Math 762- Final Exam

April 17, 2003

Must show all work for full credit!!!

I pledge that I have not violated the NJIT code of honor ____________________________

Please note that the problems with specified number and page are from the course textbook: A Course in Mathematical Statistics, By George G. Roussas - Second Edition

1. Suppose that n cylindrical shafts made by a machine are selected at random from the production of the machine and their diameters and lengths measured. It is found that $N_{11}$ have both (diameters and lengths) measurements within the tolerance limits, $N_{12}$ have satisfactory length but unsatisfactory diameters, $N_{21}$ have satisfactory diameters but unsatisfactory lengths, and $N_{22}$ are unsatisfactory as to both measurements. $\sum_{i,j=1}^{2} N_{ij} = n$. Each shaft may be regarded as a drawing from a multinomial population with density

$$p_{11}^{N_{11}} p_{12}^{N_{12}} p_{21}^{N_{21}} (1 - p_{11} - p_{12} - p_{21})^{N_{22}}, \text{ for } x_{ij} = 0, 1; \sum_{i,j=1}^{2} x_{ij} = 1,$$

having three parameters. What are the maximum-likelihood estimates of the parameters if $N_{11} = 90$, $N_{12} = 6$, $N_{21} = 3$, and $N_{22} = 1$? (10 pts)

2. Observations $X_1, \ldots, X_n$ are drawn from normal populations with the same mean $\mu$ but with different variances $\sigma_1^2, \sigma_2^2, \ldots, \sigma_n^2$. If we assume that the $\sigma_i^2$ are known, what is the maximum likelihood estimator of $\mu$? (10 pts)

3. Let $X_1, \ldots, X_n$ be a random sample from the discrete density function

$$f(x; \theta) = \begin{cases} \theta^x (1-\theta)^{1-x}, & x = 0, 1, \\ 0, & \text{elsewhere} \end{cases}, \text{ where } 0 \leq \theta \leq \frac{1}{2}. \text{ Note that } \Omega = [0, \frac{1}{2}].$$

Find a maximum likelihood estimator of $\theta$. What is the maximum likelihood estimator of $\theta^2$? (10 pts)
4. Let $X_1, \ldots, X_n$ be a random sample from the density

$$f(x; \theta) = \frac{\ln(\theta)}{\theta - 1} \theta^{-1} I_{(0,1)}(x).$$

(a) Find a complete sufficient statistic if there is one. (6pts)

(b) Find a function of $\theta$ for which there exists an unbiased estimator whose variance coincides with the Cramer-Rao lower bound if such exists (7 pts).

5. For a random sample from the Poisson distribution, find an unbiased estimator of

$$\tau(\lambda) = (1 + \lambda) e^{-\lambda}. \quad (10 \text{ pts})$$

6. Let $X_1, \ldots, X_n$ be a random sample from the density

$$f(x; \theta) = \theta^2 xe^{-\theta x} I_{(0,\infty)}(x).$$

(a) In testing $H: \theta \leq 1$ versus $A: \theta > 1$ for $n = 1$ (a sample of size one) the following test was used: Reject $H$ if and only if $X_1 \leq 1$. Find the power function and size of this test. (7 pts)

(b) Find a most powerful size-$\alpha$ test of $H: \theta = 1$ versus $A: \theta = 2$. (8 pts)

(c) Does there exist a uniformly most powerful size-$\alpha$ test of $H: \theta \leq 1$ versus $A: \theta > 1$? If so, what is it? Show work. (10 pts)

7. Problem # 13.7.2 i), page 369. Must show how the Likelihood ratio test was arrived at. (12 pts)

8. Problem # 15.2.2 page 404. (10 pts)