
Therefore the range of $X$ is $\left\{\frac{1}{4}, \frac{3}{8}, \frac{1}{2}, \frac{5}{8}, \frac{6}{8}\right\}$.

3-18. Probabilities are nonnegative and sum to one.

a) $P(X = 2) = \frac{3}{4} \times (\frac{1}{4})^2 = \frac{3}{64}$

b) $P(X \leq 2) = \frac{3}{4} \times [1 + \frac{1}{4} + (\frac{1}{4})^2] = \frac{63}{64}$

c) $P(X > 2) = 1 - P(X \leq 2) = \frac{1}{64}$

d) $P(X \geq 1) = 1 - P(X \leq 0) = 1 - (\frac{3}{4}) = \frac{1}{4}$

3-26. $X =$ number of components that meet specifications

- $P(X=0) = (0.05)(0.02) = 0.001$
- $P(X=1) = (0.05)(0.98) + (0.95)(0.02) = 0.068$
- $P(X=2) = (0.95)(0.98) = 0.931$

3-36. The sum of the probabilities is 1 and all probabilities are greater than or equal to zero; pmf: $f(1) = 0.7$, $f(4) = 0.2$, $f(7) = 0.1$

a) $P(X \leq 4) = 0.9$

b) $P(X > 7) = 0$

c) $P(X \leq 5) = 0.9$

d) $P(X > 4) = 0.1$

e) $P(X \leq 2) = 0.7$

3-46. Mean and variance for exercise 3-20

\[
\mu = E(X) = 0 \times f(0) + 1 \times f(1) + 2 \times f(2) + 3 \times f(3)
\]
\[
= 0(8 \times 10^{-6}) + 1(0.0012) + 2(0.0576) + 3(0.9412)
\]
\[
= 2.940008
\]

\[
V(X) = \mu^2 \sum f(x_i) - (\mu)^2
\]
\[
= 0.05876096
\]

3-50. $\mu = E(X) = 350 \times 0.06 + 450 \times 0.1 + 550 \times 0.47 + 650 \times 0.37 = 565$

\[
V(X) = \sum_{i=1}^{4} f(x_i)(x - \mu)^2 = 6875
\]
\[
\sigma = \sqrt{V(X)} = 82.92
\]

3-59. The range of $Y$ is 0, 5, 10, ..., 45. $E(X) = (0+9)/2 = 4.5$

\[
E(Y) = 0(1/10) + 5(1/10) + ... + 45(1/10)
\]
\[
= 5[0(0.1) + 1(0.1) + ... + 9(0.1)]
\]
\[
= 5E(X)
\]
\[
= 5(4.5)
\]
\[
= 22.5
\]
\[ V(X) = 8.25, V(Y) = 5^2(8.25) = 206.25, \sigma_Y = 14.36 \]

3-68. \( n = 3 \) and \( p = 0.5 \)

\[ F(x) = \begin{cases} 
0 & x < 0 \\
0.125 & 0 \leq x < 1 \\
0.5 & 1 \leq x < 2 \\
0.875 & 2 \leq x < 3 \\
1 & 3 \leq x 
\end{cases} \quad \text{where} \quad f(0) = \left( \frac{1}{2} \right)^3 = \frac{1}{8} \]

\[ f(1) = 3 \left( \frac{1}{2} \right)^2 = \frac{3}{8} \]

\[ f(2) = 3 \left( \frac{1}{4} \right) = \frac{3}{8} \]

\[ f(3) = \left( \frac{1}{4} \right)^3 = \frac{1}{8} \]

3-76. \( n = 20, p = 0.13 \)

(a) \( P(X = 3) = \binom{20}{3} p^3 (1 - p)^{17} = 0.235 \)

(b) \( P(X \geq 3) = 1 - P(X < 3) = 0.492 \)

(c) \( \mu = E(X) = np = 20 \times 0.13 = 2.6 \)

\[ V(X) = np(1 - p) = 2.262 \]

\[ \sigma = \sqrt{V(X)} = 1.504 \]