MATH 333: Probability & Statistics. $\boxed{\textbf{Final Examination}}$ (Spring 2005)

May 11, 2005 (A)   **NJIT**

Name:	SSN:	Section #

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➔ $\boxed{\textbf{Must show all work to receive full credit.}}$

I pledge my honor that I have abided by the Honor System. _____
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1. Suppose that 0.10% of all computers of a certain type experience CPU failure during the warranty period. A random sample of 10,000 computers is selected.

   a. What are expected value and standard deviation of the number of computers in the sample that experience CPU failure? (4 pts)
   b. What is the probability that more than 4 sampled computers experience CPU failure? (4 pts)
   c. What is the probability that no computers in the sample experience CPU failure? (2 pts)

2. Suppose that in a large population, 1 in 500 adults is afflicted with a disease for which a diagnostic test has been developed. The test is such that when an adult has the disease, a positive result occurs 99% of the time. On the other hand, when an adult does not have the disease, a positive result occurs 2% of the time. An adult is chosen at random for testing.

    a. What is the probability that the test result for this adult is positive? (4 pts)
    b. If the test result is positive, what is the probability that this adult has the disease? (4 pts)
    c. If the test result is negative, what is the probability that this adult does not have the disease? (4 pts)

3. Let X = the time between calls to a service center. It is known that X follows an exponential distribution with a mean of 10 minutes.

   (a) What is the probability that X is greater than 30 minutes? (3 pts)
   (b) Find the 90th percentile of X [Hint: P(X $\leq$ 90th percentile of X) = 0.9]. (3 pts)
   (c) What is the median time between calls to the service center? (3 pts)

4. A chemical supply company currently has in stock 100 pounds of a certain chemical. It sells the chemical to customers in 5 pound bags. Let X = the number of bags ordered by a randomly chosen customer, where X has the following probability mass function (pmf):

x	1	2	3	4
f(x)	.2	.3	.3	.2

    a) Compute E(X). (3 pts)
    b) Compute V(X). (3 pts)
    c) Let Y = the number of pounds of chemical left after the first customer's order is shipped. Find E(Y) [Hint: The number of pounds left is a linear function of X]. (3 pts)
    d) Find V(Y). (3 pts)

5. The strength of steel grade is normally distributed with mean of 43 and standard deviation of 4. Find the probability that the strength is:

    (a) Greater than 43? (3 pts)
    (b) Less than 40? (3 pts)
    (c) Less than or equal to 40? (3 pts)
    (d) Between 40 and 43? (3 pts)

6. A random sample of 10 breakdown voltages yielded a sample mean of 32.1 V and a sample standard deviation of 3.0 V. You may assume that breakdown voltage is normally distributed.
    (a) Compute a 95% confidence interval (C.I.) for the population mean. (4 pts)
    (b) Find a 90% C.I. for the population standard deviation. (4 pts)
    (c) What is the minimum sample size required to ensure that the width of the 95% C.I. for the population mean is no more than 2 V? (4 pts)

7. The New Jersey Department of Motor Vehicles reported that 70% of all vehicles undergoing emissions test passed the test on the first try. Essex county claims that it has a higher rate of vehicles passing emissions test on the first try than the state-wide average. In order to test this claim, a random sample of emission test results of 200 vehicles from Essex county was examined and it was found that 160 vehicles passed the test on the first try.
   (a) Formulate your hypotheses and explain why you chose those hypotheses. (5 pts)
   (b) Does the sample data support Essex county claim when the significance level is $\alpha = .05$? (5 pts)

8. The results of a Wagner turbidity test performed on 15 samples were (in microamperes):
   26.7, 25.8, 24.0, 24.9, 26.4, 25.9, 24.4, 21.7, 24.1, 25.9, 27.3, 26.9, 27.3, 24.8, 23.6
   You may assume that turbidity follows a normal distribution.

   (a) Find a 95% confidence interval on the population mean. (6 pts)
   (b) Find a 90% confidence interval on the population variance. (5 pts)

9. The drying time of paint follows a normal distribution with a mean of 75 minutes and standard deviation of 5 minutes. A chemist has proposed a new mix of the paint designed to decrease the drying time. A sample of 25 test specimens of the new mix of the paint was chosen and the sample average drying time was found to be 72.3 minutes.

    a. Formulate your hypotheses and explain why you chose those hypotheses. (4 pts)
    b. Does the sample data support the chemist's claim when the significance level is α =.01? (4 pts)
    c. What is the P-value for the test in part (b)? (4 pts)

END