Math 341, Exam II, Spring 2009, Name____________
Student # ______________

Wednesday, April 28. *Please show the complete solution (with all steps) to each problem to receive full credit.*

1. An experimenter wishes to compare the effectiveness of two methods of training industrial employees to perform an assembly operation. The selected employees are to be divided into two groups of equal size, the first receiving training method1 and the second receiving training method 2. After training, each employee will perform the assembly operation, and the length of assembly time will be recorded. The experimenter expects the measurements for both groups to have a range of approximately 8 minutes. If the estimate of the difference in mean assembly times is to be correct to within 1 minute with probability 0.99, how many workers must be included in each training group? (15 points)

2. Solid copper produced by sintering (heating without melting) a powder under specified environmental conditions is then measured for porosity (the volume fraction due to voids) in a laboratory. A sample of $n_1 = 4$ independent porosity have mean $\overline{y}_1 = 0.22$ and sample variance $s_1^2 = 0.001$. A second laboratory repeats the same process on solid copper formed from an identical powder and gets $n_2 = 5$ independent porosity measurements with $\overline{y}_2 = 0.17$ and $s_2^2 = 0.002$ (assume that the population variances are equal). Estimate the true difference between the population means $\mu_1 - \mu_2$ by obtaining a confidence interval for these two laboratories with confidence coefficient 0.99. Base on the computed confidence interval can we conclude that the two population means are equal? Why or Why not? (15 points)

3. The following data, with measurements in hundreds of hours, represent the lengths of life of ten identical electronic components operating in a guidance control system for missiles:

0.64	1.53	0.73	2.26	2.36
1.6	0.15	1.83	1.87	1.13

The length of life of a component of this type is assumed to follow a Weibull distribution with density function given by $f(y|\theta) = \begin{cases} \frac{2y}{\theta} e^{-y^2/\theta}, & y > 0, \\ 0, & elsewhere. \end{cases}$ Let $Y_1, \ldots, Y_n$ be a random sample of size n from this Weibull distribution.

a. Find a sufficient statistics for θ. (10 points)

b. Find the distribution of $T_i = \frac{2Y_i^2}{\theta}$. (5 points)

c. Use the sufficient statistics to form a pivotal quantity. Thus compute a 95 % confidence interval for θ. (15 points)

4. Let $X_1, X_2, X_3, \ldots$ be independent Bernoulli random variables such that P($X_i = 1$) = p and P($X_i = 0$) = 1- p for each i = 1, 2, 3,…. Let the random variable Y denote the number of trials necessary to obtain the first success – that is, the value of i for which $X_i = 1$ occurs for the first time. Then the Y has a geometric distribution with P($Y = y$) = $(1-p)^{y-1} p$, for y = 1, 2, 3, ….
   a. Find the method-of-moments estimator of p based on this single observation Y. (8 points)
   b. Show that $\overline{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i$ is a consistent estimator of $\frac{1}{p}$. (12 points)

5. Let $Y_1, \ldots, Y_n$ be a random sample of observations from a uniform distribution with probability density function $f(y_i|\theta) = \begin{cases} \frac{1}{\theta}, & 0 \le y_i \le \theta, \\ 0, & elsewhere, \end{cases}$ and i = 1, 2, …, n.
   a. Find the MLE of θ. (15 points)
   b. Find the MLE of E (Y_1^2). (5 points)