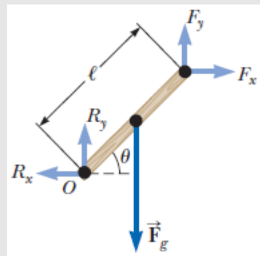
PHYSICS 111 HOMEWORK SOLUTION #10

April 22, 2013

0.1

Consider the following figure: 

- Select the necessary conditions for equilibrium of the object shown in the figure above and the torque about an axis through point O.

  1. $F_x + F_y = 0$
  2. $F_y + R_y - F_g = 0$
  3. $R_x + R_y = 0$
  4. $F_y l \cos\theta - F_g(l/2)\cos\theta - F_x l \sin\theta = 0$
  5. $F_x - R_x = 0$
  6. $F_y(l/2)\cos\theta - R_y(l/2)\cos\theta - F_x(l/2)\sin\theta + R_x(l/2)\sin\theta = 0$

Conditions of equilibrium are : $\sum F_i = ma$ and $\sum \tau_i = 0$

Projecting the first on the x-axis and y-axis should give:

$$F_x - R_x = 0$$

and

$$F_y + R_y - F_g = 0$$

For the torques, we can choose the counter-clock direction as the positive one. F_g doesn't contribute as it's on the rotation axle. We have:

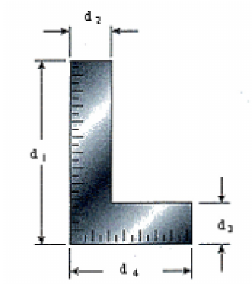
$$\begin{aligned} F_y \frac{l}{2}\sin(\frac{\pi}{2} - \theta) - R_y \frac{l}{2}\sin(\frac{\pi}{2} + \theta) - F_x \frac{l}{2}\sin\theta + R_x \frac{l}{2}\sin(\pi - \theta) &= 0 \\ F_y(l/2)\cos\theta - R_y(l/2)\cos\theta - F_x(l/2)\sin\theta + R_x(l/2)\sin\theta &= 0 \end{aligned}$$

The statements that hold true are 2)-5) and 6)

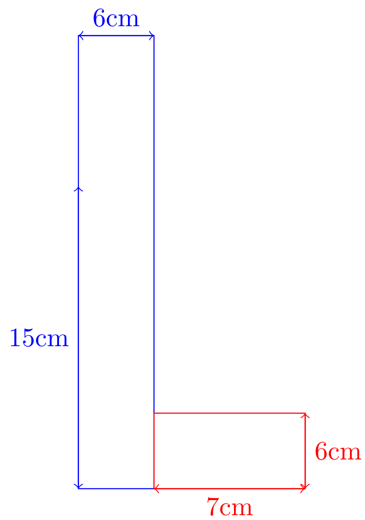
0.2

A carpenter's square has the shape of an L, where d_1 = 15.0 cm, d_2 = 6.00 cm, d_3 = 6.00 cm, d_4 = 13.0 cm. Locate its center of gravity. (Take (x, y) = (0, 0) at the intersection of d_1 and d_4.)

([ ] 5.07 , [ ] 6.07) cm



The L-shape carpenter square can be thought of as the combination of two rectangles (blue and red in the picture below). The blue one has a mass of $m_1 = 15 \times 6 = 90$ units, while the red one is $m_2 = 7 \times 6 = 42$ units (the squares are supposes to be uniform and their masses scale linearly with their area).



The blue rectangle has its center of mass at ($x_1 = 3, y_1 = 7.5$), the red square has its center of mass at ($x_2 = 9.5, y_2 = 3$).
The overall center of mass coordinates are then calculated as follows:

$$\begin{aligned} x &= \frac{m_1}{m_1+m_2}x_1 + \frac{m_2}{m_1+m_2}x_2 \\ &= \frac{90}{132}\times 3 + \frac{42}{132}\times 9.5 \\ &= 5.07\text{cm} \end{aligned}$$

and

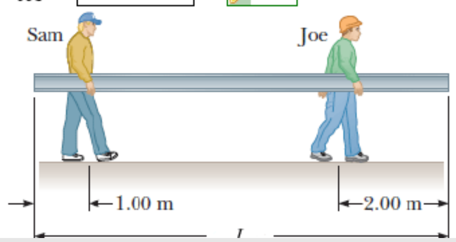
$$\begin{aligned} y &= \frac{m_1}{m_1+m_2}y_1 + + \frac{m_2}{m_1+m_2}y_2 \\ &= \frac{90}{132}\times 7.5 + \frac{42}{132}\times 3 \\ &= 6.07\text{cm} \end{aligned}$$

0.3

A uniform beam of length L = 7.65 m and weight 5.45 × 10[2] N is carried by two workers, Sam and Joe, as shown in the figure below. Determine the force that each person exerts on the beam.

F_{Sam} = 214 N

F_{Joe} = 331 N



We use Newton's 2nd law: $\sum F_i = ma$ and $\sum \tau_i = 0$

$$F_{sam} + F_{Joe} = mg = 5.45 \times 10^2$$

If we choose the center of gravity (for convenience so that the weight won't contribute), we can evaluate the torque of all forces about this point. We take $r_1 = \frac{7.65}{2} - 1 = 2.825$ m as the distance from the center to Sam and

$r_2 = \frac{7.65}{2} - 2 = 1.825$ m the distance from the center to Joe.

$$\begin{aligned} F_{sam} \times r_1 &= F_{joe} \times r_2 \\ F_{sam} &= F_{joe} \frac{r_2}{r_1} \\ F_{sam} &= 1.548 F_{joe} \end{aligned}$$

We can now plug in the first equation to get :

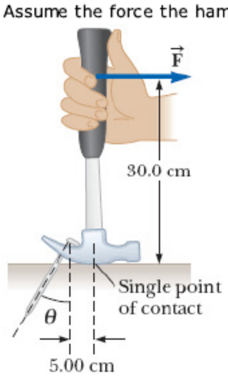
$$\begin{aligned} 1.548 F_{joe} + F_{joe} &= 545 \\ F_{joe} &= \frac{545}{1.548 + 1} \\ &= 214 \text{ N} \end{aligned}$$

and

$$\begin{aligned} F_{sam} &= 1.548 F_{joe} \\ &= 1.548 \times 214 \\ &= 331 \text{ N} \end{aligned}$$

0.4

The figure below shows a claw hammer being used to pull a nail out of a horizontal board where θ = 25.2°. The mass of the hammer is 1.00 kg. A force of 180 N is exerted horizontally as shown, and the nail does not yet move relative to the board. Assume the force the hammer exerts on the nail is parallel to the nail.



(a) Find the force exerted by the hammer claws on the nail.

magnitude [ ] ✗ 1.19 kN

direction [ ] ✗ 64.8 ° above the horizontal

(b) Find the force exerted by the surface on the point of contact with the hammer head.

a)

According to the action/reaction principle, the force exerted by the hammer claws on the nail is of the same magnitude as the force exerted by the nail on the hammer claws. Let's take the hammer as our system which is in equilibrium. The forces exerted are:

- The weight : $m\vec{g}$
- The force exerted by the nail: $\vec{f}$
- The applied force : $\vec{F}$
- The friction force from the surface on point O: $\vec{R}$

By choosing the clockwise direction as positive,the torques about point O are:

- $F \times 0.30 = 180 \times 0.30 = 54$ N.m
- $-f \times d = -f \times 0.05 \cos\theta$
- $\vec{R}$ and $m\vec{g}$ will not contribute.

Equilibrium requires:

$$\begin{aligned} 54 - f \times 0.05 \cos\theta &= 0 \\ f &= \frac{54}{0.05 \cos 25.2} \\ &= 1193.6 \text{ N} \\ &= 1.19 \text{ kN} \end{aligned}$$

Its direction is just $90 - 25.2 = 64.8°$

N.B. Vector force $\vec{f}$ can be written in the x-y coordinate system as :

$$\begin{aligned} \vec{f} &= -1193.6 \sin\theta \vec{i} - 1193.6 \cos\theta \vec{j} \\ &= -508\vec{i} - 1080\vec{j} \end{aligned}$$

b)

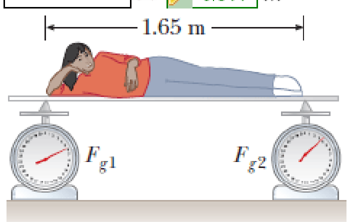
The hammer is still the system we are studying at equilibrium:

$$\begin{aligned} \vec{R} + m\vec{g} + \vec{F} + \vec{f} &= \vec{0} \\ \vec{R} &= -(m\vec{g} + \vec{F} + \vec{f}) \\ &= -(-1 \times 9.81\vec{j} + 180\vec{i} - 508\vec{i} - 1080\vec{j}) \\ &= 328\vec{i} + 1090\vec{j} \end{aligned}$$

0.5

In exercise physiology studies, it is sometimes important to determine the location of a person's center of mass. This determination can be done with the arrangement shown in the figure below. A light plank rests on two scales, which read F_{g1} = 370 N and F_{g2} = 300 N. A distance of 1.65 m separates the scales. How far from the woman's feet is her center of mass?

0.911 m



The lady is under three forces: F_{g1} , F_{g2} and gravity which will have no contribution to the net torque if we take the arbitrary point of rotation 0 as the center of mass. r_1 and r_2 are the distances from the first and second scale to the center of mass respectively. Under equilibrium we have:

$$\begin{aligned} F_{g1} \times r_1 &= F_{g2} \times r_2 \\ 370 \times r_1 &= 300 \times r_2 \end{aligned}$$

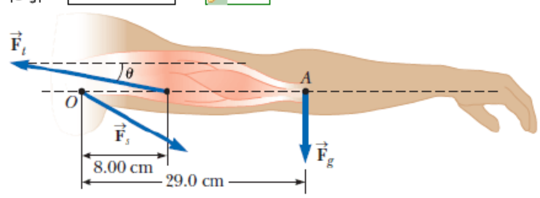
With the condition : $r_1 + r_2 = 1.65$ m
We can easily solve to get : $r_1 = 0.739$ m and $r_2 = 0.911$ m.
0.911 m is the distance from the center of mass to the lady's feet.

0.6

The arm in the figure below weighs 36.6 N. The gravitational force on the arm acts through point *A*. Determine the magnitudes of the tension force $\vec{\mathbf{F}}_t$ in the deltoid muscle and the force $\vec{\mathbf{F}}_s$ exerted by the shoulder on the humerus (upper-arm bone) to hold the arm in the position shown. (Let θ = 13.5°.)

$|\vec{\mathbf{F}}_t|$ = ✗ 568 N

$|\vec{\mathbf{F}}_s|$ = ✗ 561 N



Tension Force $\vec{F_t}$:

To keep the arm in the position of the figure, the net torque about Point O should be zero:

$$\begin{aligned}
F_g \times 0.29 &= F_t \times 0.08 \times \sin(\pi - \theta) \\
&= F_t \times 0.08 \times \sin(\theta) \\
F_t &= \frac{0.29 \times F_g}{0.08 \times \sin\theta} \\
&= \frac{0.29 \times 36.6}{0.08 \times \sin 13.5} \\
&= 568 \text{ N}
\end{aligned}$$

Force $\vec{F_s}$

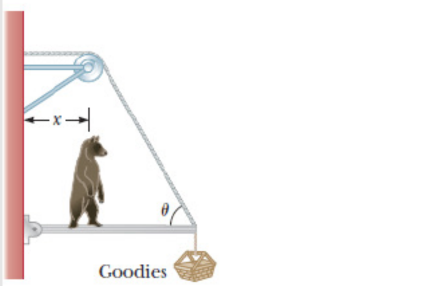
The arm is in equilibrium: $\sum \vec{F_i} = \vec{0}$:

$$\begin{aligned}
\vec{F_s} + \vec{F_t} + \vec{F_g} &= \vec{0} \\
\vec{F_s} &= -(\vec{F_t} + \vec{F_g}) \\
F_s^2 &= F_t^2 + F_g^2 + 2\vec{F_t} \cdot \vec{F_g} \\
F_s^2 &= F_t^2 + F_g^2 + 2 \times F_t \times F_g \times \cos(\vec{F_t}, \vec{F_g}) \\
F_s^2 &= F_t^2 + F_g^2 + 2 \times F_t \times F_g \times \cos(\frac{\pi}{2} + \theta) \\
F_s^2 &= F_t^2 + F_g^2 - 2 \times F_t \times F_g \times \sin\theta
\end{aligned}$$

$$
\begin{aligned}
F_s &= \sqrt{F_t^2 + F_g^2 - 2 \times F_t \times F_g \times \sin\theta} \\
&= \sqrt{568^2 + 36.6^2 - 2 \times 568 \times 36.6 \times \sin 13.5} \\
&= 561 \text{ N}
\end{aligned}
$$

0.7

A hungry bear weighing 750 N walks out on a beam in an attempt to retrieve a basket of goodies hanging at the end of the beam (see figure below). The beam is uniform, weighs 200 N, and is 5.50 m long, and it is supported by a wire at an angle of θ = 60.0°. The basket weighs 80.0 N.

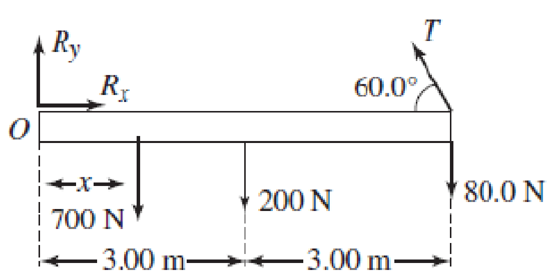


(a) Draw a free-body diagram for the beam.

(b) When the bear is at x = 1.10 m, find the tension in the wire supporting the beam and the components of the force exerted by the wall on the left end of the beam.

(c) If the wire can withstand a maximum tension of 775 N, what is the maximum distance the bear can walk before the wire breaks?

a) Free body diagram



Some of the numerical values in this free body diagram are different that the numerical values of the problem.

b)

- The tension in the wire can be evaluated from the equilibrium condition: $\sum \tau_i = 0$

$$\begin{aligned}
T \times 5.50 \times \sin 60 &= 200 \times 2.75 + 750 \times x + 80 \times 5.50 \qquad (**) \\
T &= \frac{200 \times 2.75 + 750 \times 1.10 + 80 \times 5.50}{5.50 \times \sin 60} \\
&= 381 \text{ N}
\end{aligned}$$

- R_x can be obtained from $\sum F_{xi} = 0$

$$\begin{aligned}
R_x - T\cos 60 &= 0 \\
R_x &= T\cos 60 \\
&= 381 \cos 60 \\
&= 191 \text{ N}
\end{aligned}$$

- R_y can be obtained from $\sum F_{yi} = 0$

$$\begin{aligned}
R_y + T\sin 60 &= 750 + 200 + 80 \\
R_y &= 750 + 200 + 80 - 381 \sin 60 \\
&= 700 \text{ N}
\end{aligned}$$

c)

In equation (**) above, for a maximum tension of $T_{max} = 775$ N the bear can reach a distance x_{max} evaluated as follows:

$$\begin{aligned}
T_{max} \times 5.50 \times \sin 60 &= 200 \times 2.75 + 750 \times x_{max} + 80 \times 5.50 \\
x_{max} &= \frac{T_{max} \times 5.50 \times \sin 60 - 200 \times 2.75 - 80 \times 5.50}{750} \\
&= \frac{775 \times 5.50 \times \sin 60 - 200 \times 2.75 - 80 \times 5.50}{750} \\
&= 3.60 \text{ m}
\end{aligned}$$