

MATH 762: Statistical Inference **Midterm Exam** (Spring 2007)

March 17, 2007 NJIT

Name:	SSN:	Section #
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Instructors : S. Dhar

➔ **Must show all steps for each problem to receive full credit.**

I pledge my honor that I have abided by the Honor System. _____
(Signature)

Score	
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1. Let X_1, \dots, X_n be a random sample from a distribution with mean μ and variance σ_X^2 .

Assume that $EX_1^4 < \infty$. Show that $S_n^2 = \frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n-1}$ is

- a. an unbiased estimator of σ_X^2 and (10 points)

- b. consistent estimator of σ_X^2 . (10 points)

2. Let X be a random variables with mean and variance $\mu_1 = -5, \sigma_1^2 = 4$ and Y be a random variables with mean and variance $\mu_2 = 1, \sigma_2^2 = 6$ and the correlation ρ between X and Y is $\rho = 0.5$. Find the mean and variance of $Z = 2X - 3Y + 1$. (15 points)

3. Let Y_n denote the maximum of a random sample from a distribution of the continuous type that has cdf $F(x)$ and pdf $f(x) = F'(x)$. Find the limiting distribution of $Z_n = n[1 - F(Y_n)]$. (20 points)
4. Let $f(x) = \frac{1}{x^2}, 1 < x < \infty$, zero elsewhere, be the pdf of a random variable X . Consider a random sample of size 60 from the distribution having this pdf. Compute the approximate probability that at most 40 of the observations of the random sample are less than 3. (15 points)
5. When 100 tacks () were thrown on a table 64 of them landed point up. Obtain a 95% confidence interval for the probability that a tack of this type will land point up. Assume independence. Interpret your confidence interval. (15 points)

6. Let $Y_1 < Y_2 < Y_3 < Y_4$ be the order statistics of a random sample of size $n = 4$ from a distribution with pdf $f(x; \theta) = \frac{1}{\theta}, 0 < x < \theta$, zero elsewhere, where $0 < \theta$. The hypothesis $H_0 : \theta = 1$ is rejected and $H_1 : \theta > 1$ is accepted if the observation $Y_4 \geq c$.
- Find the constant c so that the significance level is $\alpha = 0.01$. (5 points)
 - Determine the power function of the test. (10 points)

END