MATH 333A: Probability & Statistics. **Final Examination** **(Fall 2006)**

December 20, 2006 **NJIT**

Score	
#1	
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#4	
#5	
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Total	

Name: **SSN:** **Section #**

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➔ **Must show all steps for each problem to receive full credit.**

I pledge my honor that I have abided by the Honor System. ______________________
(Signature)

1. At a certain gas station, 40% of the customers buy regular gas, 35% buy plus gas, and 25% buy premium gas. Of those customers buying regular gas, 30% fill their tanks. Of those customers buying plus gas, 40% fill their tanks. Of those customers buying premium gas, 70% fill their tanks. (4 points each)

   a. What is the probability that the next customer will buy plus gas and fill the tank?
   b. What is the probability that the next customer fills the tank?
   c. If the next customer fills the tank, what is the probability that regular gas is bought?
   d. Let G denote the event that the next customer buys plus gas and F denote the event that the next customer fills the tank. Are F and G independent events? Provide justification for your answer.

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2. The following summary was prepared when a person was testing a claim that the population mean drying time for a paint has decreased from 120 minutes to a lower value:

   Sample size = 64
   Sample mean = 110 minutes
   Sample standard deviation = 39 minutes
   Rejection region for the null hpothesis: Sample mean less than or equal to 112 minutes.

   a. State the null and the alternative hypotheses?
      For the given data, what is your inference about the claim stated above? (4 points)
   b. What is α, the Type I error? (4 points)
   c. What is the Type II error, if the true population mean is 115 minutes? (5 points)
   d. What is the minimum sample size to reduce the type II error to 0.1, when the true population mean is 115 and the Type I error is kept at the value found in Part b? (4 points)

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3. The reaction time X, measured in seconds, to a certain stimulus has the following probability density function: (4 points each)

$$f(x) = kx^{-2}, \qquad \text{if} \quad 1 < x < 3.$$

a. Find the value of the constant k.
b. Obtain the cumulative distribution function of X.
c. Find the range that contains the middle 50% values of the random variable X.
d. Find the mean, median, and standard deviation of the random variable X.

4. The manufacturer of a certain brand of car battery claims that its batteries will last 3 years on an average, with a standard deviation of one year. Assume that the battery lifetime (T), expressed in years, has a Normal distribution. (4 points each)

   a. Compute the probability that the lifetime of a battery will be negative. Based on this probability, do you believe that Normal distribution is **a** reasonable **model** for the battery lifetime?

   b. Find the warranty period (t_0) such that only 5% of all batteries of this brand go bad before the warranty period expires**.**

   c. Suppose a new battery costs $100 and refund on batteries that go bad before t_0, are prorated as described by the "refund function" R(T),

   $$R(T) = \$\ 100\ (1 - \frac{T}{t_0}), \quad \text{if } 0 < T < t_0$$

   (where t_0 is the warranty period found in Part b above).

   What proportion of batteries sold do not get a refund?

   d. What proportion of batteries sold get a refund and the refund exceeds $30?

5. Consider the brand of batteries as described in Problem # 4, whose lifetimes (unit = year) can be modeled by a Normal distribution $N(\mu, \sigma^2)$, where the true population mean μ and standard deviation σ are both unknown. The observed lifetimes (in years) of 5 randomly chosen batteries were recorded as follows:

   1.9, 2.4, 4.2, 6.5, 3.5

   a. At 5% level of significance, is there support for the claim that the true standard deviation of battery lifetimes is more than 1 year? (5 points)

   b. Use the sample data to compute the P-value for the hypothesis in Part a. Hint: Use chi-square table. (5 points)

   c. Use the sample data to get a 95% confidence interval for the population standard deviation σ. (5 points)

6. Based on past experience, the mean lifetime, μ, of bulbs in overhead projectors is 750 hours and $\sigma = 60$. A manufacturer is claiming that $\mu > 750$. A random sample of 25 bulb-lifetime is collected and it yields a sample mean of 780 hours. It is reasonable to assume that the bulb-lifetime follows a Normal distribution. (5 points each)

   a. Formulate and test your hypotheses at $\alpha = 0.01$.
   b. Compute the P-value of your test in Part a.
   c. Find the Type II error, β, when $\mu = 770$.
   d. What is the minimum sample size needed, if it is desired to keep β at 0.05, when $\mu = 770$?

END