In equilibrium, \( \Sigma F = ma = 0 \)

**General solution** \( \vec{v} = \text{constant} \) (constant could be \( = 0 \))

**Example:** inclined plane — at what angle \( \theta \) will the masses not move if they are initially at rest?

1. Draw free body diagram for each mass
2. Choose coordinate system for each mass
3. Apply \( \Sigma F = ma \) for each mass
4. Solve equations

\[
T - m_1 g = a \quad m_1 a = 0
\]

\[
N - m_2 g \cos \Theta = 0
\]

\[
m_2 g \sin \Theta - T = m_2 a = 0
\]

Combine

\[
m_2 g \sin \Theta - m_1 g = 0
\]

\[
\sin \Theta = \frac{m_1}{m_2}
\]
Instead, let's assume that $\sin \theta \neq \frac{m_1}{m_2}$ so that masses will move. $\Rightarrow$ Calculate acceleration.

\[ T - m_1 g = m_1 a \]
\[ m_2 g \sin \theta - T = m_2 a \]

accelerations of masses the same ... why?

$\Rightarrow$ Coupled together by a string which doesn't stretch or go slack. "Distance" between blocks fixed so $|\vec{V}_1| = |\vec{V}_2|$ for both must be the same.

Combine equations.

\[ T - m_1 g = m_1 (a + g) \]
\[ m_2 g \sin \theta - m_1 (a + g) = m_2 a \]

\[ a = \frac{g (m_2 \sin \theta - m_1)}{m_1 + m_2} \]

Iclicker #2
Forces of Friction/Circular Motion

Frictional Forces oppose the "natural" motion of an object relative to surface. Frictional Forces are very familiar — they allow us to walk (friction of shoes on floor), drive a car (friction of tires on road).

Friction forces can be both static and kinetic motion.

\[ \text{Note} \quad f_s \leq \mu_s N \]

\[ f_s = \mu_s N \quad \text{only when object is just about to start moving} \]

\[ f_k = \mu_k N \quad \text{to a good approximation, kinetic force is constant once object starts moving.} \]

Friction independent of contact area between surface.

Consider:

\[ \begin{array}{c}
\text{m} \\
\text{m}
\end{array} \quad \begin{array}{c}
\text{m} \\
\text{m}
\end{array} \]

\( < \) more contact area, but weight spread out over larger area.
Coefficients of friction vary widely.

PPT Slide table 5.1

I-Clicker

Example: a block is sliding across a table.

\[ \mu_s = 0.6 \quad \mu_k = 0.3 \]

What is the acceleration of the block?

\[ \begin{align*}
N - mg &= 0 \\
-\mu_k N &= ma
\end{align*} \]

\[ \begin{align*}
\mu_k &= \mu_s N \\
&= \mu_s mg
\end{align*} \]

\[ \begin{align*}
-\mu_k mg &= \mu_s mg \\
\mu_s mg &= \mu_s mg \\
\mu &\rightarrow \text{mass cancels} \\
6 &= \mu_s mg \\
\mu &= 6/9.8 \\
a &= \mu g \\
|a| &= 0.6(9.8) = 5.88 \text{ m/s}^2
\end{align*} \]

I-Clicker

PPT Slide figure 5.16
For fixed angle $\theta$, what force $F$ is required to move blocks at constant velocity?

\[ F = \frac{\mu_k g (m_1 + m_2)}{\cos \theta + \mu_k \sin \theta} \]
Other types of friction forces — fluids.

Models for drag in liquid:

\[ f = kv \quad \Leftrightarrow \quad \text{fluid resistance at low velocity} \]

\[ f = Dv^2 \quad \Leftrightarrow \quad \text{fluid resistance at high velocity} \]

\( D \) depends on shape and size of object and density of air.

\[ mg - f = ma \]

As velocity increases, \( f \) increases until terminal velocity is reached.

At terminal velocity, \( a = 0 \)

\[ mg = D \quad \Rightarrow \quad mg = Dv^2 \]

\[ V_{\text{term}} = \sqrt{\frac{mg}{D}} \]
Circular Motion - Chapter 6

Recall that for circular motion $a_c = \frac{v^2}{r}$

so force required is $\Sigma F_c = ma_c = \frac{v^2}{r}$

---

**Active Figure**

---

**Example**  Conical Pendulum

---

what is angle $\theta$ for a given velocity $v$?

---

\[ T \cos \theta - mg = 0 \]

\[ T \sin \theta = \frac{mv^2}{r} \]

**Combine**

\[ r = L \sin \theta \]

\[ \frac{\sin \theta}{\cos \theta} = \frac{mv^2}{L \sin \theta (mg)} \]

\[ \frac{\sin^2 \theta}{\cos \theta} = \frac{v^2}{Lg} \]

**PPT Slide**
Friction on Roadways

First consider a flat road way

\[ N - mg = 0 \]
\[ \mu_s N = \frac{mv^2}{R} \Rightarrow V_{max} \]
\[ V_{max} = \frac{R \mu_s mg}{x} \]
\[ V_{max} = \sqrt{R \mu_s g} \]

In poor weather, \( \mu_s \) decreases so \( V_{max} \) is lower.

For sharp turns, \( R \) decreases so \( V_{max} \) is lower.

What about a banked curve?
Neglecting friction, a component of normal force makes car move in a circle. Without friction

\[ F_c = N \sin \theta = \frac{mv^2}{R} \]

What if \( \frac{mv^2}{R} > N \sin \theta \)? Friction needed to keep car from skidding up incline.

What if \( \frac{mv^2}{R} < N \sin \theta \)? Friction needed to keep car from skidding down incline.

6.2 Non-uniform circular motion

Non-uniform circular motion

Example - Forces on a swing.

Note: \( \vec{N} \) not constant. \( \Rightarrow \) \( \alpha_c = \frac{mv^2}{R} \) not constant in general

So forces not constant:

\[ \Sigma \vec{F} = \Sigma \vec{F}_r + \Sigma \vec{F}_e \]

Radial forces
tangetial forces
Example: Ball on string swinging in vertical plane

- Assume \( V^2 \) constant

- What is tension in string?

At bottom,

\[
T - mg = \frac{mv^2}{r}
\]

\[
T = m(g + \frac{v^2}{r})
\]

At top,

\[
T + mg = \frac{mv^2}{r}
\]

\[
T = m\left(\frac{v^2}{r} - g\right)
\]

What happens if \( \frac{v^2}{r} < g \)?

\[
T = \frac{mv^2}{r}
\]