Section 9-5 (Week 14 Chapter 9)

9-76 a)

1) The parameter of interest is the true fraction of satisfied customers.
2) $H_0 : p = 0.9$
3) $H_1 : p \neq 0.9$
4) $\alpha = 0.05$
5) $z_0 = \dfrac{x - np_0}{\sqrt{np_0(1-p_0)}}$ or $z_0 = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}}$; Either approach will yield the same conclusion
6) Reject H_0 if $z_0 < -z_{\alpha/2}$ where $-z_{\alpha/2} = -z_{0.025} = -1.96$ or $z_0 > z_{\alpha/2}$ where $z_{\alpha/2} = z_{0.025} = 1.96$
7) $x = 850$ $n = 1000$ $\hat{p} = \dfrac{850}{1000} = 0.85$

$$z_0 = \frac{x - np_0}{\sqrt{np_0(1-p_0)}} = \frac{850 - 1000(0.9)}{\sqrt{1000(0.9)(0.1)}} = -5.27$$

8) Because -5.27<-1.96 reject the null hypothesis and conclude the true fraction of satisfied customers is significantly different from 0.9 at $\alpha = 0.05$.

The P-value: $2(1-\Phi(5.27)) \leq 2(1-1) \approx 0$

b) The 95% confidence interval for the fraction of surveyed customers is:

$$\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \leq p \leq \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$.85 - 1.96\sqrt{\frac{0.85(0.15)}{1000}} \leq p \leq .85 + 1.96\sqrt{\frac{0.85(0.15)}{1000}}$$

$$0.827 \leq p \leq 0.87$$

Because 0.9 is not included in the confidence interval, we reject the null hypothesis at $\alpha = 0.05$.

9-78 a)

1) The parameter of interest is the true fraction defective integrated circuits
2) $H_0 : p = 0.05$
3) $H_1 : p \neq 0.05$
4) $\alpha = 0.05$

5) $z_0 = \dfrac{x - np_0}{\sqrt{np_0(1-p_0)}}$ or $z_0 = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}}$; Either approach will yield the same conclusion

6) Reject H_0 if $z_0 < -z_{\alpha/2}$ where $-z_{\alpha/2} = -z_{0.025} = -1.96$ or $z_0 > z_{\alpha/2}$ where $z_{\alpha/2} = z_{0.025} = 1.96$

7) x = 13 n = 300 $\hat{p} = \dfrac{13}{300} = 0.043$

$$z_0 = \frac{x - np_0}{\sqrt{np_0(1-p_0)}} = \frac{13 - 300(0.05)}{\sqrt{300(0.05)(0.95)}} = -0.53$$

8) Since $-0.53 > -1.65$, do not reject null hypothesis and conclude the true fraction of defective integrated circuits is not significantly less than 0.05, at α = 0.05.

The P-value: 2(1-Φ(0.53))=2(1-0.70194)=0.59612

b) The 95% confidence interval is:

$$\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \le p \le \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$.043 - 1.96\sqrt{\frac{0.043(0.957)}{300}} \le p \le 043 + 1.96\sqrt{\frac{0.043(0.957)}{300}}$$

$$0.02004 \le p \le 0.065$$

Since $0.02004 \le p = 0.05 \le 0.065$, then we fail to reject the null hypothesis.

9-82 a) 1) The parameter of interest is the true proportion of engineering students planning graduate studies

2) $H_0 : p = 0.50$

3) $H_1 : p \neq 0.50$

4) $\alpha = 0.05$

5) $z_0 = \dfrac{x - np_0}{\sqrt{np_0(1-p_0)}}$ or $z_0 = \dfrac{\hat{p} - p_0}{\sqrt{\dfrac{p_0(1-p_0)}{n}}}$; Either approach will yield the same conclusion

6) Reject H_0 if $z_0 < -z_{\alpha/2}$ where $-z_{\alpha/2} = -z_{0.025} = -1.96$ or $z_0 > z_{\alpha/2}$ where $z_{\alpha/2} = z_{0.025} = 1.96$

7) x = 117 n = 484 $\hat{p} = \dfrac{117}{484} = 0.2423$

$$z_0 = \frac{x - np_0}{\sqrt{np_0(1-p_0)}} = \frac{117 - 484(0.5)}{\sqrt{484(0.5)(0.5)}} = -11.36$$

8) Since $-11.36 > -1.65$, reject the null hypothesis and conclude the true proportion of engineering students planning graduate studies is significantly different from 0.5, at α = 0.05.

P-value =2[1 − Φ(11.36)] ≅ 0

b.) $\hat{p} = \dfrac{117}{484} = 0.242$

$$\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \le p \le \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$0.242 - 1.96\sqrt{\frac{0.242(0.758)}{484}} \le p \le 0.242 - 1.96\sqrt{\frac{0.242(0.758)}{484}}$$

$$0.204 \le p \le 0.280$$

Since the 95% confidence interval does not contain the value 0.5, then conclude that the true proportion of engineering students planning graduate studies is significantly different from 0.5.