PHYSICS 111 HOMEWORK SOLUTION, week 4, chapter 5, sec 1-7

February 13, 2013

0.1

A 2.00-kg object undergoes an acceleration given by
$$\vec{a} = (6.00\hat{i} + 4.00\hat{j})m/s^2$$

- a) Find the resultatnt force acting on the object

- b) Find the magnitude of the resultant force

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a) Newton's Second Law: $\sum \vec{F} = m\vec{a}$

with $m = 2.00 kg$ and $\vec{a} = (6.00\hat{i} + 4.00\hat{j})m/s^2$

$$
\begin{aligned}
\sum \vec{F} &= 2.00 \times (6.00\hat{i} + 4.00\hat{j}) \\
\sum \vec{F} &= (12.00\hat{i} + 8.00\hat{j})N
\end{aligned}
$$

b) Magnitude of the resultant force

$$
\begin{aligned}
|\vec{F}| &= \sqrt{12^2 + 8^2} \\
|\vec{F}| &= \sqrt{208} \\
|\vec{F}| &= 14.42N
\end{aligned}
$$

0.2

The average speed of a nitrogen molecule in air is about $6.70 \times 10^2 m/s$, and its mass is about $4.68 \times 10^{-26} kg$

- a) If it takes $4.00 \times 10^{-13}s$ for a nitrogen molecule to hit a wall and rebound with the same speed but moving in an opposite direction (assumed to be the negative direction), what is the average acceleration of the molecule during this time interval ?

- b) What average force does the molecule exert on the wall?

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a) Average acceleration of the molecule:

Velocity vector is changing sign but keep a constant magnitude: $\vec{v}_{before} = v\hat{i}$ and $\vec{v}_{after} = -v\hat{i}$ the change in velocity is:

$$
\begin{aligned}
\Delta \vec{v} &= -v\hat{i} - v\hat{i} \\
&= -2v\hat{i}
\end{aligned}
$$

the change in magnitude is just : $\Delta v = 2v$, with average acceleration of :

$$
\begin{aligned}
a &= \frac{\Delta v}{\Delta t} \\
&= \frac{2v}{\Delta t} \\
&= \frac{2 \times 6.70 \times 10^2}{4.00 \times 10^{-13}} \\
&= 3.35 \times 10^{15} m/s^2
\end{aligned}
$$

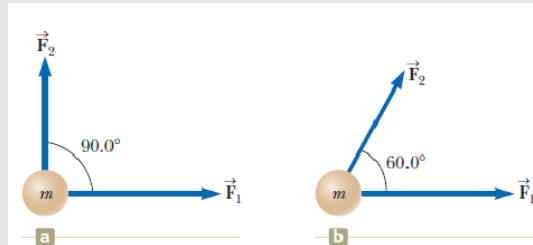### b) Average force exerted :

Newton's Second Law : $\sum \vec{F} = m\vec{a}$, with magnitude

$$
\begin{aligned}
\sum F &= ma \\
&= 4.68 \times 10^{-26} \times 3.35 \times 10^{15} \\
&= 1.57 \times 10^{-10} N
\end{aligned}
$$

3

Two forces $\vec{F}_1$ and $\vec{F}_2$ act on a 4.80-kg object. $F_1 = 30.0N$ and $F_2 = 16.0N$.

- a) Find the acceleration of the object for the configuration of forces shown in Figure (a).

- b) Find the acceleration of the object for the configuration of forces shown in Figure (b).

a) Acceleration magnitude and direction (from the horizontal x-axis)

Newton's Second Law: $\sum \vec{F} = m\vec{a}$
$\vec{F}_1 = 30\hat{i}$ and $\vec{F}_2 = 16\hat{j}$ resulting in $\sum \vec{F} = 30\hat{i} + 16\hat{j}$

$$
\begin{aligned}
\vec{a} &= \frac{\sum \vec{F}}{m} \\
&= \frac{30\hat{i} + 16\hat{j}}{4.80} \\
&= 6.25\hat{i} + 3.33\hat{j}
\end{aligned}
$$

Magnitude :

$$
\begin{aligned}
a &= \sqrt{6.25^2 + 3.33^2} \\
&= 7.08 m/s^2
\end{aligned}
$$

and direction angle:

$$
\theta = \arctan\left(\frac{3.33}{6.25}\right) = 28°
$$

b)

$\vec{F_1} = 30\hat{i}$ But

$$
\begin{aligned}
\vec{F_2} &= |\vec{F_2}| \times [\cos 60°\hat{i} + \sin 60°\hat{j}] \\
&= 16(0.5\hat{i} + 0.87\hat{j}) \\
&= 8\hat{i} + 13.86\hat{j}
\end{aligned}
$$

$$
\begin{aligned}
\vec{a} &= \frac{\sum \vec{F}}{m} \\
&= \frac{38\hat{i} + 13.86\hat{j}}{4.80} \\
&= 7.91\hat{i} + 2.89\hat{j}
\end{aligned}
$$

Magnitude :

$$
\begin{aligned}
a &= \sqrt{7.91^2 + 2.89^2} \\
&= 8.42 m/s^2
\end{aligned}
$$

and direction angle:

$$
\theta = \arctan\left(\frac{2.89}{7.91}\right) = 20°
$$

A 17.0-*lb* block rests on the floor.

- a) What force does the floor exert on the block?

- b) If a rope is tied to the block and run vertically over a pulley, and the other end is attached to a free-hanging 10.5-*lb* weight, what is the force exerted by the floor on the 17.0-*lb* block?

- If we replace the 10.5-*lb* weight in part (b) with a 36.0-*lb* weight, what is the force exerted by the floor on the 17.0-*lb* block?
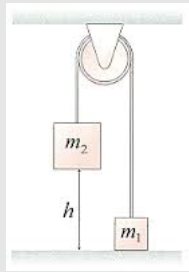
a)

$$1\text{-}lb = 0.453592kg$$

The block is at rest on the horizontal floor, gravity force $\vec{F}_g$ (pointing downward) is balanced by a normal force (pointing upward) $\vec{F}_n$ exerted by the floor to keep the block at rest, the two are equal in magnitude:

$$F_n = F_g = mg = 17.0 \times 0.4535 \times 9.81$$

$$F_n = 75.6N$$

Its direction is : upward

b)

The rope tied to the block vertically will generate a force $\vec{T}$ pointing upward and equal in magnitude to the gravity force acting on the free-hanging block: $T = m'g = 10.5 \times 0.4535 \times 9.81 = 46.7N$ The forces acting on the block in this situation are :

$$\vec{F}_g = -75.6\hat{j} \qquad \text{pointing downward}$$

$$\vec{T} = 46.7\hat{j} \qquad \text{pointing upward}$$

$$\vec{F}_n \qquad \text{pointing upward}$$

Forces should balance eachother to keep the block in rest: $\vec{F}_n + \vec{T} = -\vec{F}_g$

$$\begin{aligned} \vec{F}_n &= -\vec{T} - \vec{F}_g \\ &= -46.7\hat{j} + 75.6\hat{j} \\ &= 28.9\hat{j} \end{aligned}$$

magnitude is : 28.9 N direction is : upward

c)

The new free hanging weight is heavier than the block (36-lb vs 17-lb), the block will be lifted up from the floor and there will be no contact. The force exerted by the floor will not exist. Remember that forces of this type are contact forces unlike gravity.

0.5

The distance between two telephone poles is d = 48.5 m. When a 0.950-kg bird lands on the telephone wire midway between the poles, the wire sags h = 0.198 m.

- a)Draw a free-body diagram of the bird. (Do this on paper. Your instructor may ask you to turn in this diagram.)

- b) How much tension does the bird produce in the wire? Ignore the weight of the wire.

a) The free body diagram of the bird

d=48.5m

$\vec{T_2}$ $\vec{T_1}$

α

h

$d/2$

Free Body Diagram

$\vec{F_g} = m\vec{g}$
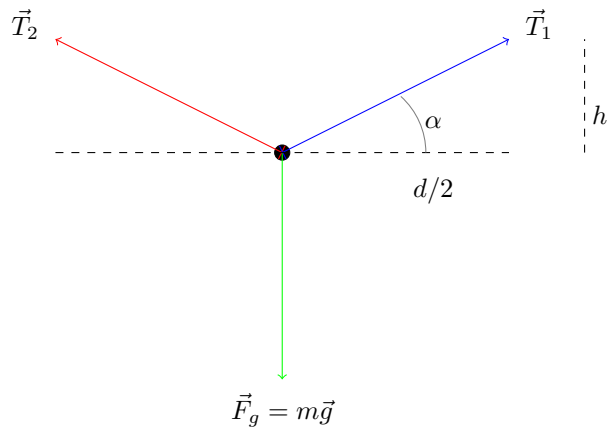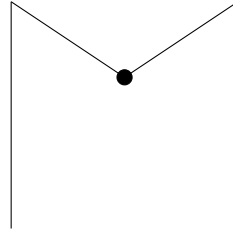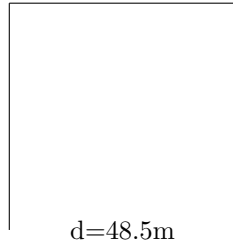
b)

The tension produced by the bird in the wire is the same tension produced by the wire on the bird (action-reaction law).

For symetry considerations $\vec{T_1}$ and $\vec{T_2}$ are of same magnitude.

Newtons Law : $\sum \vec{F} = \vec{F_g} + \vec{T_1} + \vec{T_2} = \vec{0}$

A projection on the vertical axis should give:

$T_1 \sin \alpha + T_2 \sin \alpha - mg = 0$ with $T_1 = T_2 = T$

on the other hand

$\tan \alpha = \frac{h}{d/2} = \frac{0.198}{24.25} = 0.082$ thus $\alpha = 0.47°$

$$
\begin{aligned}
T &= \frac{mg}{2 \sin \alpha} \\
&= \frac{0.950 \times 9.81}{2 \times \sin 0.47°} \\
&= 568N
\end{aligned}
$$

8

0.6

An object of mass $m_1 = 3.80\text{-}kg$ placed on a frictionless, horizontal table is connected to a string that passes over a pulley and then is fastened to a hanging object of mass $m_2 = 7.20\text{-}kg$ as shown in the figure.

- a) Draw free-body diagrams of both objects.

- b)Find the magnitude of the acceleration of the objects.

-) Find the tension in the string.
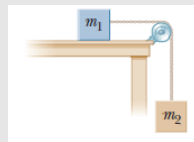
a) Free body diagrams

- Object
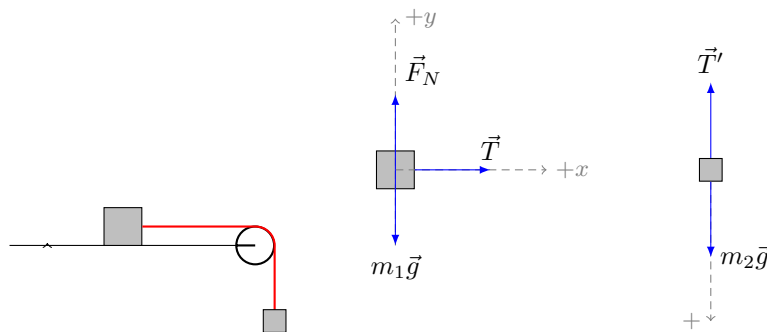
b)

- For object m_1

$$\sum \vec{F} = m_1\vec{g} + \vec{T} + \vec{F}_N = m_1\vec{a_1} \; ; \qquad \vec{a}_1 = a_1\hat{i} \text{ (horizontal motion)}$$

Prejection on the x-axis should give:

$$0 + T + 0 = m_1 a_1 \qquad\qquad (1)$$

9

- For object m_2

  $$\sum \vec{F} = m_2\vec{g} + \vec{T'} = m_2\vec{a_2} \; ; \qquad \vec{a_2} = a_2\hat{j} \text{ (vertical motion)}$$

  Prejection on the y-axis (+ sign downward only for convenience)should give:

  $$m_2g - T' = m_2a_2 \qquad\qquad (2)$$

- the two objects are connected together and the magnitude of their accelerations is the same: $a_1 = a_2 = a$

  the string tensions are also the same $T = T'$

  We can derive :

  $$\begin{aligned} m_1a &= T \\ &= m_2g - m_2a \end{aligned}$$

  and finally

  $$\begin{aligned} a &= \frac{m_2g}{m_1 + m_2} \\ &= \frac{7.20 \times 9.81}{7.20 + 3.80} \\ &= 6.42 m/s^2 \end{aligned}$$

c) Tension in the strings

$$\begin{aligned} T &= m_1a \\ &= 3.80 \times 6.42 \\ &= 24.4N \end{aligned}$$

0.7

In the Atwood machine shown below, $m_1 = 2.00$-kg and $m_2 = 5.60$-kg. The masses of the pulley and string are negligible by comparison. The pulley turns without friction and the string does not stretch. The lighter object is released with a sharp push that sets it into motion at $v_i = 2.15$ m/s downward.

- a) How far will m_1 descend below its initial level?

- b) Find the velocity of m_1 after 1.80 s.

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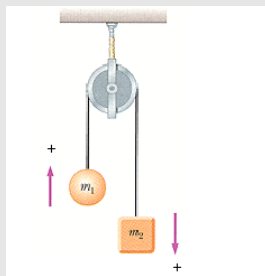
a)

We can start by drawing the body diagrams for both masses with the directions of choice:
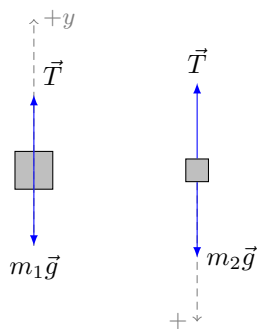
- m_1 descending

$$-m_1 g + T = m_1 a$$

- m_2 ascending

$$m_2 g - T = m_2 a$$

- this gives :

$$(m_2 - m_1)g = (m_1 + m_2)a$$

and

$$
\begin{aligned}
a &= \frac{(m_2 - m_1)g}{m_1 + m_2} \qquad (*)\\
&= \frac{5.60 - 2}{2 + 5.60} 9.81\\
&= 4.65 m/s^2
\end{aligned}
$$

- we can use the time-independent equation to derive the distance crossed by mass m_1 at the moment its velocity vanishes and about to start rising

$$v_f^2 - v_i^2 = 2ad$$

then

$$
\begin{aligned}
d &= \frac{v_f^2 - v_i^2}{2a}\\
&= \frac{0 - 2.15^2}{2 \times (4.65)}\\
&= -0.497m\\
|d| &= 49.7cm
\end{aligned}
$$

b)

to get the velocity of m_1 we use : $v = v_i + at = -2.15 + 4.65t$ at $t = 1.80s$ the velocity is $v = -2.15 + 4.65 \times 1.80 = 6.22 m/s$

its direction is upward

0.8

Three blocks are in contact with one another on a frictionless, horizontal surface as shown in the figure below. A horizontal force is applied to m_1. Take $m_1 = 2.00kg$, $m_2 = 3.00$ kg, $m_3 = 4.55$ kg, and $F = 22.5$ N.

- a) Draw a separate free-body diagram for each block.

- b) Find the acceleration of the blocks

- c) Find the resultant force on each block.

- d) Find the magnitudes of the contact forces between the blocks.

- e) You are working on a construction project. A coworker is nailing up plasterboard on one side of a light partition, and you are on the opposite side, providing "backing" by leaning against the wall with your back pushing on it. Every hammer blow makes your back sting. The supervisor helps you put a heavy block of wood between the wall and your back. Using the situation analyzed in parts (a) through (d) as a model, explain how this change works to make your job more comfortable.
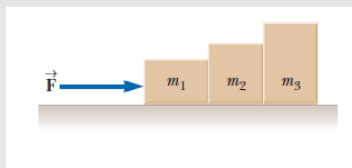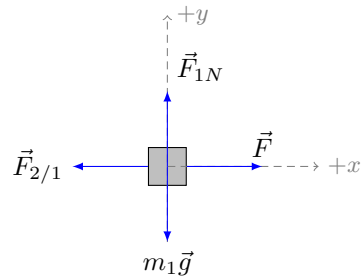
a) The free Body diagrams
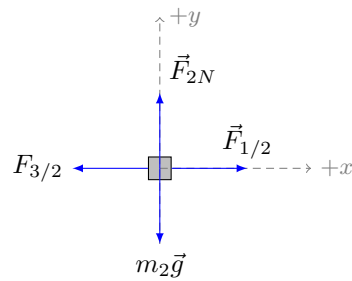
We use the following notations:

$F_{i/j}$ for the force exxerted by mass i on mass j.

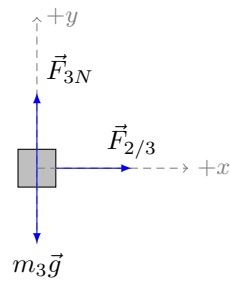F_{iN} for the normal force exerted by the surface on mass i (no friction)

- mass m_1

- mass m_2

- mass m_3

b) acceleration of the blocks

all the block will be moving with the same acceleration and velocit. $\sum \vec{F} = m_1 a$
projected on the x-axis

- on each mass we get the set of the following equations:

$F_{2/3} = m_3 a$ $\qquad$ $F_{1/2} - F_{3/2} = m_2 a$ $\qquad$ $F - F_{2/1} = m_1 a$

- action/reaction principle requires $F_{1/2} = F_{2/1}$ $\qquad$ and $\qquad$ $F_{2/3} = F_{3/2}$

- $m_3 a = F_{1/2} - m_2 a = F - m_1 a - m_2 a$;

$$(m_1 + m_2 + m_3)a = F$$

-

$$
\begin{aligned}
a &= \frac{F}{m_1 + m_2 + m_3} \\
&= \frac{22.5}{2 + 3 + 4.55} \\
&= 2.36 m/s^2
\end{aligned}
$$

The acceleration has the direction : Left $\longrightarrow$ Right

c) Resultant force on each block

$\sum \vec{F_i} = m_i \vec{a}$

- on m_1 $\qquad$ $\sum F_1 = m_1 a = 2 \times 2.36 = 4.72 N$

- on m_2 $\qquad$ $\sum F_2 = m_2 a = 3 \times 2.36 = 7.08 N$

- on m_3 $\qquad$ $\sum F_3 = m_3 a = 4.55 \times 2.36 = 10.74 N$

d)magnitudes of the contact forces

From the equations of section b)

- $F_{2/1} = F_{1/2} = F - m_1 a = 22.5 - 2 \times 2.36 = 17.8 N$

- $F_{3/2} = F_{2/3} = F_{1/2} - m_2 a = 17.8 - 3 \times 2.36 = 10.7 N$

d)

Notice that the horizontal forces are reducing in magnitude going from block to block towards the right side:

$$F > F_{1/2} > F_{2/3}$$

The impact of the nailing will be less and less as we put more blocks. By stacking plasterboard blocks we can make a coworker's job much more comfortable.