Section 8-4

8-38 $\chi^2_{0.05,10} = 18.31$ $\chi^2_{0.025,15} = 27.49$ $\chi^2_{0.01,12} = 26.22$

$\chi^2_{0.005,25} = 46.93$ $\chi^2_{0.95,20} = 10.85$ $\chi^2_{0.99,18} = 7.01$ $\chi^2_{0.995,16} = 5.14$

8-42 95% confidence interval for σ

$n = 17 \quad s = 0.09$

$\chi^2_{\alpha/2,n-1} = \chi^2_{0.025,16} = 28.85$ and $\chi^2_{1-\alpha/2,n-1} = \chi^2_{0.975,16} = 6.91$

$$\frac{16(0.09)^2}{28.85} \le \sigma^2 \le \frac{16(0.09)^2}{6.91}$$

$$0.0045 \le \sigma^2 \le 0.0188$$

$$0.067 < \sigma < 0.137$$

8-46 a) 99% two-sided confidence interval on σ^2

$n = 10 \quad s = 1.913 \quad \chi^2_{0.005,9} = 23.59$ and $\chi^2_{0.995,9} = 1.73$

$$\frac{9(1.913)^2}{23.59} \le \sigma^2 \le \frac{9(1.913)^2}{1.73}$$

$$1.396 \le \sigma^2 \le 19.038$$

b) 99% lower confidence bound for σ^2

For α = 0.01 and n = 10, $\chi^2_{\alpha,n-1} = \chi^2_{0.01,9} = 21.67$

$$\frac{9(1.913)^2}{21.67} \le \sigma^2$$

$$1.5199 \le \sigma^2$$

c) 90% lower confidence bound for σ^2

For α = 0.1 and n = 10, $\chi^2_{\alpha,n-1} = \chi^2_{0.1,9} = 14.68$

$$\frac{9(1.913)^2}{14.68} \le \sigma^2$$

$$2.2436 \le \sigma^2$$

$$1.498 \le \sigma$$

d) The lower confidence bound of the 99% two-sided interval is less than the one-sided interval. The lower confidence bound for σ^2 is in part (c) is greater because the confidence is lower.

8-48 a) 95% Confidence Interval on the proportion of such tears that will heal.

$\hat{p} = 0.676 \quad n = 37 \; z_{\alpha/2} = 1.96$

$$\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \le p \le \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$0.676 - 1.96\sqrt{\frac{0.676(0.324)}{37}} \le p \le 0.676 + 1.96\sqrt{\frac{0.676(0.324)}{37}}$$

$$0.5245 \le p \le 0.827$$

b) 95% lower confidence bound on the proportion of such tears that will heal.

$$\hat{p} - z_{\alpha}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \le p$$

$$0.676 - 1.64\sqrt{\frac{0.676(0.33)}{37}} \le p$$

$$0.549 \le p$$

8-52 a) 95% Confidence Interval on the true proportion of helmets showing damage

$\hat{p} = \frac{18}{50} = 0.36 \qquad n = 50 \qquad z_{\alpha/2} = 1.96$

$$\hat{p} - z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \le p \le \hat{p} + z_{\alpha/2}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$0.36 - 1.96\sqrt{\frac{0.36(0.64)}{50}} \le p \le 0.36 + 1.96\sqrt{\frac{0.36(0.64)}{50}}$$

$$0.227 \le p \le 0.493$$

b) $n = \left(\frac{z_{\alpha/2}}{E}\right)^2 p(1-p) = \left(\frac{1.96}{0.02}\right)^2 0.36(1-0.36) = 2212.76$

$n \cong 2213$

c) $n = \left(\frac{z_{\alpha/2}}{E}\right)^2 p(1-p) = \left(\frac{1.96}{0.02}\right)^2 0.5(1-0.5) = 2401$.