Math 341, Final Exam, Spring 2009, Name_____________

Student # _______________

Wednesday, May 12. *Please show the complete solution (with all steps) to each problem to receive full credit.*

1. The braking distance of two types (1 and 2) of automobiles was compared. Random samples of $n_1 = 61$ automobiles of type 1 and $n_2 = 41$ automobiles of type 2 were tested. The recorded measurement was the distance required to stop when the brakes were applied at 40 miles per hour. Do the data provide sufficient evidence to indicate a difference in the variance stopping distances of the two types of automobiles? Test at $\alpha = 0.01$. Clearly specify the null and alternate hypotheses and the rejection region (RR). (15 points)

2. An anthropologist wishes to estimate the average height of men for a certain race of people. If the population standard deviation is assumed to be 2.5 inches and if she randomly samples 81 men, find the probability that the absolute difference between the sample mean and the true population mean will not exceed 0.5 inch. (10 points)

3. Let $Y_1,\ldots,Y_n$ be a random sample of size n from a population whose density is given by $f(y)=\begin{cases}3\beta^3 y^{-4}, & \beta \le y,\\ 0, & elsewhere\end{cases}$, where $\beta > 0$ is unknown. Consider the estimator $\hat{\beta}=\min(Y_1,\ldots,Y_n)$. Derive the bias and the mean squared error (MSE) of the estimator $\hat{\beta}$. Simplify the MSE as a ratio of factors. Find the limits of the bias and the MSE as n goes to infinity. (15 points)

4. The number of persons coming through a blood bank until the first person with type A blood is found is a random variable Y with a geometric distribution. If p denotes the probability that any one randomly selected person will posses type A blood, the $E(Y) = \frac{1}{p}$ and $V(Y) = \frac{1-p}{p^2}$.

   a. Find a function of Y that is an unbiased estimator of $V(Y)$. (8 points)

   b. Form a reasonable 2-standard–error bound on the error of estimation when Y is used to estimate $\frac{1}{p}$. (7 points)

5. a. – b. Let $Y_1, \ldots, Y_n$ denote a random sample from a population with a uniform distribution on the interval $(0, \theta)$. Let $Y_{(n)} = \max(Y_1, \ldots, Y_n)$ and $U = (1/\theta)Y_{(n)}$.

   a. Show that U has distribution function $F_U(u) = \begin{cases} 0, & u < 0, \\ u^n, & 0 \le u \le 1, \\ 1, & u > 1. \end{cases}$ (5 points)

b. Use U from 5 a. above to develop a 95% lower confidence bound for θ. (5 points)

6. The display below give readings in foot-pounds of the impact strength of two kinds of packing material, type A and type B. Determine whether the data suggests a difference in mean strength between the two kinds of material. Assume that the population variances are equal. Test at $\alpha = 0.05$ level of significance. Clearly specify the null and alternate hypotheses and the RR. Compute the bounds on the p-value of the test and use it also to draw conclusion.

A	1.25	1.16	1.33	1.15	1.23	1.2	1.32	1.28	1.21
B	0.89	1.01	0.97	0.95	0.94	1.02	0.98	1.06	0.98

(20 points)

7. An experimenter has prepared a drug dosage level that she claims will induce sleep for 80% of people suffering from insomnia. After examining the dosage, we feel that her claims regarding the effectiveness of the dosage are inflated. In an attempt to disprove her claim we administer het prescribed dosage to 40 insomniacs and we observe Y, the number for whom the drug dose induces sleep. We wish to test the hypothesis H_0: $p = 0.80$ versus the alternative H_a: $p < 0.80$. Assume that the rejection region $\{y \leq 24\}$ is used. (15 points)
    a. Find α. Is the approximation good?

    b. Find β, when $p = 0.6$. Is the approximation good?