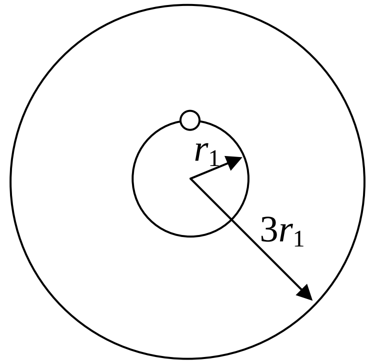
Section 8.4

Problem 8.A: The orbits of two stars about their common center of mass are circles with radii r_1 and $3r_1$, as shown in the figure at right. **(a)** The position of one star is shown. Sketch the location of the other star at the same moment. **(b)** If the mass of the more massive star is m_1, what is the mass of the less massive star in terms of m_1? **(c)** The equivalent one-dimensional orbit problem has an effective potential energy



$$U_{\text{eff}}(r) = -\frac{Gm_1m_2}{r} + \frac{\ell^2}{2\mu r^2}.$$

Identify the meaning of μ and r used in this equation, and give their values in terms of m_1 and r_1. [Note: I am not asking you to solve the above equation.]

Problem 8.B: [computer] In class I sketched some orbits for a comet far from the Sun, for different values of angular momentum. Let's take a quantitative look at this situation. **(a)** Consider a comet in a circular orbit. The effective potential, as always, is as shown in problem 8.A above. Plot this curve for the case $Gm_1m_2 = 1$, and $\ell^2/2\mu = 1$. This amounts to choosing special units for r. Allow r to range from 0 to 20, i.e. r = [0: 0.01: 20] in Matlab. If you allow the plot to scale itself, the y range is much too large to see the shape, so when you plot it, restrict the y range from -0.5 to 2. You should see a plot like that of Fig. 8.4 or 8.5 in the text. Since this is a circular orbit, use the same approach as you did in prob. 8.12 (*a*) of the text to find the value of $r = r_c$ of the orbit. For a circular orbit, $E = U_{\text{eff}}(r_c)$, so what is the total energy E for the orbit? **(b)** Now we strap rockets on the comet and fire them to slow the comet "instantaneously," removing angular momentum such that the new angular momentum is $\sqrt{2}$ smaller, i.e. $\ell^2/2\mu = 1/2$. Plot the new U_{eff} on the same graph (i.e. use *hold on* in Matlab). Because the comet slowed instantaneously, the comet is at the same $r = r_c$ as before the "burn." What is the new value of energy E (again = $U_{\text{eff}}(r_c)$)? Has E increased or decreased? Writing the equation $E = U_{\text{eff}}(r)$, solve to find the two turning points of the orbit (one of which is obviously r_c). Sketch these two turning points on your second curve. Do they make sense? The orbit in part (a) was circular. What shape of orbit is this one? **(c)** Instead of firing the rocket to slow the comet, say we had fired in the opposite direction to instantaneously speed up the comet and increase angular momentum by $\sqrt{2}$, i.e. $\ell^2/2\mu = 2$. Plot this U_{eff} on the same graph as before. What is the total energy $E = U_{\text{eff}}(r_c)$ of the orbit now? Writing the equation $E = U_{\text{eff}}(r)$, solve again to find the two turning points of the orbit (one of which is obviously still r_c). What shape of orbit is this?