Math 341-001, Exam I, Spring 2014, Name______________

Student # _______________

Monday, March 03. *Please show the complete solution (with all steps) to each problem to receive perfect score.*

1. The management at a fast-food outlet is interested in the joint behavior of the random variables Y_1 defined as the total time between a customer's arrival at the store and departure from the service window, and Y_2, the time a customer waits in line before reaching the service window. Since, Y_1 includes the time a customer waits in line, we must have $Y_1 \geq Y_2$. The joint distribution of observed values of Y_1 and Y_2 can be modelled by the probability density function with time measured in minutes. $f(y_1, y_2) = \begin{cases} e^{-y_1}, 0 \leq y_2 \leq y_1 < \infty, \\ 0, \quad \text{elsewhere.} \end{cases}$ Find the probability time spend at the service window Y_1 - Y_2 is at most 2 minutes, i.e. $P(Y_1 - Y_2 \leq 2)$.
(Reference problem 5.15, page 235) (15 points)

2. In problem 1. above, find the probability the total time Y_1 is at most 2 minutes given that the customer already waited in line exactly 1 minute, i.e., Y_2=1. (a) Thus, Find $P(Y_1 \leq 2 | Y_2 = 1)$.
(b) Are Y_1 and Y_2 independent? Why? Or why not?
(c) Find E(Y_2) and V(Y_2). (Reference problems 5.29 p. 244 & 5.77 p. 262) (25 points)

3. Let W be a normal random variable with mean 0 and variance 1. Derive the distribution of $U = W^2$. (Reference example 6.11, page 319) (15 points)

4. Let $Y_1, \ldots, Y_{10}$ be independent Exponential distributed with mean = 5.

a) Find the distribution of $Y_{(1)} = min(Y_1, ...Y_{10})$.

b) Identify the distribution obtained in 4. a).

c) Find $P(Y_{(1)} \leq 3)$.

(Reference problem # 6.81, page 339) (25 points)

5. The joint distribution for the length of life of two different types of components operating in a system is given by $f(y_1, y_2) = \begin{cases} \frac{1}{2} e^{-y_1} e^{-y_2/2}, \ y_2 > 0, y_1 > 0, \\ 0, \quad \text{elsewhere.} \end{cases}$

Find the probability density function for the relative efficiency of the two types of components measured by $U = Y_2 / Y_1$. (20 points)