Math 762- Exam I

Name: ________________

March 3, 2003 **Student #:** ______________

Must show all work for full credit!!!

I pledge that I have not violated the NJIT code of honor____________________

1. Prove or disprove following (7 pts. each):
   (a) If two c.d. f.'s of the random variables X and Y satisfy $F_X(z) = F_Y(z)$ for all z, then P(X = Y) = 1.
   (b) If E[X] > E[Y], then P(X > Y) = 1.

2. Suppose that $X_1, \ldots, X_n$ form a random sample from a uniform (0, 1).
   Let Y_1 and Y_n be the first and last order statistics, respectively.
   Determine the value of $P(Y_1 \le 0.1 \text{ and } Y_n \le 0.9)$.

(15 pts)

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3. Let X_n be a t-distributed random variable, with n degrees of freedom. Show that X_n converges in distribution to standard normal random variable as n goes to ∞.

(11 pts)

4. Let $X_n, n = 1, 2, \ldots,$ be random variables having negative binomial distribution with p_n and r_n such that $p_n \to 1, r_n \to \infty,$ so that $r_n(1 - p_n) = \lambda_n \to \lambda\,(>0).$ Show that $X_n \underset{n\to\infty}{\overset{d}{\to}} X,$ where X is a random variable distributed as P(λ), i.e., the Poisson r. v. with parameter λ .

(15 pts)

5. Suppose that $X_1, \ldots, X_n$ form a random sample from a distribution whose mean is μ and variance ∞ > σ^2 > 0. Show that $\sqrt{n}\left(\frac{\bar{X} - \mu}{S}\right) \overset{d}{\to} Z,$ as n goes to ∞, where Z denotes the standard normal random variable and $S^2 = \frac{\sum_{i=1}^{n}(X_i - \bar{X})^2}{n}.$

(15 pts)

6. Let X_1, X_2 be independent r. v.'s distributed as U ($\alpha, \alpha+1$). Then

i) Derive the p. d. f.'s of the r.v.'s $X_1 + X_2$ and $X_1 - X_2$;

ii) Determine whether these r. v.'s are independent or not.

(15 pts)

7. Let X be the uniform random variable on the interval (-1, 3). Compute the p. d. f. of $Y = X^2$.

(15 pts)