Fall 2016 MATH 333 Formula Sheet

1. Sample Mean: $\bar{x} = \dfrac{\sum_{i=1}^{n} x_i}{n}$
2. Sample variance: $s^2 = \dfrac{\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2}{n-1}$ $\quad or \quad$ $s^2 = \dfrac{\sum_{i=1}^{n} x_i^{\,2} - \dfrac{(\sum_{i=1}^{n} x_i)^2}{n}}{n-1}$
3. Addition rule: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
   $$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$
4. Multiplication Rule: P (A $\cap$ B) = P(B|A) P(A) = P(A|B) P(B)
5. Conditional Probability of an event B given an event A, $P(B \mid A) = \dfrac{P(A \cap B)}{P(A)}$ $\; for \;\; P(A) > 0$
6. Independence (Two events): Two events are independent if any one of the following equivalent statements is true: 1) P (A|B) = P (A) 2) P (B|A) = P (B) 3) P(A $\cap$ B) = P(A)P(B)
7. For positive integers n and r, with n ≥ r,
   a. Permutations rule: $P_r^n = \dfrac{n!}{(n-r)!}$
   b. Combination rule: $C_r^n = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$
8. a) Total Probability Rule (two events): For any events A & B, $P(B) = P(B \cap A) + P(B \cap A') = P(B \mid A)P(A) + P(B \mid A')P(A')$

   b) Total Probability Rule (for multiple events): Assume E_1, E_2, .., E_k are k mutually exclusive and exhaustive subsets. Then $P(B) = P(B \cap E_1) + P(B \cap E_2) + \ldots\ldots + P(B \cap E_k) = P(B \mid E_1)P(E_1) + P(B \mid E_2)P(E_2) + \ldots. + P(B \mid E_k)P(E_k)$
9. Bayes' Theorem: If E_1, E_2, … E_k are k mutually exclusive and exhaustive events and B is any event
   $$P(E_1 \mid B) = \frac{P(B \mid E_1)P(E_1)}{P(B \mid E_1)P(E_1) + P(B \mid E_2)P(E_2) + \ldots + P(B \mid E_k)P(E_k)} \quad for \;\; P(B) > 0$$
10. The cdf of a discrete random variable X, $F(x) = P(X \le x) = \sum_{x_i \le x} f(x_i)$
11. Mean or Expected value of a discrete random variable X: $\mu = E(X) = \sum_x x f(x)$
12. Variance and standard deviation of a discrete random variable X:
    $$\sigma^2 = V(X) = E(X-\mu)^2 = \sum_x (x-\mu)^2 f(x) = \sum_x x^2 f(x) - \mu^2 \qquad \sigma = \sqrt{\sigma^2}$$
13. Expected value of a function of a discrete random variable X: $E[h(X)] = \sum_x h(x) f(x)$
14. Discrete Uniform Distribution on the consecutive set of integers a, a+1, a+2, …,b for a ≤ b:
    $$f(x) = \frac{1}{(b-a+1)}, \qquad \mu = E(X) = \frac{b+a}{2}, \qquad \sigma^2 = \frac{(b-a+1)^2 - 1}{12}$$
15. Binomial Distribution: $f(x) = C_x^n p^x (1-p)^{n-x}$, $\quad x = 0, 1, .., n \quad$ where $C_x^n = \dfrac{n!}{x!(n-x)!}$

    For Binomial random variable X, X ~ Bin(n, p): $\mu = E(X) = np$, $\sigma^2 = V(X) = np(1-p)$

16. Geometric Distribution: $f(x) = (1-p)^{x-1} p \qquad x = 1, 2, 3.......$

For a Geometric random variable X with parameter p, $\mu = E(X) = \dfrac{1}{p}, \qquad \sigma^2 = V(X) = \dfrac{(1-p)}{p^2}$

17. Poisson Distribution: $f(x) = \dfrac{e^{-\lambda}\lambda^x}{x!} \qquad x = 0, 1, 2, ...$ and $\lambda > 0$

For a Poisson random variable X, X ~ Poisson(λ), $\mu = E(X) = \lambda, \quad \sigma^2 = V(X) = \lambda$

18. The cdf of a continuous random variable X: $F(x) = P(X \le x) = \int_{-\infty}^{x} f(u)du \qquad for\ -\infty < x < \infty$

19. Mean or Expected value of a continuous random variable X: $\mu = E(X) = \int_{-\infty}^{\infty} x f(x)dx$

20. Variance and standard deviation of a continuous random variable X:

$$\sigma^2 = V(X) = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx = \int_{-\infty}^{\infty} x^2 f(x)dx - \mu^2 \qquad \sigma = \sqrt{\sigma^2}$$

21. Expected value of a function of a continuous random variable X, $E[h(X)] = \int_{-\infty}^{\infty} h(x) f(x)dx$

22. Continuous Uniform Distribution: $f(x) = \dfrac{1}{(b-a)}, \qquad a \le x \le b$

For a continuous uniform random variable X over $a \le x \le b$, $\mu = E(X) = \dfrac{(a+b)}{2}$, $\sigma^2 = V(X) = \dfrac{(b-a)^2}{12}$

23. Exponential Distribution: $f(x) = \lambda e^{-\lambda x}$ for $0 \le x < \infty$, $\lambda > 0$; $F(x) = P(X \le x) = 1 - e^{-\lambda x}$, $x \ge 0$

For exponential random variable X with parameter λ, $\mu = E(X) = \dfrac{1}{\lambda}$, $\sigma^2 = V(X) = \dfrac{1}{\lambda^2}$

24. Lack of Memory Property: For an exponential random variable X, $P(X < t_1 + t_2 \mid X > t_1) = P(X < t_2)$

25. The cumulative distribution function of a standard normal random variable: $\Phi(z) = P(Z \le z)$

26. The standard normal random variable $Z = \dfrac{X-\mu}{\sigma}$ with $E(Z) = 0$ and $V(Z) = 1$

27. Normal approximation to the Binomial Distribution:

If X ~ Bin(n, p), $Z = \dfrac{X - np}{\sqrt{np(1-p)}}$ is approximately a standard normal random variable

Continuity correction: $P(X \le x) \cong P\left(Z \le \dfrac{x + 0.5 - np}{\sqrt{np(1-p)}}\right)$ and $P(x \le X) \cong P\left(\dfrac{x - 0.5 - np}{\sqrt{np(1-p)}} \le Z\right)$

28. Normal approximation to the Poisson Distribution:

If X ~ Poisson(λ), $Z = \dfrac{X-\lambda}{\sqrt{\lambda}}$ is approximately a standard normal random variable

Continuity correction: $P(X \le x) \cong P\left(Z \le \dfrac{x + 0.5 - \lambda}{\sqrt{\lambda}}\right)$ and $P(x \le X) \cong P\left(\dfrac{x - 0.5 - \lambda}{\sqrt{\lambda}} \le Z\right)$

29. Z score for sample mean $\overline{X}$ for a sample of size n is $Z = \dfrac{\overline{X} - \mu}{\sigma / \sqrt{n}}$

30. 100 (1- α)% Confidence Interval (CI): (For one sided confidence bound, use appropriate lower or upper confidence limit and replace $z_{\alpha/2}$ with z_α, $t_{\alpha/2,n-1}$ with $t_{\alpha,n-1}$, $\chi^2_{\alpha/2,n-1}$ with $\chi^2_{\alpha,n-1}$, and $\chi^2_{1-\alpha/2,n-1}$ with $\chi^2_{1-\alpha,n-1}$)

$\bar{x} - z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}} \le \mu \le \bar{x} + z_{\alpha/2}\cdot\frac{\sigma}{\sqrt{n}}$
$\bar{x} - t_{\alpha/2,n-1}\cdot\frac{s}{\sqrt{n}} \le \mu \le \bar{x} + t_{\alpha/2,n-1}\cdot\frac{s}{\sqrt{n}}$
$\frac{(n-1)S^2}{\chi^2_{\alpha/2,n-1}} \le \sigma^2 \le \frac{(n-1)S^2}{\chi^2_{1-\alpha/2,n-1}}$
$\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \le p \le \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$
$\bar{x}_1 - \bar{x}_2 - z_{\alpha/2}\cdot\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}} \le \mu_1 - \mu_2 \le \bar{x}_1 - \bar{x}_2 + z_{\alpha/2}\cdot\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}$
$\bar{d} - t_{\alpha/2,n-1}\cdot\frac{s_D}{\sqrt{n}} \le \mu_D \le \bar{d} + t_{\alpha/2,n-1}\cdot\frac{s_D}{\sqrt{n}}$

31. Sample size determination:

$n = \left(\frac{z_{\alpha/2}\sigma}{E}\right)^2$
$n = \left(\frac{z_{\alpha/2}}{E}\right)^2 p(1-p)$
$n = \frac{(z_{\alpha/2}+z_\beta)^2\sigma^2}{\delta^2}$ where $\delta = \mu - \mu_0$
$n = \frac{(z_\alpha+z_\beta)^2\sigma^2}{\delta^2}$ where $\delta = \mu - \mu_0$

32. Single Sample Hypothesis Testing

Test Statistic
$Z_0 = \frac{\bar{X} - \mu_0}{\sigma/\sqrt{n}}$
$T_0 = \frac{\bar{X} - \mu_0}{S/\sqrt{n}}$
$Z_0 = \frac{X - np_0}{\sqrt{np_0(1-p_0)}}$ or $Z_0 = \frac{\hat{P} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}}$

33. Probability of Type II error for the test on the mean, variance known

$\beta = \Phi\left(z_{\alpha/2} - \frac{\delta\sqrt{n}}{\sigma}\right) - \Phi\left(-z_{\alpha/2} - \frac{\delta\sqrt{n}}{\sigma}\right)$	$\beta = 1 - \Phi\left(-z_\alpha - \frac{\delta\sqrt{n}}{\sigma}\right)$	$\beta = \Phi\left(z_\alpha - \frac{\delta\sqrt{n}}{\sigma}\right)$

34. Two Sample Hypothesis Testing

Test Statistic	
$Z_0 = \frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}}$	$T_0 = \frac{\bar{D} - \Delta_0}{S_D/\sqrt{n}}$

35. For the estimated regression line $\hat{\beta}_1 = \frac{\sum xy - \frac{\sum x \sum y}{n}}{\sum x^2 - \frac{(\sum x)^2}{n}}$ *or* $\hat{\beta}_1 = \frac{n\sum xy - \sum x \sum y}{n\sum x^2 - (\sum x)^2}$ $\hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X}$