PHYSICS 111 HOMEWORK SOLUTION #8

March 24, 2013

0.1

A particle of mass m moves with momentum of magnitude p.

- a) Show that the kinetic energy of the particle is: $K = \frac{p^2}{2m}$ (Do this on paper. Your instructor may ask you to turn in this work.)
- b) Express the magnitude of the particle's momentum in terms of its kinetic energy and mass.

a)

By definition, the momentum of a partcile moving with velocity $\vec{v}$ is : $\vec{p} = m\vec{v}$. Its magnitude is $p = mv$.

Kinetic energy is :

$$\begin{aligned} K &= \frac{mv^2}{2} \\ &= \frac{m(\frac{p}{m})^2}{2} \\ &= \frac{p^2}{2m} \end{aligned}$$

b)

$K = \frac{mv^2}{2}$, therefore $v = \sqrt{\frac{2K}{m}}$

$$\begin{aligned} p &= mv \\ &= m\sqrt{\frac{2K}{m}} \\ &= \sqrt{m^2\frac{2K}{m}} \\ &= \sqrt{2Km} \end{aligned}$$

0.2

An object has a kinetic energy of 239 J and a momentum of magnitude 27.3 kgm/s. Find the speed and the mass of the object.

- Let's use the expressions of problem 1

$$\begin{aligned} \frac{K}{p} &= \frac{mv^2}{2mv} \\ &= \frac{v}{2} \\ v &= \frac{2K}{p} \\ &= \frac{2 \times 239}{27.3} \\ &= 17.51\text{m/s} \end{aligned}$$

- The mass cam be then obtained from momentum as :

$$\begin{aligned} m &= \frac{p}{v} \\ &= \frac{27.3}{17.51} \\ &= 1.56\text{kg} \end{aligned}$$

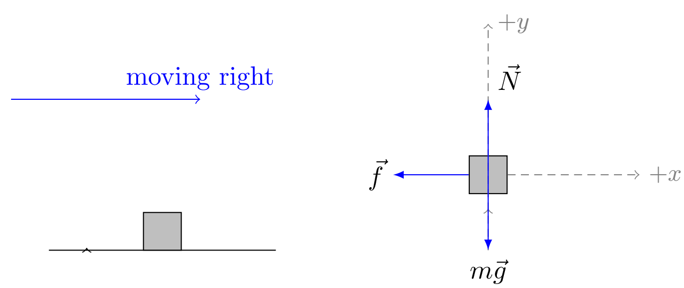
0.3

At one instant, a 17.0-kg sled is moving over a horizontal surface of snow at 3.10 m/s. After 7.00 s has elapsed, the sled stops. Use a momentum approach to find the magnitude of the average friction force acting on the sled while it was moving.

- The sled started with a speed of 3.10 m/s and will come to stop after 7.00s. This change in speed or momentum is due to friction.

We can rewrite Newton's second law by taking momentum change into consideration as follows:

$$\begin{aligned}\sum \vec{F}_i &= m\vec{a} \\ &= m\frac{d\vec{v}}{dt} \\ &= \frac{d(m\vec{v})}{dt} \\ &= \frac{d\vec{p}}{dt}\end{aligned}$$



If we project this on the horizontal surface , the only force that remains is friction f. In average:

$$\begin{aligned}-f &= \frac{p_f - p_i}{\Delta t} \\ &= m\frac{v_f - v_i}{\Delta t} \\ &= 17 \times \frac{0 - 3.10}{7} \\ f &= 7.53\text{N}\end{aligned}$$

0.4

A 45.2-kg girl is standing on a 159-kg plank. Both originally at rest on a frozen lake that constitutes a frictionless, flat surface. The girl begins to walk along the plank at a constant velocity of 1.48 $\vec{i}$ m/s relative to the plank.

- a) What is the velocity of the plank relative to the ice surface?
- b) What is the girl's velocity relative to the ice surface?

a)

The system $\{girl{+}plank\}$ is in a frictionless environment for which momentum should be conserved during motion.
We adopt the following notations :

- the girl has mass m and velocity $\vec{v}$ relative to surface
- the plank has mass M and velocity $\vec{V}$ relative to surface
- the girl's velocity relative to the plank is $\vec{v}_{grl/plk} = 1.48\vec{i}$

At rest vector momentum is $\vec{0}$, during the motion this momentum is $m\vec{v} + M\vec{V}$. We should have : $m\vec{v} + M\vec{v} = \vec{0}$ or consequently $m\vec{v} = -M\vec{V}$. This already indicates that the girl and the plank are moving in opposite directions. On the other hand, the girl's velocity relative to surface is an addition of her velocity relative to the plank and the velocity of the plank relative to the surface :

$$\vec{v} = \vec{v}_{grl/plk} + \vec{V}$$

$$\begin{aligned}
m\vec{v} &= -M\vec{V} \\
m(\vec{v}_{grl/plk} + \vec{V}) &= -M\vec{V} \\
-(m+M)\vec{V} &= m\vec{v}_{grl/plk} \\
\vec{V} &= -\frac{m}{m+M}\vec{v}_{grl/plk} \\
&= -\frac{45.2}{159+45.2} \times 1.48\vec{i} \\
&= -0.33\vec{i}
\end{aligned}$$

The plank is moving with speed 0.33 m/s in the opposite direction.

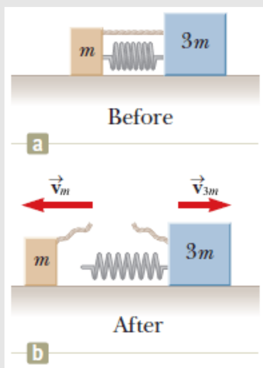
b)

The girl's velocity relative to the ice surface is:

$$\begin{aligned}\vec{v} &= \vec{v}_{grl/plk} + \vec{V} \\ &= 1.48\vec{i} - 0.33\vec{i} \\ &= 1.15\vec{i} \\ v &= 1.15\text{m/s}\end{aligned}$$

0.5

Two blocks of masses m and 3m are placed on a frictionless, horizontal surface. A light spring is attached to the more massive block, and the blocks are pushed together with the spring between them as shown in the figure below. A cord initially holding the blocks together is burned; after that happens, the block of mass 3m moves to the right with a speed of $\vec{V}_{3m} = 2.60\vec{i}$ m/s



- a)What is the velocity of the block of mass m? (Assume right is positive and left is negative.)
- b) Find the system's original elastic potential energy, taking m = 0.460 kg.
- c) Is the original energy in the spring or in the cord?
- d) Explain your answer to part (c).
- e) Is the momentum of the system conserved in the bursting-apart process?
- f) Explain how that is possible considering there are large forces acting.
- g)Explain how that is possible considering there is no motion beforehand and plenty of motion afterward?

a)

Momentum is conserved and we have :

$$\begin{aligned} m\vec{v}_m + 3m\vec{V}_{3m} &= \vec{0} \\ \vec{v}_m &= -\frac{3m}{m}\vec{V}_{3m} \\ &= -3\vec{V}_{3m} \\ &= -3 \times 2.60\vec{i} \\ &= -7.80\vec{i} \\ |\vec{v}_m| &= 7.80\text{m/s} \end{aligned}$$

b)

Total energy is conserved, gravitation potential energy doesnt change but elastic potential energy changes after the spring is released.

$$\begin{aligned} U_{el_f} - U_{el_i} &= -(K_f - K_i) \\ 0 - U_{el_f} &= -(\frac{1}{2}mv_m^2 + \frac{1}{2}3mV_{3m}^2) \\ U_{el_i} &= \frac{m}{2}(v_m^2 + 3V_{3m}^2) \\ &= \frac{0.460}{2}(7.80^2 + 3 \times 2.60^2) \\ &= 18.7\text{J} \end{aligned}$$

c)

the original energy is in the spring.

d)

A force had to be exerted over a distance to compress the spring, transferring energy into it by work. The cord exerts force, but over no distance.

e)

the momentum of the system is conserved in the bursting-apart process and that's what we used in the first question.

f)

The forces on the two blocks are internal forces, which cannot change the momentum of the system the system is isolated.

e)

Even though there is motion afterward, the final momenta are of equal magnitude in opposite directions so the final momentum of the system is still zero.

0.6

After a 0.390-kg rubber ball is dropped from a height of 1.70 m, it bounces off a concrete floor and rebounds to a height of 1.55 m.

- a) Determine the magnitude and direction of the impulse delivered to the ball by the floor.
- b) Estimate the time the ball is in contact with the floor to be 0.07 seconds. Calculate the average force the floor exerts on the ball.

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a)

Hitting the floor then bouncing up will cause a momentum change , the impulse delivered to the ball by the floor is just this momentum change.

$$\begin{aligned} Impulse &= \Delta Momentum \\ &= m\vec{v}_{after} - m\vec{v}_{before} \end{aligned}$$

Falling from a 1.70 m will give the speed at the moment the ball hits the floor:

$$\begin{aligned} v_{before}^2 - 0 &= 2a\Delta H \\ v_{before}^2 &= 2g\Delta H \\ v_{before} &= \sqrt{2g\Delta H} \\ &= \sqrt{2 \times 9.81 \times 1.70} \\ &= 5.77\text{m/s} \\ \vec{v}_{before} &= -5.77\vec{j} \end{aligned}$$

Similarly, Bouncing up to 1.55m will give the speed at the moment the ball bounces up:

$$\begin{aligned} v_{after} &= \sqrt{2 \times 9.81 \times 1.55} \\ &= 5.51\text{m/s} \\ \vec{v}_{after} &= 5.51\vec{j} \end{aligned}$$

Finally,

$$\begin{aligned} Impulse &= \Delta Momentum \\ &= m\vec{v}_{after} - m\vec{v}_{before} \\ &= 0.390(5.51\vec{j} + 5.77\vec{j}) \\ &= 4.40\vec{j} \end{aligned}$$

The impulse delivered amounts to 4.40 kg.m/s with direction up.

b)

In problem 2 we rewrote Newton's 2nd Law as: $\sum \vec{F}_i = \frac{d\vec{P}}{dt}$.
The average force the floor exerts on the floor is then

$$\begin{aligned} \vec{F} &= \frac{\Delta P}{\Delta t} \\ &= \frac{4.40}{0.07}\vec{j} \\ &= 62.9\vec{j} \end{aligned}$$

with maginutude 62.9 N and direction up.

0.7

A tennis player receives a shot with the ball (0.060 0 kg) traveling horizontally at 59.5 m/s and returns the shot with the ball traveling horizontally at 37.5 m/s in the opposite direction. (Assume the initial direction of the ball is in the x direction.)

- a) What is the impulse delivered to the ball by the tennis racquet?
- b)What work does the racquet do on the ball?

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a)

We will use the same procedure as in problem 6.
The impulse delivered to the ball is :

$$
\begin{aligned}
Impulse &= \Delta Momentum \\
&= m\vec{v}_{after} - m\vec{v}_{before} \\
&= 0.060(37.5\vec{i} - (-59.5)\vec{i}) \\
&= 5.82\vec{i}\ \text{kg.m/s}\ \ or\ \text{N.s}
\end{aligned}
$$

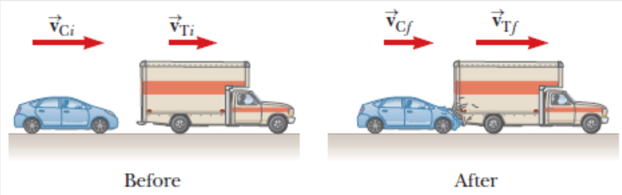
b)

The work done by the raquet on the ball can be calculated from the kinetic enrgy change as:

$$
\begin{aligned}
W &= \Delta K \\
&= \frac{1}{2}m(v_f^2 - v_i^2) \\
&= \frac{1}{2} \times 0.06(37.5^2 - 59.5^2) \\
&= -64\text{J}
\end{aligned}
$$

0.8

A 1 240.0 kg car traveling initially with a speed of 25.000 m/s in an easterly direction crashes into the back of a 8 100.0 kg truck moving in the same direction at 20.000 m/s. The velocity of the car right after the collision is 18.000 m/s to the east.



- a)What is the velocity of the truck right after the collision? (Give your answer to five significant figures.)
- b) What is the change in mechanical energy of the cartruck system in the collision?
- c) Account for this change in mechanical energy.

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a)

Conservation of momentum of the {car+truck} system can be expressed as:

$$\begin{aligned} m\vec{V}_{ci} + M\vec{V}_{ti} &= m\vec{V}_{cf} + M\vec{V}_{tf} \\ \vec{V}_{tf} &= \frac{m}{M}(\vec{V}_{ci} - \vec{V}_{cf}) + \vec{V}_{ti} \\ &= \frac{1240}{8100}(28 - 18)\vec{i} + 20\vec{i} \\ &= 21.0716\vec{i} \end{aligned}$$

The truck will keep moving east with speed of 21.0716 m/s

b)

The change in mechanical energy is computed through the kinetic energy change as no potential energy change takes place:

$$\begin{aligned} \Delta E &= \frac{1}{2}m(V_{cf}^2 - V_{ci}^2) + \frac{1}{2}M(V_{tf}^2 - V_{ti}^2) \\ &= \frac{1}{2} \times 1240 \times (18^2 - 25^2) + \frac{1}{2} \times 8100(21.072^2 - 20^2) \\ &= -8301\text{J} \end{aligned}$$

c)

Most of the energy was transformed to internal energy with some being carried away by sound.

0.9

A 9.6-g bullet is fired into a stationary block of wood having mass m = 4.90 kg. The bullet imbeds into the block. The speed of the bullet-plus-wood combination immediately after the collision is 0.603 m/s. What was the original speed of the bullet? (Express your answer with four significant figures.)

We use momentum conservation and we take into account that the bullet and the block have the same speed after collision

$$m\vec{v}_i + M\vec{V}_i = m\vec{v}_f + M\vec{V}_f$$

$$\begin{aligned} m\vec{v}_i + 0 &= (m+M)\vec{V}_f \\ v_i &= \frac{m+M}{m} V_f \\ &= \frac{0.0096 + 4.90}{0.0096} 0.603 \\ &= 308.4 \mathrm{m/s} \end{aligned}$$

0.10

A neutron in a nuclear reactor makes an elastic, head-on collision with the nucleus of a carbon atom initially at rest.

- a) What fraction of the neutron's kinetic energy is transferred to the carbon nucleus? (The mass of the carbon nucleus is about 12.0 times the mass of the neutron.)
- b) The initial kinetic energy of the neutron is 1.1010^{-13} J. Find its final kinetic energy and the kinetic energy of the carbon nucleus after the collision.

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a

Let's adopt the following notations :

- for the neutron, mass m, v_i and v_f are the initial and final velocity respectively.
- for the atom , mass M , V_i and V_f are the initial and final velocity respectively.

The fraction of the neutron's kinetic energy that's transferred to the carbon nucleus is just :

$$\frac{\frac{1}{2}MV_F^2}{\frac{1}{2}mv_i^2}$$

Conservation of momentum on the head-on collision gives:

$$\begin{aligned} mv_i &= mv_f + MV_f \\ v_i - v_f &= \frac{M}{m} V_f \\ &= 12V_f \end{aligned}$$

Conservation of kinetic energy gives :

$$\begin{aligned}
\frac{1}{2}mv_i^2 &= \frac{1}{2}mv_f^2 + \frac{1}{2}MV_f^2 \\
v_i^2 - v_f^2 &= \frac{M}{m}V_f^2 \\
&= 12V_f^2 \\
&\textit{and} \\
\frac{v_i^2 - v_f^2}{v_i - v_f} = v_i + v_f &= \frac{12V_f^2}{12V_f} \\
&= V_f
\end{aligned}$$

We get the set of equations :

$$v_i - v_f = 12V_f$$

and

$$v_i + v_f = V_f$$

to finally obtain

$$v_i = \frac{13}{2}V_f$$

and the fraction transfered to the atom:

$$\begin{aligned}
\frac{\frac{1}{2}MV_F^2}{\frac{1}{2}mv_i^2} &= 12 \times (\frac{2}{13})^2 \\
&= 0.284
\end{aligned}$$

b)

The kinetic energy of the carbon nucleus is:

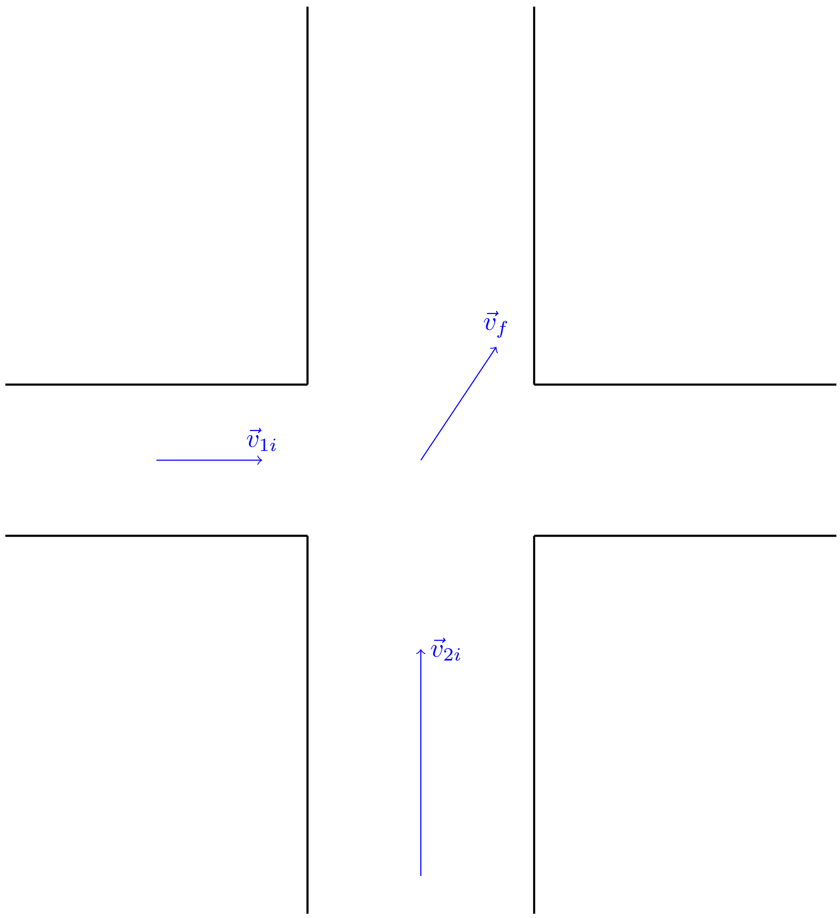
$$\begin{aligned}
\frac{1}{2}MV_F^2 &= 0.284Ki_{initial} \\
&= 0.284 \times 1.10 \times 10^{-13} \\
&= 3.12 \times 10^{-14}\text{J}
\end{aligned}$$

The final kinetic energy of the neutron is :

$$\begin{aligned}
Ki_{final} &= 1.10 \times 10^{-13} - 3.12 \times 10^{-14} \\
&= 7.88 \times 10^{-14}\text{J}
\end{aligned}$$

0.11

Two automobiles of equal mass approach an intersection. One vehicle is traveling with velocity 13.0 m/s toward the east and the other is traveling north with speed v2 . Neither driver sees the other. The vehicles collide in the intersection and stick together, leaving parallel skid marks at an angle of 61.5° north of east. Determine the initial speed v_{2i} of the northward-moving vehicle.
The speed limit for both roads is 35 mi/h and the driver of the northward-moving vehicle claims he was within the speed limit when the collision occurred. Is he telling the truth?



Momentum conservation :

$$
\begin{aligned}
m\vec{v}_{1i} + m\vec{v}_{2i} &= 2m\vec{v}_f \\
\vec{v}_{1i} + \vec{v}_{2i} &= 2\vec{v}_f
\end{aligned}
$$

We can project the last equation on the West-East Direction to get:

$$v_{1i} = 2v_f \cos 61.5$$

and on the South-North direction to get :

$$v_{2i} = 2v_f \sin 61.5$$

$$\frac{v_{2i}}{v_{1i}} = \tan 61.55$$

$$\begin{aligned} v_{2i} &= v_{1i}\tan 61.55 \\ &= 13\tan 61.5 \\ &= 23.9\text{m/s} \\ &= 53.5\text{mile/hour} \end{aligned}$$

Second driver is definitely lying about his claim.

0.12

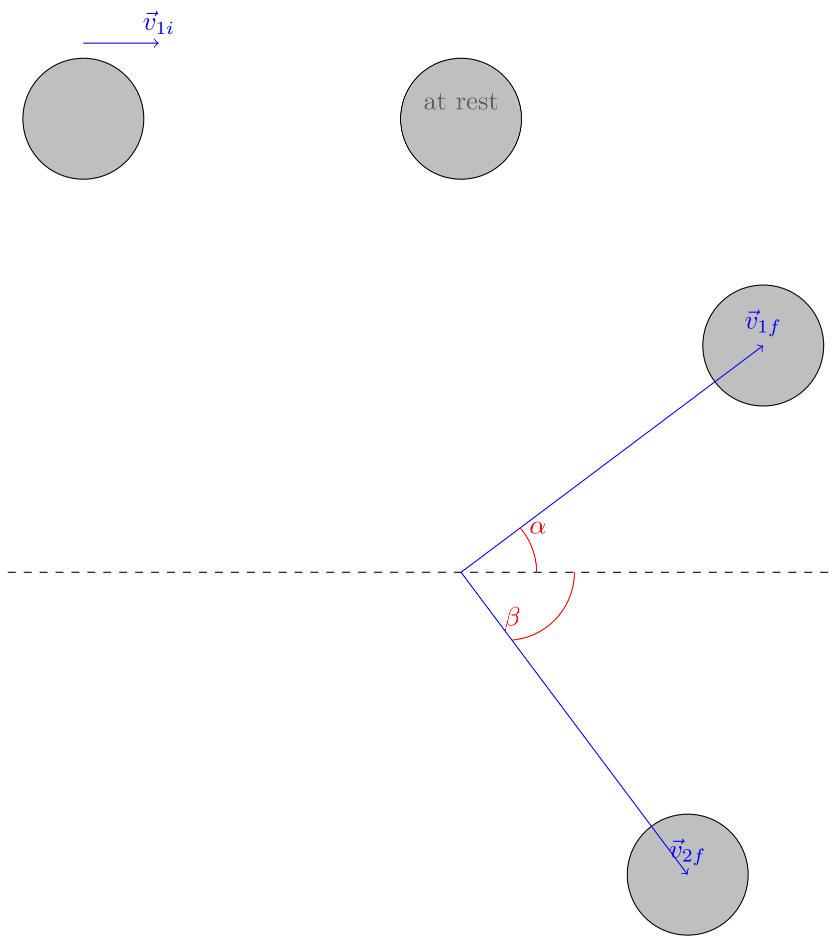
An object of mass 2.99 kg, moving with an initial velocity of $5.01\vec{i}$, collides with and sticks to an object of mass 2.31 kg with an initial velocity of -2.96$\vec{j}$ m/s. Find the final velocity of the composite object.

Momentum conservation of the two-object system will give us the final velocity after collision:

$$\begin{aligned} (m_1+m_2)\vec{V}_f &= m_1\vec{v}_{1i}+m_2\vec{v}_{2i} \\ \vec{V}_f &= \frac{1}{m_1+m_2}(m_1\vec{v}_{1i}+m_2\vec{v}_{2i}) \\ &= \frac{1}{2.99+2.31}(2.99\times 5.01\vec{i}-2.31\times 2.96\vec{j}) \\ &= 2.83\vec{i}-1.29\vec{j}\text{ m/s} \end{aligned}$$

0.13

A billiard ball moving at 5.20 m/s strikes a stationary ball of the same mass. After the collision, the first ball moves at 4.67 m/s at an angle of 26.0° with respect to the original line of motion. Assuming an elastic collision (and ignoring friction and rotational motion), find the struck ball's velocity after the collision.



Momentum is again conserved :

$$\begin{aligned} m\vec{v}_{1i} &= m\vec{v}_{1f} + m\vec{v}_{2f} \\ \vec{v}_{1i} &= \vec{v}_{1f} + \vec{v}_{2f} \end{aligned}$$

Projections on the x and y-axis:

$$\begin{aligned} v_{1i} &= v_{1f}\cos\alpha + v_{2fx} \\ v_{2fx} &= v_{1i} - v_{1f}\cos\alpha \\ &= 5.20 - 4.67\cos 26 \\ &= 1.00263 \end{aligned}$$

and

$$\begin{aligned} 0 &= v_{1f} \sin\alpha + v_{2fy} \\ v_{2fy} &= -v_{1f} \sin\alpha \\ &= -4.67 \sin 26 \\ &= -2.04719 \end{aligned}$$

Speed of the struck ball is:

$$\begin{aligned} v_{2f} &= \sqrt{1.003^2 + 20047^2} \\ &= 2.97\text{m/s} \end{aligned}$$

and making an angle β of:

$$\begin{aligned} \beta &= \arctan\frac{-2.047}{1.003} \\ &= -63.9^\circ \end{aligned}$$

N.B: $\alpha + |\beta| = 90^\circ$. This can be derived easily by looking at conservation of monentum and kinetic energy:

$$\begin{aligned} \vec{v}_{1i} &= \vec{v}_{1f} + \vec{v}_{2f} \\ v_{1i}^2 &= v_{1f}^2 + v_{2f}^2 + 2\vec{v}_{1f} \cdot \vec{v}_{2f} \end{aligned}$$

and

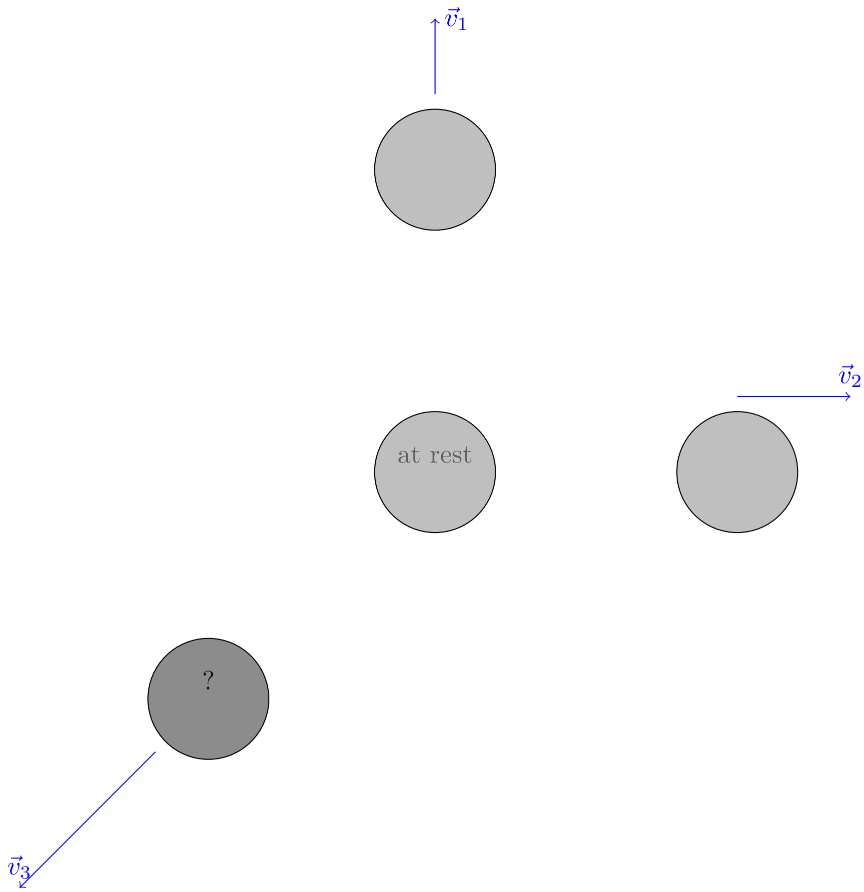
$$\begin{aligned} \frac{1}{2}mv_{1i}^2 &= \frac{1}{2}mv_{1f}^2 + \frac{1}{2}mv_{2f}^2 \\ v_{1i}^2 &= v_{1f}^2 + v_{2f}^2 \end{aligned}$$

This implies $\vec{v}_{1f} \cdot \vec{v}_{2f} = 0$ or $\alpha + |\beta| = 90^\circ$.

0.14

An unstable atomic nucleus of mass 1.80×10^{-26}kg initially at rest disintegrates into three particles. one of the particles of mass 5.16×10^{-27} kg, moves in the y direction with a speed of 6.00×10^{6} m/s. Another particle of mass 8.46×10^{-27} kg, moves in the x direction with a speed of 4.00×10^{6} m/s.

- Find the velocity of the third particle
- Find the total kinetic energy increase in the process.



Momentum is conserved :

$$\begin{aligned}
\vec{0} &= m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3 \\
\vec{v}_3 &= -\frac{1}{m_3}(m_1\vec{v}_1 + m_2\vec{v}_2) \\
&= -\frac{1}{0.438 \times 10^{-26}}(0.516 \times 6 \times 10^6\vec{j} + 0.846 \times 4 \times 10^6\vec{i}) \times 10^{-26} \\
&= -7.73 \times 10^6\vec{i} - 7.07 \times 10^6\vec{j}
\end{aligned}$$

its maginitude is $v_3 = \sqrt{7.73^2 + 7.07^2} = 10.47 \times 10^6 \mathrm{m/s}$

b)

The kinetic energy increase of the system is then computed as:

$$\Delta Ki = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \frac{1}{2}m_3v_3^2$$

$$
\begin{aligned}
&= \frac{0.516 \times 6^2 + 0.846 \times 4^2 + 0.438 \times 10.47^2}{2} \times 10^1 2 \times 10^{-26} \\
&= 4.01 \times 10^{-13} \mathrm{J}
\end{aligned}
$$