10-2  a)

1) The parameter of interest is the difference in means $\mu_1 - \mu_2$. Note that $\Delta_0 = 0$.

2) $H_0 : \mu_1 - \mu_2 = 0$ or $\mu_1 = \mu_2$

3) $H_1 : \mu_1 - \mu_2 < 0$ or $\mu_1 < \mu_2$

4) $\alpha = 0.05$

5) The test statistic is

$$z_0 = \frac{(\bar{x}_1 - \bar{x}_2) - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

6) Reject $H_0$ if $z_0 < -z_{\alpha} = -1.645$

7) $\bar{x}_1 = 14.2$ $\bar{x}_2 = 19.7$

$\sigma_1 = 10$ $\sigma_2 = 5$

$n_1 = 10$ $n_2 = 15$

$$z_0 = \frac{(14.2 - 19.7)}{\sqrt{\frac{(10)^2}{10} + \frac{(5)^2}{15}}} = -1.61$$

8) Because $-1.61 > -1.645$, do not reject the null hypothesis. There is not sufficient evidence to conclude that the two means differ at $\alpha = 0.05$.

$P$-value = $\Phi(-1.61)$ = 0.0537

b) $\mu_1 - \mu_2 \leq (\bar{x}_1 - \bar{x}_2) + z_{\alpha} \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$

$\mu_1 - \mu_2 \leq (14.2 - 19.7) + 1.645 \sqrt{\frac{(10)^2}{10} + \frac{(5)^2}{15}}$

$\mu_1 - \mu_2 \leq 0.12$

With 95% confidence, we believe the true difference in the means is less than 0.12. Because 0 is contained in this interval, we can conclude there is no significant difference between the means. We fail to reject the null hypothesis.

c)

$$\beta = 1 - \Phi \left( -z_{\alpha} - \frac{\delta}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \right)$$

$$= 1 - \Phi \left( -1.65 - \frac{4}{\sqrt{\frac{(10)^2}{10} + \frac{(5)^2}{15}}} \right)$$

$$= 1 - \Phi(-0.4789) = 1 - 0.316 = 0.684$$

Power = 1 - 0.684 = 0.316
10-3

10-10  a) 1) The parameter of interest is the difference in mean, \( \mu_1 - \mu_2 \)
2) \( H_0 : \mu_1 - \mu_2 = 0 \) or \( \mu_1 = \mu_2 \)
3) \( H_1 : \mu_1 - \mu_2 \neq 0 \) or \( \mu_1 \neq \mu_2 \)
4) \( \alpha = 0.05 \)
5) The test statistic is
\[
t_0 = \frac{\overline{x}_1 - \overline{x}_2 - \Delta_0}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}
\]
6) Reject the null hypothesis if \( t_0 < -t_{\alpha/2, n_1 + n_2 - 2} \) where \( t_{0.025, 28} = -2.048 \) or \( t_0 > t_{\alpha/2, n_1 + n_2 - 2} \) where \( t_{0.025, 28} = 2.048 \)
7) \( \overline{x}_1 = 4.7 \quad \overline{x}_2 = 7.8 \)
\[
s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}} = \sqrt{\frac{14(4) + 14(6.25)}{28}} = 2.26
\]
\[
n_1 = 15 \quad n_2 = 15
\]
\[
t_0 = \frac{(4.7 - 7.8)}{2.26 \sqrt{\frac{1}{15} + \frac{1}{15}}} = -3.75
\]
8) Because \(-3.75 < -2.048\), reject the null hypothesis at \( \alpha = 0.05 \).
\[
P\text{-value} = 2P(t > 3.75) < 2(0.0005), \quad P\text{-value} < .001
\]
b) 95% confidence interval: \( t_{0.025, 28} = 2.048 \)
\[(\bar{x}_1 - \bar{x}_2) - t_{\alpha/2, n_1 + n_2 - 2} (s_p) \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \leq \mu_1 - \mu_2 \leq (\bar{x}_1 - \bar{x}_2) + t_{\alpha/2, n_1 + n_2 - 2} (s_p) \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}\]

\[(4.7 - 7.8) - 2.048(2.26) \sqrt{\frac{1}{15} + \frac{1}{15}} \leq \mu_1 - \mu_2 \leq (4.7 - 7.8) + 2.048(2.26) \sqrt{\frac{1}{15} + \frac{1}{15}}\]

\[-4.79 \leq \mu_1 - \mu_2 \leq -1.41\]

Because zero is not contained in this interval, we are 95% confident that the means are different.

10-32

a) According to the normal probability plots, the assumption of normality does not appear to be violated since the data fall approximately along a straight line.

Normal Probability Plot

![Normal Probability Plot](image)

b) \(\bar{d} = 0.667\)  \(s_d = 2.964, n = 12\)

95% confidence interval:

\[\bar{d} - t_{\alpha/2, n-1} \left(\frac{s_d}{\sqrt{n}}\right) \leq \mu_d \leq \bar{d} + t_{\alpha/2, n-1} \left(\frac{s_d}{\sqrt{n}}\right)\]

\[0.667 - 2.201 \left(\frac{2.964}{\sqrt{12}}\right) \leq \mu_d \leq 0.667 + 2.201 \left(\frac{2.964}{\sqrt{12}}\right)\]

\[-1.216 \leq \mu_d \leq 2.55\]

Because zero is contained within this interval, there is no significant indication that one design language is preferable at a 5% significance level.

10-36

a) 1) The parameter of interest is the mean difference in impurity level, \(\mu_d\)

where \(d_i = \text{Test 1} - \text{Test 2}\).

2) \(H_0 : \mu_d = 0\)

3) \(H_1 : \mu_d \neq 0\)

4) \(\alpha = 0.01\)

5) The test statistic is

\[t_0 = \frac{\bar{d}}{s_d / \sqrt{n}}\]

6) Reject the null hypothesis if \(t_0 < -t_{0.005,7}\) or \(t_0 > t_{0.005,7}\) where \(t_{0.005,7} = 3.499\)

7) \(\bar{d} = -0.2125\)
$s_d = 0.1727$

$n = 8$

$t_0 = \frac{-0.2125}{0.1727 / \sqrt{8}} = -3.48$

8) Because $-3.499 < -3.48 < 3.499$ cannot reject the null hypothesis. There is not sufficient evidence that the tests give different mean impurity levels at $\alpha = 0.01$.

b) 1) The parameter of interest is the mean difference in impurity level, $\mu_d$ where $d_i = \text{Test 1} - \text{Test 2}$.
2) $H_0: \mu_d + 0.1 = 0$
3) $H_1: \mu_d + 0.10 < 0$
4) $\alpha = 0.05$
5) The test statistic is

$$t_0 = \frac{\bar{d} + 0.1}{s_d / \sqrt{n}}$$

6) Reject the null hypothesis if $t_0 < t_{0.05,7}$ where $t_{0.05,7} = 1.895$

7) $\bar{d} = -0.2125$

$s_d = 0.1727$

$n = 8$

$$t_0 = \frac{-0.2125 + 0.1}{0.1727 / \sqrt{8}} = -1.8424$$

8) Because $-1.895 < -1.8424$ we fail to reject the null at the 0.05 level of significance.