Math 762- Final Exam

April 17, 2003 **Name: ___________________**
Student #: _______________

Must show all work for full credit!!!

I pledge that I have not violated the NJIT code of honor__________________________

Please note that the **problems with specified number** and page are **from the course** textbook: **A Course in Mathematical Statistics, By George G. Roussas - Second Edition**

1. Suppose that n cylindrical shafts made by a machine are selected at random from the production of the machine and their diameters and lengths measured. It is found that N_{11} have both (diameters and lengths) measurements within the tolerance limits, N_{12} have satisfactory length but unsatisfactory diameters, N_{21} have satisfactory diameters but unsatisfactory lengths, and N_{22} are unsatisfactory as to both measurements. $\sum_{i,j=1}^{2} N_{ij}$ = n. Each shaft may be regarded as a drawing from a multinomial population with density

$$p_{11}^{x_{11}} p_{12}^{x_{12}} p_{21}^{x_{21}} (1 - p_{11} - p_{12} - p_{21})^{x_{22}}, \, for \; x_{ij} = 0,1; \sum_{i,j=1}^{2} x_{ij} = 1,$$

having three parameters. What are the maximum-likelihood estimates of the parameters if N_{11} = 90, N_{12} = 6, N_{21} = 3, and N_{22} = 1? (10 pts)

2. Observations $X_1, \ldots, X_n$ are drawn from normal populations with the same mean μ but with different variances $\sigma_1^2, \sigma_2^2, ..., \sigma_n^2$. If we assume that the σ_i^2 are known, what is the maximum likelihood estimator of μ? (10 pts)

3. Let $X_1, \ldots, X_n$ be a random sample from the discrete density function $f(x;\theta) = \begin{cases} \theta^{x}(1-\theta)^{1-x}, x = 0,1, \\ 0, \qquad elsewhere \end{cases}$, where $0 \le \theta \le \frac{1}{2}$. Note that $\Omega = [0, \frac{1}{2}]$. Find a maximum likelihood estimator of θ. What is the maximum likelihood estimator of θ^{2}? (10 pts)

4. Let $X_1,\ldots,X_n$ be a random sample from the density $f(x;\theta)=\frac{\ln(\theta)}{\theta-1}\theta^{x}I_{(0,1)}(x)$. (a) Find a complete sufficient statistic if there is one. (6pts) (b) Find a function of θ for which there exists an unbiased estimator whose variance coincides with the Cramer-Rao lower bound if such exists (7 pts).

5. For a random sample from the Poisson distribution, find an unbiased estimator of $\tau(\lambda)=(1+\lambda)e^{-\lambda}$. (10 pts)

6. Let $X_1,\ldots,X_n$ be a random sample from the density $f(x;\theta)=\theta^{2}xe^{-\theta x}I_{(0,\infty)}(x)$. (a) In testing H: $\theta\le 1$ *versus* A: $\theta>1$ for n =1 (a sample of size one) the following test was used: Reject H if and only if $X_1\le 1$. Find the power function and size of this test. (7 pts) (b) Find a most powerful size-α test of H: $\theta=1$ versus A: $\theta=2$. (8 pts) (c) Does there exits a uniformly most powerful size-α test of H: $\theta\le 1$ *versus* A: $\theta>1$? If so, what is it? Show work. (10 pts)

7. Problem # **13.7.2 i)**, page 369. Must show how the Likelihood ratio test was arrived at. (12 pts)

8. Problem # **15.2.2** page 404. (10 pts)