Math 341, Quiz 3, Spring 2009, Name_____________

Student # _______________

Wednesday, March 11. *Please show the complete solution (with all steps) to the problem to receive full credit.*

1. Let Y have probability density function

$$f_Y(y) = \begin{cases} \dfrac{3(\theta - y)^2}{\theta^3}, 0 < y < \theta \\ 0, \qquad \textit{elsewhere.} \end{cases}$$

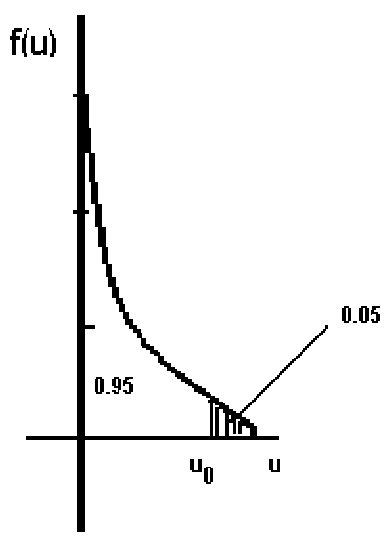
a. Show that $\dfrac{Y}{\theta}$ is a pivotal quantity. (5 points)

Let $U = \dfrac{Y}{\theta}$ then the change of variable method gives the density of U as follows.
$Y = \theta U$,

$$f_U(u) = \begin{cases} \dfrac{3(1-u)^2}{\theta}\left|\dfrac{dy}{du}\right|, 0 < u < 1 \\ 0, \qquad \textit{elsewhere,} \end{cases}$$

because $0 < u\theta < \theta$ is same as $0 < u < 1$, which itself is obtained by dividing by θ.

Now, $\dfrac{dy}{du} = \theta$, hence



$$f_U(u) = \begin{cases} 3(1-u)^2, 0 < u < 1 \\ 0, \qquad \textit{elsewhere.} \end{cases}$$

Since, the density of U does not depend on θ and $U = \dfrac{Y}{\theta}$ is a function of Y and θ alone $\dfrac{Y}{\theta}$ is a pivotal quantity.

b. Use the pivotal quantity from part (a.) to find a 95% lower confidence limit for θ. (5 points)

From figure we see that

$P(0 < \dfrac{Y}{\theta} < u_0) = 0.95$. Hence,

$P(0 < \dfrac{Y}{u_0} < \theta) = 0.95$. Here,

$\int_0^{u_0} 3(1-u)^2 du = 0.95$ this implies

$\left.\dfrac{(1-u)^3}{-1}\right|_0^{u_0} = 0.95$, i.e.

$(1-u_0)^3 = 0.05$, $u_0 = 1 - (0.05)^{1/3} =$ 0. 63159685.

Hence the lower confidence limit for θ is $\dfrac{Y}{0.63159685} = 1.583288454Y.$