Week 13(4/9 - 4/13) Chapter 9: #30, #42, #44, #56, #64, #68

9-42 a) 1) The parameter of interest is the true mean hole diameter, μ.

2) $H_0 : \mu = 1.50$

3) $H_1 : \mu \neq 1.50$

4) $\alpha = 0.01$

5) $z_0 = \dfrac{\bar{x} - \mu}{\sigma / \sqrt{n}}$

6) Reject H_0 if $z_0 < -z_{\alpha/2}$ where $-z_{0.005} = -2.58$ or $z_0 > z_{\alpha/2}$ where $z_{0.005} = 2.58$

7) $\bar{x} = 1.4975$, $\sigma = 0.01$

$$z_0 = \frac{1.4975 - 1.50}{0.01/\sqrt{25}} = -1.25$$

8) Since $-2.58 < -1.25 < 2.58$, do not reject the null hypothesis and conclude the true mean hole diameter is not significantly different from 1.5 in. at α = 0.01.

b) p-value$=2(1-\Phi(|Z_0|))=2(1-\Phi(1.25)) \cong 0.21$

c)

$$\beta = \Phi\left(z_{0.005} - \frac{\delta\sqrt{n}}{\sigma}\right) - \Phi\left(-z_{0.005} - \frac{\delta\sqrt{n}}{\sigma}\right)$$

$$= \Phi\left(2.58 - \frac{(1.495-1.5)\sqrt{25}}{0.01}\right) - \Phi\left(-2.58 - \frac{(1.495-1.5)\sqrt{25}}{0.01}\right)$$

$= \Phi(5.08) - \Phi(-0.08) = 1 - .46812 = 0.53188$

power$=1-\beta=0.46812$.

d) Set $\beta = 1 - 0.90 = 0.10$

$$n = \frac{(z_{\alpha/2} + z_\beta)^2 \sigma^2}{\delta^2} = \frac{(z_{0.005} + z_{0.10})^2 \sigma^2}{(1.495 - 1.50)^2} \cong \frac{(2.58 + 1.29)^2 (0.01)^2}{(-0.005)^2} = 59.908,$$

$n \cong 60$.

e) For $\alpha = 0.01$, $z_{\alpha/2} = z_{0.005} = 2.58$

$$\bar{x} - z_{0.005}\left(\frac{\sigma}{\sqrt{n}}\right) \le \mu \le \bar{x} + z_{0.005}\left(\frac{\sigma}{\sqrt{n}}\right)$$

$$1.4975 - 2.58\left(\frac{0.01}{\sqrt{25}}\right) \le \mu \le 1.4975 + 2.58\left(\frac{0.01}{\sqrt{25}}\right)$$

$1.4923 \le \mu \le 1.5027$

The confidence interval constructed contains the value 1.5, thus the true mean hole diameter could possibly be 1.5 in. using a 99% level of confidence. Since a two-sided 99% confidence interval is equivalent to a two-sided hypothesis test at $\alpha = 0.01$, the conclusions necessarily must be consistent.

9-44 a) α=0.01, n=20, the critical values are ± 2.861
b) α=0.05, n=12, the critical values are ± 2.201
c) α=0.1, n=15, the critical values are ± 1.761

9-56 a)
1) The parameter of interest is the true mean sodium content, μ.

2) $H_0 : \mu = 300$
3) $H_1 : \mu > 300$
4) $\alpha = 0.05$
5) $t_0 = \dfrac{\bar{x} - \mu}{s/\sqrt{n}}$
6) Reject H_0 if $t_0 > t_{\alpha,n-1}$ where $t_{\alpha,n-1} = 1.943$
7) $\bar{x} = 315$, s = 16 n=7

$$t_0 = \frac{315 - 300}{16/\sqrt{7}} = 2.48$$

8) Since 2.48>1.943, reject the null hypothesis and conclude that there is sufficient evidence that the leg strength exceeds 300 watts at $\alpha = 0.05$. The p-value is between .01 and .025

b) $d = \dfrac{\delta}{\sigma} = \dfrac{|\mu - \mu_0|}{\sigma} = \dfrac{|305 - 300|}{16} = 0.3125$
Using the OC curve, Chart VII g) for $\alpha = 0.05$, d = 0.3125, and n = 7, $\beta \cong 0.9$ and power = 1–0.9 = 0.1.

c) if 1-β>0.9 then β<0.1 and n is approximately 100

d) Lower confidence bound is $\bar{x} - t_{\alpha,n-1}\left(\dfrac{s}{\sqrt{n}}\right) = 303.2$

because 300< 303.2 reject the null hypothesis

9-64 a) α=0.01, n=20, from table V we find $\chi^2_{\alpha,n-1} = 36.19$
b) α=0.05, n=12, from table V we find $\chi^2_{\alpha,n-1} = 19.68$
c) α=0.10, n=15, from table V we find $\chi^2_{\alpha,n-1} = 21.06$

9-68 a) 0.1<*P*-value<0.5
b) 0.1<*P*-value<0.5
c) 0.99<*P*-value<0.995