2.1 Position, velocity and speed.

For now, we are only interested in translational motion, so model all objects as a point object. All mass of object is at the point.

Consider allowing object to move, and then recording position of object as a function of time.

Show Active Figure 2.1

The recorded data is a position vs time plot.

Note: coordinate system and origin are defined by you the observer.

From position vs time ($x(t)$) plot, one can define displacement

$$\Delta x = x_f - x_i$$

"change in" $x$ from initial position to final position.

- for displacement, actual path of particle not important — only initial and final position
- displacement can be positive or negative $\Rightarrow$ eg. object moves "left" or "right" but this is relative to defined coordinate system.

Define Average Velocity

$$V_{av} = \frac{\Delta x}{\Delta t}$$

Example: as car in Fig. 2.1 moves from P1 to P2

$$\Delta x = 52 - 30 = 22 \text{ m}$$

$$\Delta x = 277 - 19 = 258 \text{ m}$$
\[ \Delta t = t_b - t_c \]

\[ \Delta t = 4 - 1 = 3 \]

\[ V_{\text{avg}} = \frac{22}{6} = 3.67 \text{ m/s} \]

\[ \frac{\Delta x}{\Delta t} = \frac{255}{3} = 85 \text{ m/s} \]

If we want to know the instantaneous velocity at each moment in time (as opposed to an average), we need to use a calculus definition:

\[ V_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \text{tangent slope to the } x(t) \text{ curve.} \]

Speed and velocity are not the same.

Speed is the magnitude of the velocity.

For constant velocity motion, how are \( V_x, x, t \) related?

\[ V_x = V_{\text{avg}} = \frac{\Delta x}{\Delta t} = \text{slope of } x(t) \text{ plot} \]

For constant velocity motion, the graph is a straight line.

\[ \text{Slope} = \frac{\Delta x}{\Delta t} = V_x \]
\[ V_x = \frac{\Delta x}{\Delta t} \implies V_x = \frac{x_f - x_i}{t_f - t_i} \]

\[ x_f = x_i + V_x \Delta t \quad \text{if} \quad V_x \text{ constant} \]

What if \( V_x \) not constant so \( V_x(t) \) varies in time?

**Define acceleration as slope to \( V(t) \) plot**

\[ a_{avg} = \frac{\Delta V_x}{\Delta t} = \frac{V_{x_f} - V_{x_i}}{t_f - t_i} \quad \text{in average} \]

**Instantaneously** \( \implies \)

\[ a_x = \lim_{\Delta t \to 0} \frac{\Delta V_x}{\Delta t} = \frac{dV_x}{dt} \]

\[ a_x = \frac{dV_x}{dt} = \frac{d}{dt} \left( \frac{dx}{dt} \right) = \frac{d^2x}{dt^2} \quad \text{c - 2nd derivative} \]

**Example**: \( x(t) = Ae^{-t} + Bt \)

what is \( V(t) \)?

\[ V(t) = \frac{dx(t)}{dt} = -Ae^{-t} + B \]

what is \( a(t) \)?

\[ a(t) = \frac{dV(t)}{dt} = Ae^{-t} \]
Note from the iclicker example, \( V(t=1) > 0 \) but \( a(t=1) < 0 \). How are they related?

2.5.1 Motion diagrams

- The lengths of \( \vec{V} \) and \( \vec{a} \) indicated their relative magnitude.
- Note that vector direction of \( \vec{V} \) and \( \vec{a} \) determine if object is slowing down or speeding up.

2.6.1 Constant acceleration formulas.

Let’s restrict ourselves to \( \vec{a} = \text{constant} \)

\[
A_x = \frac{V_f - V_i}{t - 0} = \frac{\Delta V}{\Delta t}
\]

(Eq. 1) \[ V_f(t) = V_i + a \cdot t \] \( \text{(constant } a_x) \)

\( t_i = 0 \)

for constant \( a \) \[ V_{avg} = \frac{V_i + V_f}{2} \]

why?

\[ V_f(t) = V_i + a_x \cdot t \]

but \( V_{avg} = \frac{x_f - x_i}{t - 0} \)
Combine equation

\[ x_6 = x_i + v_i t + \frac{a_x t^2}{2} \]

(Eq 2)

\[ V_6(t) = V_i + a_x t \]

\[ Q u a d r a t i c \ e q u a t i o n \]

\[ a_x \] constant

Now combine (Eq 1) and (Eq 2) to eliminate \( t \).

\[ x_6 - x_i = V_i t + \frac{a_x t^2}{2} \]

\[ x_6 - x_i = V_i \left( \frac{V_i - V_i}{a_x} \right) + \frac{a_x (V_6 - V_i)^2}{2} \]

\[ a_x (V_6 - V_i) = V_i \sqrt{V_i} - V_i + \frac{V_6 + V_i}{2} - 2 \sqrt{V_i} \]

\[ \Rightarrow V_6^2 = V_i^2 + 2a_x (x_6 - x_i) \]

Show PPT with equation Summary

Example Problem

Note: page 23 of textbook gives a general problem solving strategy. To become proficient, practice, practice, practice.

A car is travelling at 45 m/s. The driver steps on the brakes. a) What acceleration (magnitude and direction) are required for the car to come to a stop in 10 sec? b) How far does the car travel after applying the brakes?
Step 1: draw a diagram.
\[ v_i = 45 \text{ m/s} \]
\[ v_f = ? \]

- define a coordinate system.
- define origin.

Step 2: From the problem, identify the type of physics problem.
\[ \Rightarrow \text{1-D motion, constant acceleration.} \]

Step 3: Identify the appropriate equations/concepts which apply.
\[ v_f = v_i + at \]
why +45 and not -45?

Step 4: solve equations.
\[ 0 = 45 + a \cdot 10 \]
\[ a = \frac{0 - 45}{10} \]
\[ a = -4.5 \text{ m/s}^2 \]

\[ \Rightarrow \text{velocity in positive defined direction} \]

what does this mean?

opposite to direction defined as positive

how far does car travel?

1. can use
\[ x_f = x_i + v_i \cdot t + \frac{1}{2} a t^2 \]
2. \[ v_f^2 = v_i^2 + 2a(x_f - x_i) \]
   \[ \Rightarrow \text{now known.} \]
   \[ 0 = 45^2 + 2(-4.5)(x_f - x_i) \]
   \[ x_f - x_i = \frac{45^2}{9} = 225 \text{ m.} \]
2.7) free falling objects

Objects fall due to gravity \( |a| = 9.8 \text{ m/s}^2 \equiv g \)

Sign of \( a \) depends on orientation of coordinate system.

Note: all masses fall to ground at same acceleration.

If released together, a snow flake and rock will fall together to earth, if we neglect friction.

\[
\text{Standard problem}
\]

\( v_0 = 4 \text{ m/s} \)

How long does it take for ball thrown upward to reach ground?

\( \Rightarrow \) By convention, we define "up" as positive \( y \) direction.

\( \Rightarrow \) Origin \((0,0)\) of coordinate system is arbitrary. Can choose top of building or ground level.

\( \ast \) Once you choose a coordinate system, be consistent throughout problem.

For this example, I choose \( y \) direction to be up, and origin of coordinates to be at roof.

\[ x_0 - x_i = v_0 y + \frac{1}{2} a_y y^2 \]

\( a_y = -g \quad \Rightarrow \text{why?} \)

\( x_0 - x_i = -h \quad \Rightarrow \text{why} \)
\[ V_c = V_o \quad \text{why?} \]

so in terms of symbols

\[ \Rightarrow -h = V_o t - \frac{1}{2} gt^2 \]

\[ \Rightarrow \frac{1}{2} gt^2 - V_o t - h = 0 \]

solve for \( t \) \( \Rightarrow \) Quadratic equation

\[ t = \frac{-(-V_o) \pm \sqrt{(V_o)^2 - 4\left(\frac{1}{2}g\right)(-h)}}{2\left(\frac{1}{2}g\right)} \]

\[ t = \frac{V_o \pm \sqrt{V_o^2 + 2gh}}{g} \]

Note solve symbolically and stick in numbers at end.

\[ t = \frac{4 \pm \sqrt{4^2 + 2 \cdot 9.8 \cdot 10}}{9.8} \]

\[ t = 1.95 \text{ or } -1.15 \]

mathematically 2 roots, but only one is physically correct. \( t = 1.95 \) since we set up problem so that ball hits ground after \( t = 0 \)

2.8 derivation of constant acceleration equations from

integral calculus

Show PPT Fig 2.45 x 2
Distance travelled in one time interval is
\[ \Delta x = v_x \Delta t \]

Total distance travelled
\[ \Delta x = \sum_{n} v_{x,n} \Delta t_n \]
\[ \rightarrow \text{ sum over all time intervals.} \]

Graphically, it is area under curve of \( V(t) \)
\[ \Delta x = \lim_{\Delta t_n \rightarrow 0} \int_{t_i}^{t_f} v_x(t) \, dt \]

The integral of \( V(t) \) gives \( x(t) \).
Likewise, integral of \( a(t) \) gives \( V(t) \).

Formally:
\[ a_x = \frac{dv_x}{dt} \]
\[ dv_x = a_x \, dt \]
\[ \int_{v_i}^{v_f} dv_x = \int_{0}^{t_f} a_x \, dt \]
\[ v_f - v_i = a_x \, t_f^2 \]
\[ v_f = V(t) = v_i + a_x t^2 \]
\[ V(t) = \frac{dx}{dt} = v_i + a_x t \]
\[ \int_{x_i}^{x_f} dx = \int_{0}^{t_f} (v_i + a_x t) \, dt \]
\[ x_f - x_i = v_i \cdot t + \frac{1}{2} a_x t^2 \]