MATH 333H: Honor's Probability & Statistics. Exam 2 (Fall 2003)

1. An airport limousine can accommodate up to three passengers on any trip. The company will accept a maximum of four reservations for a trip, and a passenger must have a reservation. From previous records, 15% of all those making reservations do not appear for the trip.

   (a) If four reservations are made, what is the probability that an individual with a reservation cannot be accommodated on the trip? (4 pts)

   If X = number of people with reservations that show up for the trip, then X is B(4,.85) and P(an individual with reservations not accommodated) = P(X= 4) = ${}_4C_4\,(.15)^0(.85)^4$ = .5220

   (b) If four reservations are made, what is the expected number of available places when the limousine departs? (12 pts)

   If Y = number of available places on the limo when it leaves, so Y = 0,1,2,3. AND Z is B(4,.85). E(Y) = 0*P(Z=4) + 0*P(Z=3) + 1*P(Z=2) + 2*P(Z=1)+3*P(Z=0) = 0.12200625

2. Let X be a random variable with mean 'a' and standard deviation 'b'.
   Compute: (5 pts each)

   (a) The expected value of X – a
   E(X - a) = E(X) - a = a - a = 0.

   (b) The variance of X / b.
   $\mathrm{Var}(X/b) = (1/b)^2\mathrm{Var}(X) = (1/b)^2b^2 = 1$

   (c) The expected value of X^2 [i.e. $E(X^2)$]
   $E(X^2) = \mathrm{Var}(X) + [E(X)]^2 = b^2 + a^2$.

3. A shipment of circuit boards arrives in a factory. The factory draws a random sample of 20 circuit boards and accepts the shipment if the number of defective circuit boards in the sample is no more than one. What is the probability of accepting the shipment, if the shipment of circuit boards contains 10% defectives? (10 pts each)

   Let X be the number of defective boards. Then $X \sim$ Bin(20, 0.1), and

   P(accepting a shipment) $= P\{X \le 1\} = \sum_{k=0}^{1}\binom{20}{k}(0.1)^k(0.9)^{20-k} =$

   $= .9^{20} + 20\times.1\times.9^{19} = .1216 + .2702 = .3918$

4. Let X be the number of automobile accidents on the whole length of Interstate 95 in one day. Suppose X follows a Poisson distribution with the mean of 4 accidents per day.
   (7 pts each)

   (a) What is the probability of more than one accident in a day?

   Let X = number of automobile accidents in one day
   Then X ~ Poisson (4)

$P(X > 1) = 1 – P(X = 0) – P(X = 1) = 1 – \exp(- 4) – [\text{Exp}(- 4)] . 4 = 1 – [\exp(- 4)] . 5$
$= 1 – 5 [.0183] = 1 – 0.0916 = 0.9084$

(b) What is the probability distribution of the time interval T between two successive accidents? [i.e. What is the density function f(t)]

Let T = time interval between two successive accidents
Then given that $X \sim$ Poisson (4), $T \sim \exp(4)$ and

$f(t) = 4 \exp(- 4t)$, for $t > 0$

(c) What is the probability that the time interval between two successive accidents is more than one day?

It can be shown that when T is distributed as an exponential distribution as in Part (b), the cdf of T is given by $F(t) = 1 – \exp(-4t)$.

Therefore, $P(T > 1) = 1 – F(1) = \exp(- 4) = 0.0183$

5. The diameter of a component follows a normal distribution with mean of 1 inch and standard deviation of 0.1 inches. A component is considered good if its diameter is between 0.75 and 1.15 inches, otherwise it is defective. (6 pts each)

(a) What percentage of the components will be good?

Let X = diameter of a component
Then $X \sim N(\text{mean} = 1, \text{std dev} = 0.1)$
P(a component is good) = $P(0.75 < X < 1.15) = P([0.75 – 1]/0.1 < X < [1.15 – 1]/0.1)$
Or P(a component is good) = $P(-2.5 < Z < 1.5) = P(Z < 1.5) – P(Z < -2.5)$
$= 0.9332 – 0.0062 = 0.927$
Therefore, percentage of good components is 92.7%.

If a component is defective because its diameter is too large, it can be reworked, but if it's too small it must be scrapped.

(b) What percent of the components can be reworked?
P(a component is reworked) = $P(X > 1.15) = P(Z > 1.5) = 1 – P(Z < 1.5) = 1 – 0.9332$ $= 0.0668$. The percentage of components reworked is 6.68%

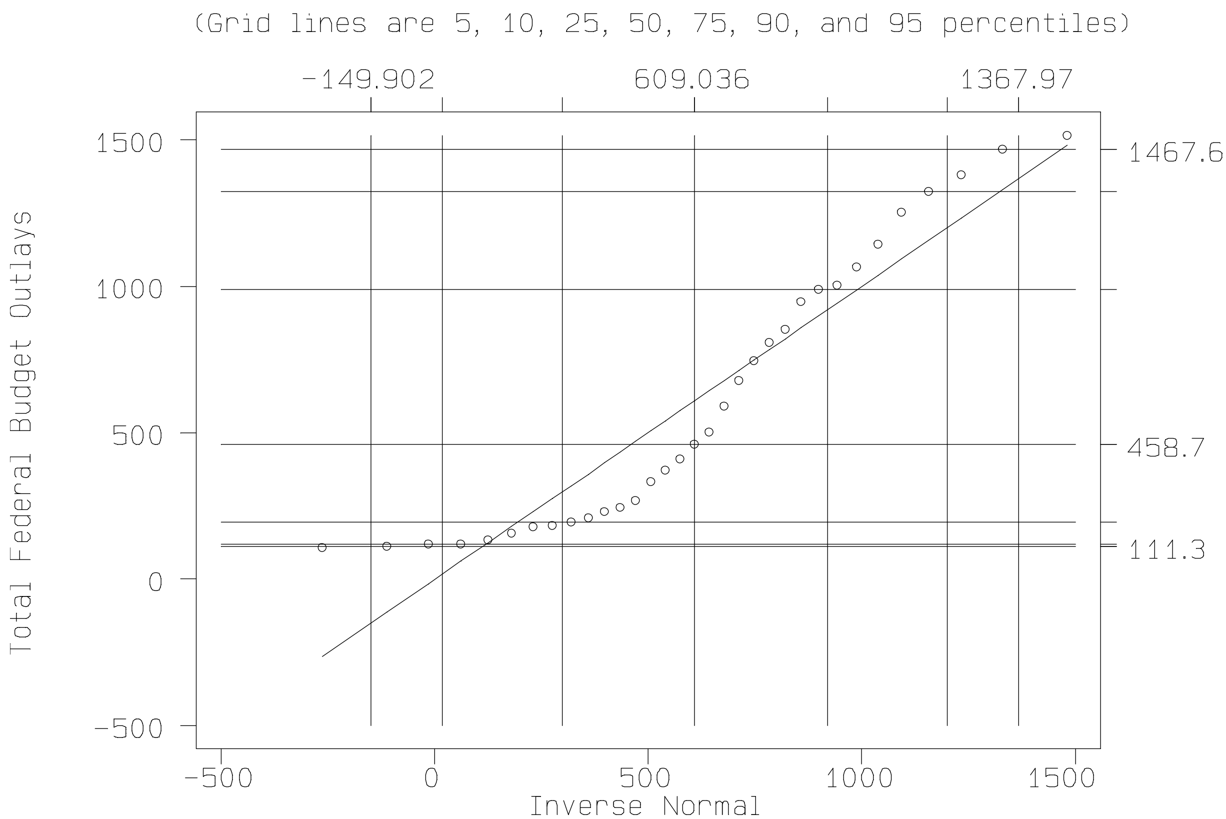
(c) If the acceptable range of the diameter is $[1 – b, 1 + b]$, for what value of b would 95% of all components have an acceptable diameter?

We want $P(1-b < X < 1+b) = 0.95$

By transforming X into a standard normal,
$P([(1 - b) – 1]/0.1 < [X – 1]/0.1 < [(1 + b) – 1]/0.1) = 0.95$
Or $P(- b/0.1 < Z < b/0.1) = 0.95$
We must have $b/0.1 = 1.96$, which gives $b = 0.196$

6. Suppose that 15 % of all steel shafts produced by a certain process are nonconforming but can be reworked (rather than having to be scrapped). Consider a random sample of 200 shafts, and let X denote the number among these that are nonconforming and can be reworked. What is the approximate probability that X is between 25 and 35 (inclusive of both end points)?(10 pts)
   Let p = probability a shaft is nonconforming = 0.15. The r. v. X~ Binomial with n = 200 and p = 0.15. Need to compute $P(25 \le X \le 35)$. Since np = 30 $\ge$ 10 and nq = 170 $\ge$ 10, we use normal approximation to Binomial. So, $P(25 \le X \le 35) = P(24.5 \le X \le 35.5) = P\{-5.5/\sqrt{30(.85)} \le Z \le 5.5/\sqrt{30(.85)}\} = P\{-1.09 \le Z \le 1.09) = \Phi(1.09) - \Phi(-1.09) =$ 0.8621-0.1379 = 0.7242.

7. Described below is the normal probability plot of thirty-three values from 1962 to1994 of the variable Total federal budget.



Is the variable Total federal budget normally distributed? Explain. (10 pts)

The normal probability plot for this variable, which is finance related variable, is not mount shape because the curve associated with the graph is S-shaped (which does not staying close to the 45 degree line) and sample size is more than 30. In fact, most "finance" related curves are positively skew and so is Total federal budget variable.