PHYSICS 111
HOMEWORK #6 SOLUTION

February 22, 2013
A block of mass \( m = 3.20 \text{ kg} \) is pushed a distance \( d = 4.60 \text{ m} \) along a frictionless, horizontal table by a constant applied force of magnitude \( F = 16.0 \text{ N} \) directed at an angle \( \theta = 26.0^\circ \) below the horizontal as shown in the figure below.

a) Determine the work done on the block by the applied force. 

\[ W_1 = \vec{F} \cdot \vec{d} = F \times d \times \cos \theta = 16 \times 4.60 \times \cos(26^\circ) = 66.2 \text{ J} \]

b) The normal force exerted by the table is perpendicular to the displacement of the block (\( \vec{F}_N \perp \vec{d} \)), the dot product should be zero \( W_2 = 0 \)
c) Gravity force \( m\vec{g} \) is also perpendicular to the displacement \((m\vec{g} \perp \vec{d})\); \( W_3 = 0 \)

d) The work done by the net force is the total work done by all forces:

\[
W = \vec{F}_{\text{net}} \cdot \vec{d} \\
= \vec{F} \cdot \vec{d} + \vec{F}_N \cdot \vec{d} + m\vec{g} \cdot \vec{d} \\
= W_1 + W_2 + W_3 \\
= 66.2 \text{ J}
\]

0.2

In 1990 Walter Arfeuille of Belgium lifted a 281.5-kg object through a distance of 17.1 cm using only his teeth. (a) How much work was done on the object by Arfeuille in this lift, assuming the object was lifted at constant velocity? 472 J (b) What total force was exerted on Arfeuille’s teeth during the lift? 2.76 kN

- a) How much work was done on the object by Arfeuille in this lift, assuming the object was lifted at constant velocity?

- b) What total force was exerted on Arfeuille’s teeth during the lift?

a) The object is lifted up at constant velocity, its acceleration should be zero and, consequently, the net force is zero. \( \vec{F} + m\vec{g} = \vec{0} \) and \( F = mg \)

\[
W = \vec{F}_{\text{net}} \cdot \vec{d} \\
= m \times g \times d \\
= 281.5 \times 9.81 \times 0.171 \\
= 472 \text{ J}
\]
b)

Action/Reaction principle requires the force exerted by Walter on the object and the force exerted by the object on Walter to be of the same magnitude (and opposite directions).

\[ F' = F = mg = 281.5 \times 9.81 = 2758 \text{ N} = 2.76 \text{ kN} \]

0.3

Vector \( \vec{A} \) has a magnitude of 5.15 units, and vector \( \vec{B} \) has a magnitude of 8.55 units. The two vectors make an angle of 46.0° with each other. Find \( \vec{A} \cdot \vec{B} \)

\[
\vec{A} \cdot \vec{B} = |\vec{A}| \times |\vec{B}| \times \cos \theta \\
= 5.15 \times 8.55 \times \cos 46.0° \\
= 30.6
\]

0.4

A force \( \vec{F} = (8\hat{i} - 3\hat{j}) \text{ N} \) acts on a particle that undergoes a displacement \( \Delta \vec{r} = (2\hat{i} + \hat{j}) \text{ m} \)

- Find the work done by the force on the particle. (Calculate all numerical answers to three significant figures.)
- What is the angle between \( \vec{F} \) and \( \Delta \vec{r} \)?
The work done by the force on the particle is:

\[ W = \vec{F} \cdot \Delta \vec{r} \]
\[ = 8 \times 2 + (-3) \times 1 \]
\[ = 13 \text{ J} \]

to find the angle we use:

\[ W = \vec{F} \cdot \Delta \vec{r} = |\vec{F}| \times |\Delta \vec{r}| \times \cos \theta \]
\[ 13 = \sqrt{8^2 + 3^2} \times \sqrt{2^2 + 1^2} \times \cos \theta \]
\[ \theta = 47.12^\circ \]

0.5

For \( \vec{A} = \hat{i} + \hat{j} - 4\hat{k} \), \( \vec{B} = -4\hat{i} + \hat{j} + 2\hat{k} \) and \( \vec{C} = 2\hat{j} - 3\hat{k} \), find
\[ \vec{C} \cdot (\vec{A} - \vec{B}) \]

\[ \vec{A} - \vec{B} = (\hat{i} + \hat{j} - 4\hat{k}) - (-4\hat{i} + \hat{j} + 2\hat{k}) \]
\[ = 5\hat{i} - 6\hat{k} \]

\[ \vec{C} \cdot (\vec{A} - \vec{B}) = (2\hat{j} - 3\hat{k}) \cdot (5\hat{i} - 6\hat{k}) \]
\[ = 0 \times 5 + 2 \times 0 + (-3) \times (-6) \]
\[ = 18 \]
An archer pulls her bowstring back 0.410 m by exerting a force that increases uniformly from zero to 213 N.

- a) What is the equivalent spring constant of the bow?
- b) How much work does the archer do on the string in drawing the bow?

If we apply Hook’s-Law to the bowstring, the force required for stretching is varying uniformly from 0 to 213 N and we should have:

$$\vec{F} = -k \vec{d}$$

which gives

$$k = \frac{F}{d} = \frac{213}{0.410} = 519.5 \text{ N/m}$$
b)

The work done by the archer through force $\vec{F}$ is equivalent to the elastic potential energy ($PE = \frac{1}{2}kd^2$) stored by the bowstring as it stretches

$$W = \frac{1}{2} kd^2$$

$$= \frac{519.5 \times 0.410^2}{2}$$

$$= 43.7 \text{ J}$$

0.7

A light spring with spring constant 1400 N/m hangs from an elevated support. From its lower end hangs a second light spring, which has spring constant 1900 N/m. An object of mass 1.50 kg is hung at rest from the lower end of the second spring.

- a) Find the total extension distance of the pair of springs.
- b) Find the effective spring constant of the pair of springs as a system. We describe these springs as in series.
Object with mass \( m \) is at rest; if spring \( k_2 \) is stretched by \( x_2 \)

\[
mg = k_2 x_2
\]

Tension in the two springs is the same; if spring \( k_1 \) is stretched by \( x_1 \)

\[
k_1 x_1 = k_2 x_2
\]

Thus,

\[
mg = k_1 x_1 = k_2 x_2
\]

Finally,

\[
x = x_1 + x_2 = \frac{mg}{k_1} + \frac{mg}{k_2} = \frac{1.5 \times 9.81}{1400} + \frac{1.5 \times 9.81}{1900} = 1.82 \times 10^{-2} \text{m}
\]

b) If we think of this set of springs as one spring with constant \( k_{eff} \) we have:

\[
mg = k_{eff} x
\]

which gives

\[
k_{eff} = \frac{mg}{x} = \frac{1.5 \times 9.81}{1.82 \times 10^{-2}} = 806 \text{N/m}
\]

N.B: The in-series springs are like resistors in parallel and their effective spring constant can be easily derived as:

\[
k_{eff} = \frac{k_1 k_2}{k_1 + k_2}
\]
A 0.300-kg particle has a speed of 2.10 m/s at point A and kinetic energy of 7.90 J at point B

- a) What is its kinetic energy at A?
- b) What is its speed at B?
- c) What is the net work done on the particle by external forces as it moves from to A to B?

a) Kinetic energy and speed are related; at point A we have:

\[ T_A = \frac{1}{2} m v_A^2 \]

\[ = \frac{1}{2} \times 0.300 \times 2.10^2 \]

\[ = 0.662 \text{J} \]

b) Speed at B:

\[ v_B = \sqrt{\frac{2T}{m}} \]

\[ = \sqrt{\frac{2 \times 7390}{0.30}} \]

\[ = 7.25 \text{m/s} \]

c) Kinetic energy change between A and B is just the net work done on the particle

\[ W = T_B - T_A \]

\[ = 7.90 - 0.662 \]

\[ = 7.24 \text{J} \]
A worker pushing a 35.0-kg wooden crate at a constant speed for 11.7 m along a wood floor does 390 J of work by applying a constant horizontal force of magnitude $F$ on the crate.

- a) Determine the value of $F$.
- b) If the worker now applies a force greater than $F$, describe the subsequent motion of the crate.
- c) Describe what would happen to the crate if the applied force is less than $F$.

\[ F = \frac{W}{d} = \frac{390}{11.7} = 33.3 \text{N} \]

b) The force $F=33.3$ N computed above was moving the grate at constant speed (Balancing friction), any increase above 33.3N will just accelerate the grate which will move with increasing speed.

c) If the applied force is less, friction will overcome the applied force which will decelerate the grate and slow it down till finally rest.
A 7.80-g bullet moving at 490 m/s penetrates a tree trunk to a depth of 4.6 cm.

• a) Use work and energy considerations to find the average frictional force that stops the bullet.

• b) (b) Assuming the friction force is constant, determine how much time elapses between the moment the bullet enters the tree and the moment it stops moving.

a)

The friction force is the only force that does work because the bullet is moving horizontally, the work done is just the variation of kinetic energy.

\[
0 - \frac{1}{2}mv^2 = W = \vec{F}_{fric} \cdot \vec{d} = -F \times d
\]

\[
F = \frac{mv^2}{d}
\]

\[
= \frac{7.80 \times 10^{-3} \times 490^2}{2 \times 4.6 \times 10^{-2}}
\]

\[
= 20356 \text{N}
\]

b)

Newton’s second law projected horizontally gives the acceleration as

\[
a = \frac{F}{m} = -\frac{20356}{7.80 \times 10^{-2}} = -2610 \times 10^3 \text{m/s}^2
\]

Notice that the bullet decelerates inside the tree trunk and that explains the minus sign for acceleration.

\[
v = at + v_0
\]

when the bullet stops \( v = 0, v_0 = 490 \text{m/s} \) and we finally get :

\[
t = \frac{v - v_0}{a} = \frac{0 - 490}{-2610 \times 10^3} = 0.000188 \text{s}
\]