

### Section 5.3

**Problem 5.B:** The position vs. time of the two-dimensional isotropic oscillator is given by equations 5.20 of the text. The “orbit” for  $\delta = \pi/4$  is shown in Fig. 5.8c. **(a)** Sketch this figure and show the point corresponding to  $t = 0$ . Also show the point on the ellipse where the distance from the origin is greatest (this is the semi-major axis of the ellipse). **(b)** Calculate the value of  $\omega t$  when the position of the oscillator position is at this point of greatest distance, for arbitrary  $A_x, A_y$ . **(c)** Calculate the angle of the semi-major axis (the tilt-angle of the ellipse) for the case  $A_x = 2 A_y$ . [Hint: The angle of any point on the “orbit” from the  $x$  axis is given by  $\phi = \tan^{-1} y/x$ .]

### Section 5.4

**Problem 5.C:** A railroad car traveling at speed  $v_0$  hits a spring bumper designed to critically damp the motion of the car. The position of the uncompressed spring is initially at  $x = 0$ . **(a)** Give the general expressions for  $x(t)$  and  $v(t)$  for these initial conditions, and sketch the curves  $x$  vs.  $t$  and  $v$  vs.  $t$ . **(b)** What is the distance,  $x_{\max}$ , of maximum compression of the spring, and at what time does it occur, in terms of the damping constant  $\beta$ ? **(c)** At what time does the minimum velocity  $v_{\min}$  occur, and what is its value? What direction is the car going at this time? **(d)** If the car continues at this speed, what fraction of energy was lost in this inelastic collision?

**Problem 5.D:** A damped oscillator has a damping constant  $\beta = 0.1 \omega_0$ . After how many periods will the oscillator amplitude be one-quarter of its original value? Give your answer to 4 significant figures.

### Section 5.6

**Problem 5.E:** A boy of mass  $m$  stands on a platform of negligible mass, supported by two springs, each of spring constant  $k$ . When the boy climbs on, the springs depress a distance  $x_0 = 0.2$  m. **(a)** Write down the equation of motion for this situation, including the weight of the boy and a damping force  $f = -bv$  (assume  $x$  positive downward). What is the total spring constant for the combined effect of the two springs? **(b)** As we saw in chapter 3, we can eliminate the gravity force from the problem by considering the equilibrium position (terms involving time derivatives are zero), and writing the equation of motion in terms of a new variable  $y = (x - x_0)$ . Find  $x_0$  in terms of  $k, m$  and  $g$ , and use it to determine  $k$  (spring constant for each spring). **(c)** The boy now begins bouncing.

What is the resonant frequency  $\omega_0$ , assuming no damping? Give your answer as a numerical answer, in  $s^{-2}$ . Assume  $g = 10$   $m/s^2$  for a nice, round number. **(d)** If the system has a damping constant  $\beta = 0.1 \omega_0$ , and the boy exerts a sinusoidal force per unit mass with amplitude  $f_0 = 1$  N/kg, what is the maximum amplitude of his bounce? For what frequency  $\omega$  does this maximum amplitude occur?

