the following (5 pts each):

(a) \( P(X = 4) \)  
(b) \( P(X \leq 1) \)  
(c) \( P(2 \leq X < 4) \)  
(d) Mean and Variance of \( X \).
2. Every day, a factory receives a shipment of components. This factory draws a random sample of 40 components without replacement from the shipment and accepts the entire shipment if and only if no more than one of the 40 components inspected is defective (10 pts each)
   (a) What is the probability of accepting the shipment of components if the shipment contains 7% defective components?
   (b) What is the probability of rejecting the shipment of components if the shipment contains 14% defective components?
3. The number of emergency room admissions in a hospital due to automobile accidents in one day (i.e., a 24-hour period) follows a Poisson probability distribution with the mean of 8 admissions. (10 pts each)
   (a) During a 12-hour period, what is the probability that the number of emergency room admissions is more than 2?
   (b) During a 36-hour period, what is the probability that the number of emergency room admissions is exactly equal to 12?
4. The time to failure (in hours) for a laser in a cytometry machine is modeled as an exponential distribution with $\lambda = 0.00006$. (watch your zeroes). (10 pts each)

(a) What is the probability that the laser will last at least 20,000 hours?
(b) What is the probability that the laser will last at most 30,000 hours?
5. The process of cell division is called mitosis. The amount of time taken for mitosis is normally distributed with an average time of 70 minutes and a standard deviation of 7 minutes. (5 pts each)

(a) What is the probability that the amount of time taken for mitosis is less than 45 minutes?
(b) What is the probability that the amount of time taken for mitosis is more than 65 minutes?
(c) What is the time interval that contains approximately 99% of the mitosis?