# MATH 333, Final Examination, Fall 2001

**December 19, 2001** **NJIT**

| **Name:** | **SSN:** | **Section #** |
|---|---|---|

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**Must show all work to receive full credit.**

**I pledge my honor that I have abided by the Honor System.** ___________________________

1. **(a)** X is a continuous random variable with probability density function,

$$f(x) = (\tfrac{1}{2})e^{-cx}, \quad 0 < x < \infty, \qquad c > 0.$$

(i) **(3 points)** Compute the value of c.

(ii) (**4 points)** Calculate E(X).

**(b) (8 points)** If X and Y are independent random variables with the same distribution (i.e., X and Y form a random sample of size two), such that E(X) = 2, V(X) = 4, then find E(3X-2Y) and V(3X-2Y).

**2.** **(a)** The time in hours during which an electrical generator is operational is a random variable that follows an exponential distribution. If the average operational time is 160 hours, what is the probability that a generator of this type will be operational for :

(i) **(3 points)** less than 40 hours ?

(ii) **(3 points)** between 60 and 160 hours ?

(See Next Page)

**2.** **(b)** The acceptability of a capillary tube for a freezer application is found by measuring the pressure drop in pounds per square inch between the two ends of the tube. The pressures obtained from a manufacturing process of capillary tubes is found to be normally distributed with an average pressure of 130 pounds per square inch and a standard deviation of 4 pounds per square inch. Determine:

(i) **(3 points)** What percent of the pressures are below 121 pounds per square inch?

(ii) **(3 points)** What pressure value is exceeded by 75% of the pressure readings?

(iii) **(3 points)** What percent of the pressure readings lie between 121 and 134 pounds per square inch ?

**3. (a) ( 9 points)** It is estimated that 3.5% of the general population will live past their $90^{th}$ birthday (*Statistical Abstract of the United States*, $112^{th}$ Edition). In a graduating class of 753 high school seniors, what is the probability that 30 or more will live beyond their $90^{th}$ birthday.

***(See Next Page)***

**3. (b) (6 points)** A random sample of size 36 from a Normal distribution with mean $\mu$ and variance $\sigma^2 = 5.2$ , has a sample mean $\overline{X} = 11.54$.

Use this data to test the null hypothesis $H_0$: $\mu = 11$, vs. $H_1{:}\mu \neq 11$, at the level of significance $\alpha = 0.05$.

**4.** To compare two brands of cigarettes, Brand A and Brand B, for their tar content, a sample of 60 was inspected from Brand A and a sample of 40 from Brand B. The results of the tests are summarized as follows:

| | sample mean | sample variance |
|---|---|---|
| Brand A | $\bar{X} = 15.4$ | $s_1^2 = 3$ |
| Brand B | $\bar{Y} = 16.8$ | $s_2^2 = 4$ |

(i) **(8 points)** Do the two brands differ in their mean tar content? Test the appropriate hypotheses with a significance level of $\alpha = 0.05$?

(ii) **(2 points)** Find the P-value.

*(See Next Page)*

**5.** The Department of Transportation is studying gas prices. Ten stations were sampled. The average gas price(per gallon) is \$2.25. The standard deviation of the ten observed gas prices is \$1.50

(i) **( 8 points)** Test the hypothesis that average price per gallon has increased from the last year's level of \$1.65 per gallon, at the .05 level. State your conclusion in terms of the business problem.

(ii) **(7 points)** If the true standard deviation is known to be $\sigma$ = \$.50, test the hypothesis that the average price per gallon has increased from the previous level of \$1.65, at the .05 level. State your conclusion in terms of the business problem.

**6.** A recent survey of 500 adult Americans, 350 said they regularly use seatbelts when driving. Let ***p*** denote the true proportion of all adult Americans who regularly use seatbelts when driving.

(i) **( 8 points)** Compute an approximate 95% confidence interval for ***p.***

(ii) **( 7 points)** How large a sample (instead of the 500 surveyed, described above) would you need, if you wanted to be 95% certain of estimating ***p*** within a 3% of its true value ?

***(See Next Page)***

**7.** Do educational toys make a difference in age at which a child learns to read? To study this question, researchers designed an experiment in which one group of pre-school children spent 2 hours each day for six months in a room well supplied with "educational" toys such as alphabet blocks, puzzles, ABC readers, coloring books featuring letters, and so forth. A control group of children spent two hours a day for six months in a "non-educational" toy room. It was anticipated that IQ differences and home environment might be uncontrollable factors unless identical twins could be used. Therefore, six pairs of identical twins of preschool age were randomly selected. From each pair, one member was randomly selected to participate in the experimental group (i.e., educational toy room, Group B) and the other in the control group (i.e., non-educational toy room, Group A). For each twin the data item recorded is the age in months when the child began reading at the primary level, please see the table below. Do the results of this study indicate that there is a difference in reading age when a preschool child is exposed to educational toys?

| | READING AGE | |
|---|---|---|
| Twin Pair number | Control Group A | Experimental Group B |
| 1 | 58 | 60 |
| 2 | 61 | 64 |
| 3 | 53 | 52 |
| 4 | 60 | 65 |
| 5 | 71 | 75 |
| 6 | 62 | 63 |

(i) **(10 points)** State and test the relevant hypotheses using $\alpha = 0.05$.

(ii) **(5 points)** Compute the bounds for the P-value of the test.

**END**