Math 341, Quiz 3, Spring 2009, Name________________

Student # ______________

Wednesday, March 11. Please show the complete solution (with all steps) to the problem to receive full credit.

1. Let $Y$ have probability density function

$$f_Y(y) = \begin{cases} \frac{3(\theta - y)^2}{\theta^3}, & 0 < y < \theta \\ 0, & \text{elsewhere} \end{cases}$$

(a) Show that $\frac{Y}{\theta}$ is a pivotal quantity. (5 points)

Let $U = \frac{Y}{\theta}$ then the change of variable method gives the density of $U$ as follows.

$$f_U(u) = \begin{cases} \frac{3(1-u)^2}{\theta}, & 0 < u < 1 \\ 0, & \text{elsewhere} \end{cases}$$

because $0 < u\theta < \theta$ is same as $0 < u < 1$, which itself is obtained by dividing by $\theta$.

Now, $\frac{dy}{du} = \theta$, hence

$$\int 3(1-u)^2 du = 0.95$$

This implies

$$3(1-u) = 0.95, \text{ i.e. } 1-u = \frac{0.95}{3} = 0.316666667$$

Thus, $u = 0.683333333$. Hence the lower confidence limit for $\theta$ is $\frac{Y}{0.683333333} = Y_{0.05}$.

(b) Use the pivotal quantity from part (a.) to find a 95% lower confidence limit for $\theta$. (5 points)

From figure we see that

$$P(0 < \frac{Y}{\theta} < u_0) = 0.95.$$ Hence,

$$P(0 < \frac{Y}{u_0} < \theta) = 0.95.$$ Here,

$$\int 3(1-u)^2 du = 0.95$$

This implies

$$\frac{(1-u)^3}{-1} = 0.95, \text{ i.e. } (1-u)^3 = 0.95.$$ Here, $u = 0.63159685$.

Hence the lower confidence limit for $\theta$ is $\frac{Y}{0.63159685} = 1.583288454Y$. 

---

![Graph of f(u)](image-url)