September 28, 2009 **Quiz I MATH 662** **Name:______________**

Must show all work to get full credit!! **Student #:______________**

1. Given the cumulative distribution function (cdf)

$$F(x) = \begin{cases} 0, & x < -2 \\ \dfrac{x+3}{5}, & -2 \le x < 1.5 \\ 1, & 1.5 \le x. \end{cases}$$

Compute (a) $P(-2 < X \le 1) = F(1) - F(-2) = \frac{4}{5} - \frac{1}{5} = \frac{3}{5}$.

(b) $P(X = -2) = F(-2) - F(-2^-) = \frac{1}{5} - 0 = \frac{1}{5}$.

(c) $P(X = 1.5)\,) = F(1.5) - F(1.5^-) = 1 - \frac{4.5}{5} = \frac{1}{10}$.

(d) $P(X > 0) = 1 - F(0) = 1 - \frac{3}{5} = \frac{2}{5}$.

(e) $P(X < 1.5) = F(1.5^-) = \frac{4.5}{5} = \frac{9}{10}$.

(5 pts)

2. If the probability density function of X is $f(x) = \frac{x^3}{4}, 0 < x < 2$, zero elsewhere. Find the cumulative distribution function of $Y = X^3$.

(5 pts)

Let us find the pdf of Y. Note that $X = Y^{1/3}$ and $\frac{dx}{dy} = \frac{1}{3}y^{-2/3}$. Using the Jacobian transformation formula we have $f_Y(y) = f(y^{1/3})\left|\frac{dx}{dy}\right| =$

$\frac{dx}{dy}\frac{y}{4}, 0 < y^{1/3} < 2 = \frac{y^{1/3}}{12}, 0 < y^{1/3} < 2 = \frac{y^{1/3}}{12}, 0 < y < 8$. {Check: Evaluate $\frac{y^{4/3}}{16}$ from 0 to 8, the difference gives 1}. Now we will get the cdf of Y.

Since support of Y is 0 to 8. $F_Y(y) = \begin{cases} 0, & y \le 0 \\ 1, & y \ge 8. \end{cases}$ Now consider 0 < y < 8,

$$F_Y(y) = \int_0^y f_Y(u)\,du = \left.\frac{u^{1/3}}{12}\right|_0^y = \frac{y^{4/3}}{16}.$$