1. A person answers each of two multiple choice questions at random. If there are four possible choices on each question, what is the conditional probability that both answers are correct given that at least one is correct?

(10 pts)

2. A laboratory blood test is 95 percent effective in detecting a certain disease when it is, in fact, present. However, the test also yields a “false positive” result for 1 percent of the health persons tested. (That is, if a healthy person is tested, then, with probability 0.01, the test result will imply he or she has the disease.) If 0.5 percent of the population actually has the disease, what is the probability a person has the disease given that the test result is positive?

(12 pts)
3. Let $X$ be a random variable whose probability mass function (pmf) is given by $p(x) = c \left( \frac{2}{3} \right)^x$, $x = 1, 2, 3, \ldots$ zero elsewhere. Find the constant $c > 0$ and the cumulative distribution function (cdf) of $X$.

(12 pts)

4. Let $X$ be a random variable whose pmf is given by $p(x) = \left( \frac{1}{2} \right)^x$, $x = 1, 2, 3, \ldots$ zero elsewhere. Let $Z = (X - 2)^2$. Find the pmf of $Z$.

(12 pts)

5. A random variable $X$ has its cdf given by $F(x) = (1 + e^{-x})^{-1}$, $-\infty < x < \infty$. Find the density function of $X$. Also find the 60th percentile for the given distribution.

(12 pts)
6. Let $X$ have the density function given by $f(x) = \begin{cases} 4x^3, & 0 < x < 1 \\ 0, & \text{elsewhere} \end{cases}$. Find the cdf and the density function of and $Y = -\ln(X^4)$. (14 pts)

7. Let $X$ and $Y$ have the joint probability density function given by $f(x, y) = \begin{cases} 3x, & 0 < y < x < 1, \\ 0, & \text{elsewhere} \end{cases}$. Are $X$ and $Y$ independent? Why or why not? Compute $E(X / y)$. (16 pts)
8. Let $X_1$ and $X_2$ have joint probability density function given by

$$f(x_1, x_2) = \begin{cases} 
2e^{-x_1-x_2}, & 0 < x_1 < x_2 < \infty, \\
0, & \text{elsewhere.}
\end{cases}$$

Find the joint density function of $Y_1 = 2X_1$ and $Y_2 = X_2 - X_1$.

(12 pts)