Section 7.3

Problem 7.A: Do problem 7.10 of the text, which asks you to write down the equations for *x, y, z* in terms of ρ and ϕ. Continue to the next step by taking time derivatives needed to determine the kinetic energy $T = \frac{1}{2}m\left(\dot{x}^2 + \dot{y}^2 + \dot{z}^2\right)$ in terms of ρ and ϕ. Also write down the potential energy, *U*, and hence the Lagrangian $\mathcal{L} = T - U$. Use the two Lagrangian equations (one for each variable ρ and ϕ), to show that the equations of motion are:

$$\ddot{\rho} = g\left(\frac{\cos\alpha}{1+\cos^2\alpha}\right)$$

$$\dot{\phi} = const.$$

Section 7.5

Problem 7.B: Do problem 7.16 of the text, to derive the equation of motion of the cylinder in terms of the generalized coordinate *x* measured along the incline. After solving that problem, now try the same problem choosing a different generalized variable, the angle ϕ of the contact point of the cylinder with the incline as it rolls without slipping. Derive the equation of motion in terms of ϕ and show that it is equivalent to that you got previously for *x*. [Hint: $x = R\phi$].

Section 7.5

Problem 7.C: Do problem 7.32 of the text, but use the following hints: Show that the CM coordinates are given by

$$x = (r+b)\sin\theta - r\theta\cos\theta$$

$$y = (r+b)\cos\theta + r\theta\sin\theta$$

where the *y* equation is just the height derived in the original problem. Take the time derivatives and combine to show that $v^2 = \dot{x}^2 + \dot{y}^2 = (b^2 + r^2\theta^2)\dot{\theta}^2$. Write down the Lagrangian and make the small angle approximation (be careful to keep more than the leading term in the *U* part. You should be able to do the rest of the problem from here.

Problem 7.D: Do the following GRE exam sample question:

74. The Lagrangian for a mechanical system is

$$L = a\dot{q}^2 + bq^4,$$

where q is a generalized coordinate and a and b are constants. The equation of motion for this system is

(A) $\dot{q} = \sqrt{\frac{b}{a}}\, q^2$

(B) $\dot{q} = \frac{2b}{a} q^3$

(C) $\ddot{q} = -\frac{2b}{a} q^3$

(D) $\ddot{q} = +\frac{2b}{a} q^3$

(E) $\ddot{q} = \frac{b}{a} q^3$