

10-2 a)

1) The parameter of interest is the difference in means $\mu_1 - \mu_2$. Note that $\Delta_0 = 0$.

2) $H_0: \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1: \mu_1 - \mu_2 < 0$ or $\mu_1 < \mu_2$

4) $\alpha = 0.05$

5) The test statistic is

$$z_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

6) Reject H_0 if $z_0 < -z_\alpha = -1.645$

7) $\bar{x}_1 = 14.2$ $\bar{x}_2 = 19.7$

$\sigma_1 = 10$ $\sigma_2 = 5$

$n_1 = 10$ $n_2 = 15$

$$z_0 = \frac{(14.2 - 19.7)}{\sqrt{\frac{(10)^2}{10} + \frac{(5)^2}{15}}} = -1.61$$

8) Because $-1.61 > -1.645$, do not reject the null hypothesis. There is not sufficient evidence to conclude that the two means differ at $\alpha = 0.05$.

$$P\text{-value} = \Phi(-1.61) = 0.0537$$

$$\text{b) } \mu_1 - \mu_2 \leq (\bar{x}_1 - \bar{x}_2) + z_\alpha \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$\mu_1 - \mu_2 \leq (14.2 - 19.7) + 1.645 \sqrt{\frac{(10)^2}{10} + \frac{(5)^2}{15}}$$

$$\mu_1 - \mu_2 \leq 0.12$$

With 95% confidence, we believe the true difference in the means is less than 0.12. Because 0 is contained in this interval, we can conclude there is no significant difference between the means. We fail to reject the null hypothesis.

c)

$$\begin{aligned} \beta &= 1 - \Phi\left(-z_\alpha - \frac{\delta}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}\right) \\ &= 1 - \Phi\left(-1.65 - \frac{-4}{\sqrt{\frac{(10)^2}{10} + \frac{(5)^2}{15}}}\right) \\ &= 1 - \Phi(-0.4789) = 1 - 0.316 = 0.684 \\ \text{Power} &= 1 - 0.684 = 0.316 \end{aligned}$$

Section 10-3

10-10 a) 1) The parameter of interest is the difference in mean, $\mu_1 - \mu_2$

2) $H_0 : \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1 : \mu_1 - \mu_2 \neq 0$ or $\mu_1 \neq \mu_2$

4) $\alpha = 0.05$

5) The test statistic is

$$t_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

6) Reject the null hypothesis if $t_0 < -t_{\alpha/2, n_1+n_2-2}$ where $-t_{0.025, 28} = -2.048$ or $t_0 > t_{\alpha/2, n_1+n_2-2}$ where

$$t_{0.025, 28} = 2.048$$

7) $\bar{x}_1 = 4.7$ $\bar{x}_2 = 7.8$

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

$$= \sqrt{\frac{14(4) + 14(6.25)}{28}} = 2.26$$

$s_1^2 = 4$ $s_2^2 = 6.25$

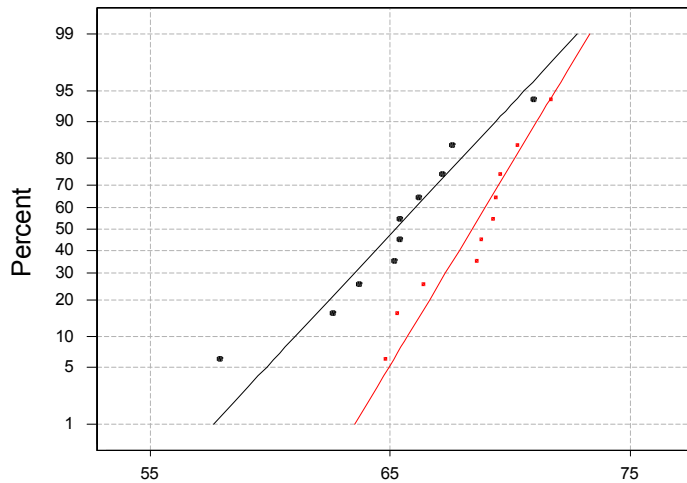
$n_1 = 15$ $n_2 = 15$

$$t_0 = \frac{(4.7 - 7.8)}{2.26 \sqrt{\frac{1}{15} + \frac{1}{15}}} = -3.75$$

8) Because $-3.75 < -2.048$, reject the null hypothesis at $\alpha = 0.05$.

$P\text{-value} = 2P(t > 3.75) < 2(0.0005)$, $P\text{-value} < .001$

b) 95% confidence interval: $t_{0.025, 28} = 2.048$



$$(\bar{x}_1 - \bar{x}_2) - t_{\alpha/2, n_1+n_2-2}(s_p) \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \leq \mu_1 - \mu_2 \leq (\bar{x}_1 - \bar{x}_2) + t_{\alpha/2, n_1+n_2-2}(s_p) \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$$

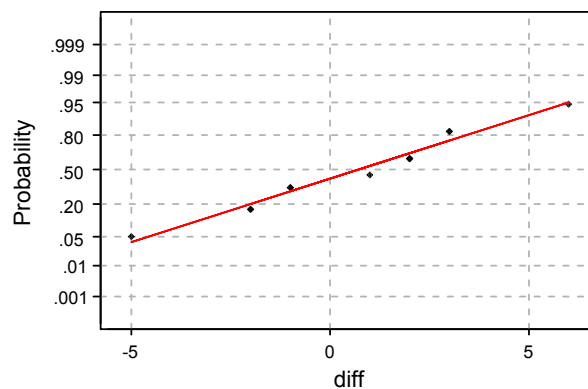
$$(4.7 - 7.8) - 2.048(2.26) \sqrt{\frac{1}{15} + \frac{1}{15}} \leq \mu_1 - \mu_2 \leq (4.7 - 7.8) + 2.048(2.26) \sqrt{\frac{1}{15} + \frac{1}{15}}$$

$$-4.79 \leq \mu_1 - \mu_2 \leq -1.41$$

Because zero is not contained in this interval, we are 95% confident that the means are different.

- 10-32 a) According to the normal probability plots, the assumption of normality does not appear to be violated since the data fall approximately along a straight line.

Normal Probability Plot



Average: 0.666667
StDev: 2.96444
N: 12

Anderson-Darling Normality Test
A-Squared: 0.315
P-Value: 0.502

- b) $\bar{d} = 0.667$ $s_d = 2.964$, $n = 12$
95% confidence interval:

$$\bar{d} - t_{\alpha/2, n-1} \left(\frac{s_d}{\sqrt{n}} \right) \leq \mu_d \leq \bar{d} + t_{\alpha/2, n-1} \left(\frac{s_d}{\sqrt{n}} \right)$$

$$0.667 - 2.201 \left(\frac{2.964}{\sqrt{12}} \right) \leq \mu_d \leq 0.667 + 2.201 \left(\frac{2.964}{\sqrt{12}} \right)$$

$$-1.216 \leq \mu_d \leq 2.55$$

Because zero is contained within this interval, there is no significant indication that one design language is preferable at a 5% significance level

- 10-36 a) 1) The parameter of interest is the mean difference in impurity level, μ_d
where $d_i = \text{Test 1} - \text{Test 2}$.

- 2) $H_0 : \mu_d = 0$
- 3) $H_1 : \mu_d \neq 0$
- 4) $\alpha = 0.01$
- 5) The test statistic is

$$t_0 = \frac{\bar{d}}{s_d / \sqrt{n}}$$

- 6) Reject the null hypothesis if $t_0 < -t_{0.005, 7}$ or $t_0 > t_{0.005, 7}$ where $t_{0.005, 7} = 3.499$

- 7) $\bar{d} = -0.2125$

$$s_d = 0.1727$$

$$n = 8$$

$$t_0 = \frac{-0.2125}{0.1727/\sqrt{8}} = -3.48$$

8) Because $-3.499 < -3.48 < 3.499$ cannot reject the null hypothesis. There is not sufficient evidence that the tests give different mean impurity levels at $\alpha = 0.01$.

b) 1) The parameter of interest is the mean difference in impurity level, μ_d
where $d_i = \text{Test 1} - \text{Test 2}$.

2) $H_0 : \mu_d + 0.1 = 0$

3) $H_1 : \mu_d + 0.10 < 0$

4) $\alpha = 0.05$

5) The test statistic is

$$t_0 = \frac{\bar{d} + 0.1}{s_d / \sqrt{n}}$$

6) Reject the null hypothesis if $t_0 < -t_{0.05,7}$ where $t_{0.05,7} = 1.895$

7) $\bar{d} = -0.2125$

$$s_d = 0.1727$$

$$n = 8$$

$$t_0 = \frac{-0.2125 + 0.1}{0.1727/\sqrt{8}} = -1.8424$$

8) Because $-1.895 < -1.8424$ we fail to reject the null at the 0.05 level of significance.