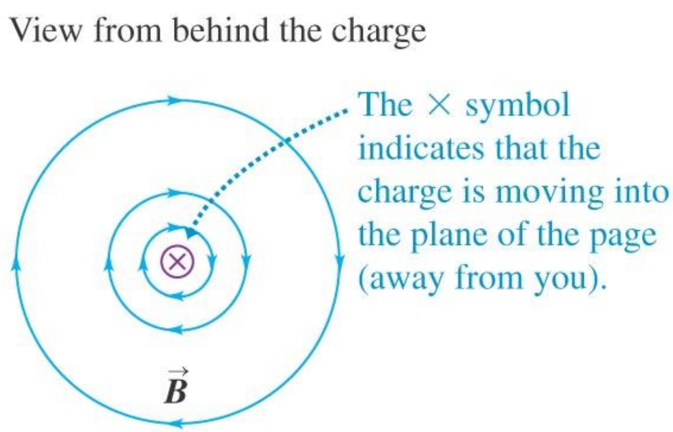
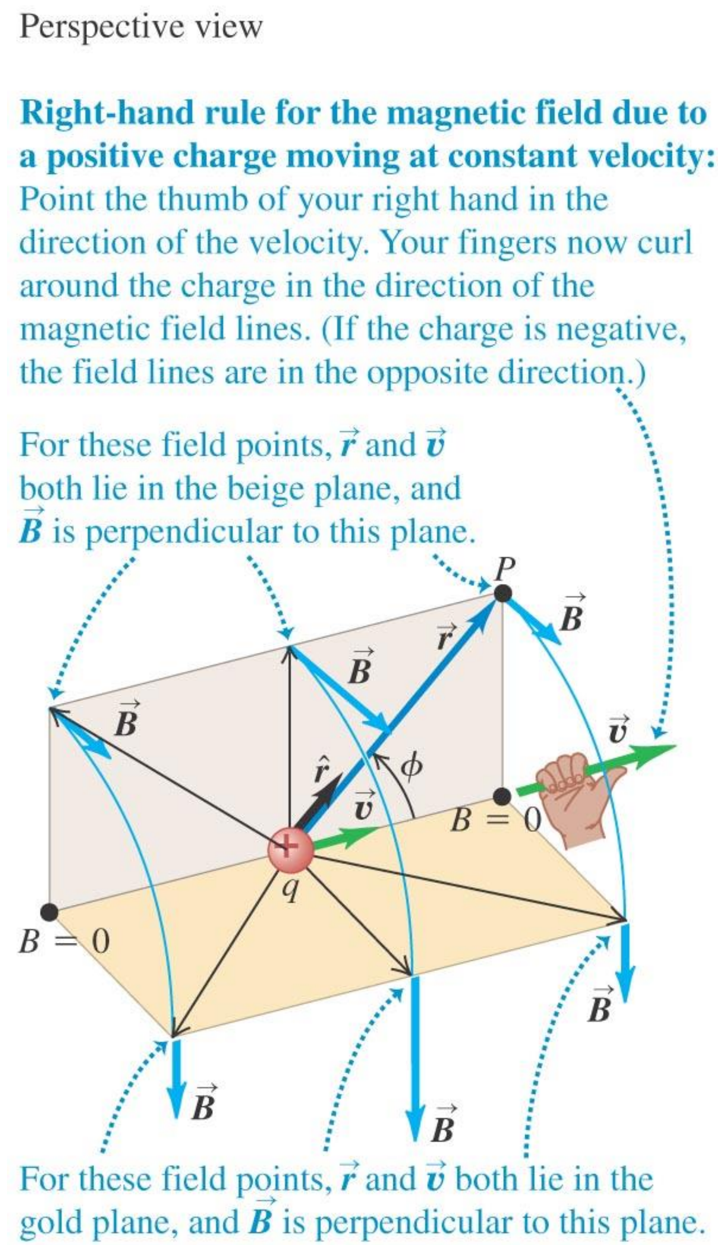
PHYS 122-Lecture 10: Sources of Magnetic Fields

- Today: Chap 28: a) Sources of Magnetic Fields
  b) Biot-Savart Law ▶ Coulomb's Law
  c) Ampere's Law ▶ Gauss' Law

The magnetic field of a moving charge

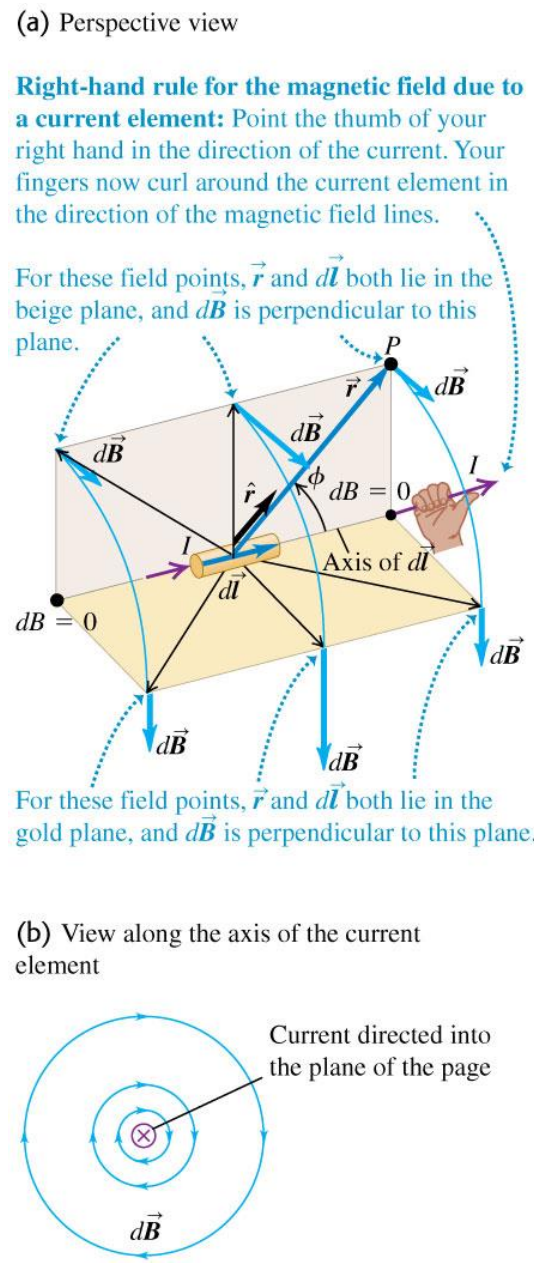
- A moving charge generates a magnetic field that depends on the velocity of the charge.
- Figure 28.1 shows the direction of the field.





Magnetic field of a current element

- The total magnetic field of several moving charges is the vector sum of each field.
- Follow the text discussion of the vector magnetic field due to a current element. Refer to Figure 28.3 at the right. The formula derived is called the *law of Biot and Savart*.

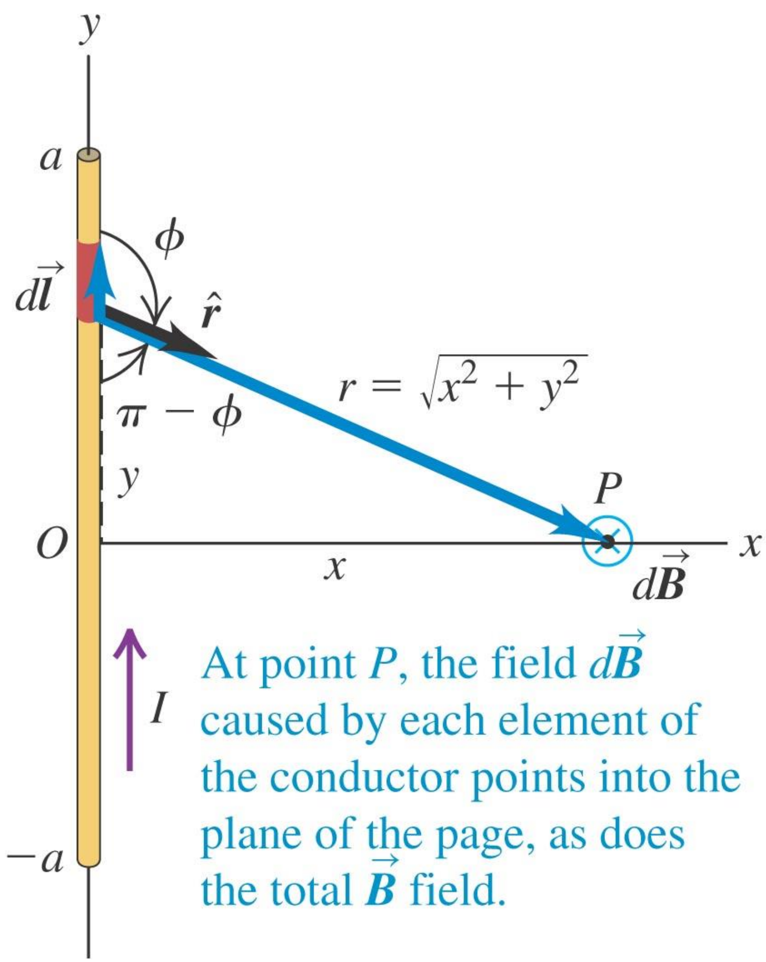


Biot-Savart Law

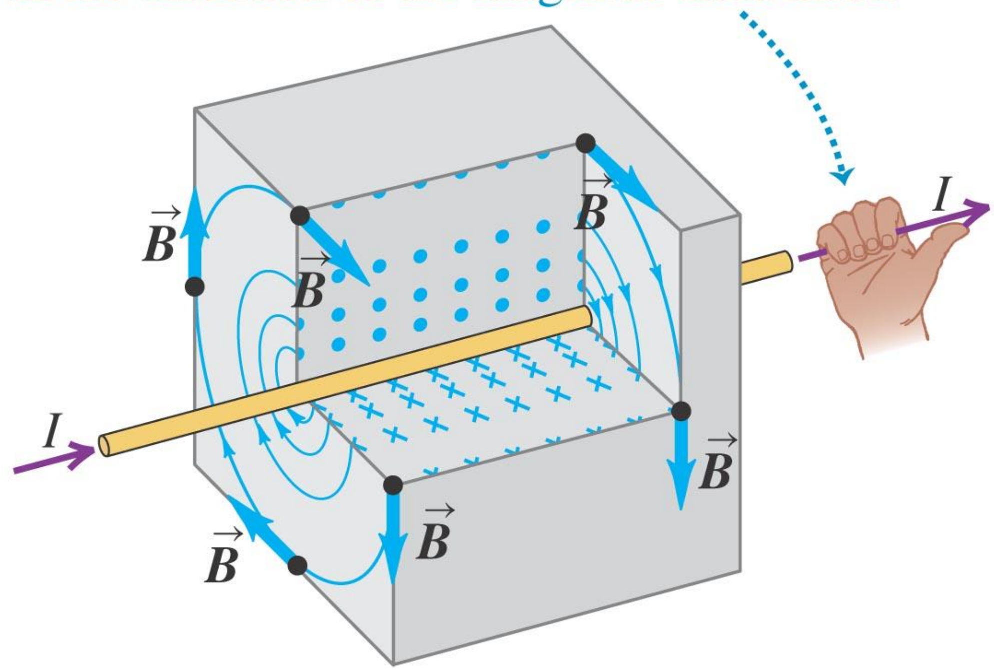
$$d\vec{B} = \frac{\mu}{4\pi}\frac{I\ \boldsymbol{d\ell}\times\hat{\boldsymbol{r}}}{r^2}$$

Magnetic field of a straight current-carrying conductor

- If we apply the law of Biot and Savart to a long straight conductor, the result is $B = \mu_0 I/2\pi x$. See Figure 28.5 below left. Figure 28.6 below right shows the right-hand rule for the direction of the force.



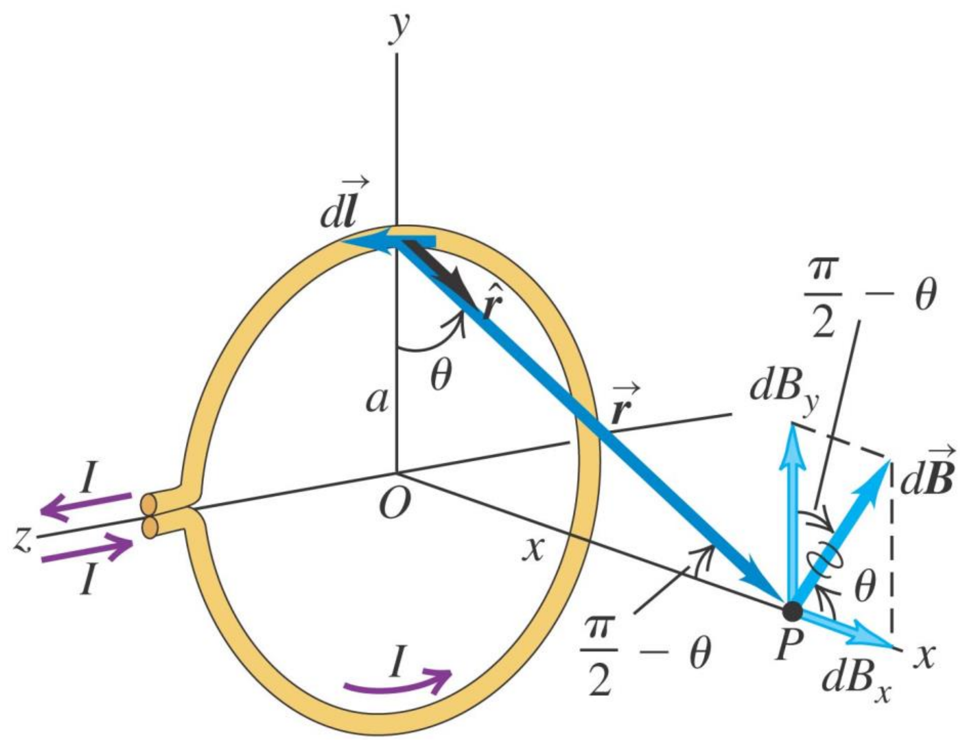
Right-hand rule for the magnetic field around a current-carrying wire: Point the thumb of your right hand in the direction of the current. Your fingers now curl around the wire in the direction of the magnetic field lines.



$$B = \frac{\mu_o I}{2\pi r}$$

Magnetic field of a circular current loop

- The Biot Savart law gives $B_x = \mu_0 I a^2/2(x^2 + a^2)^{3/2}$ on the axis of the loop. Follow the text derivation using Figure 28.12 at the right.

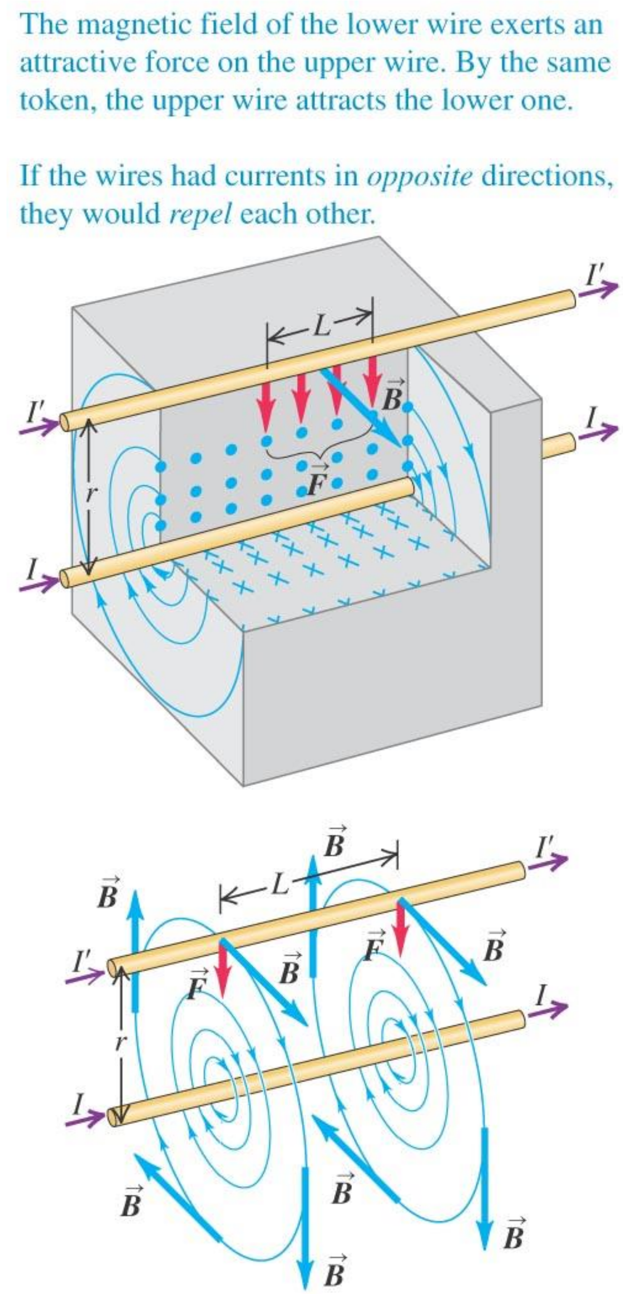


- At the center of N loops, the field on the axis is $B_x = \mu_0 NI/2a$.

Magnetic Field of a Circular Current Loop

Force between parallel conductors

- The force per unit length on each conductor is $F = \mu_0 I I' L / 2\pi r$. (See Figure 28.9 at the right.)
- The conductors attract each other if the currents are in the same direction and repel if they are in opposite directions.



Force Between Two Wires

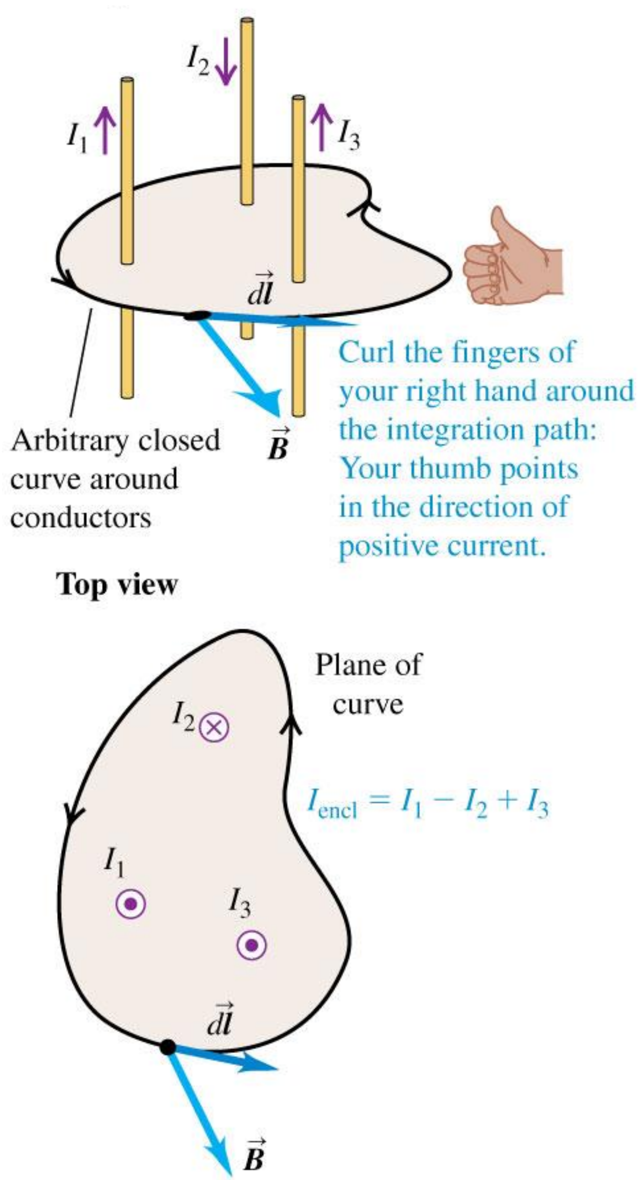
$$\overrightarrow{\mathbf{F_B}} = \mathbf{q}\left(\vec{\mathbf{v}} \times \overrightarrow{\mathbf{B}}\right)$$

$$\text{F} = I'L \times \frac{\mu_o I}{2\pi r}$$

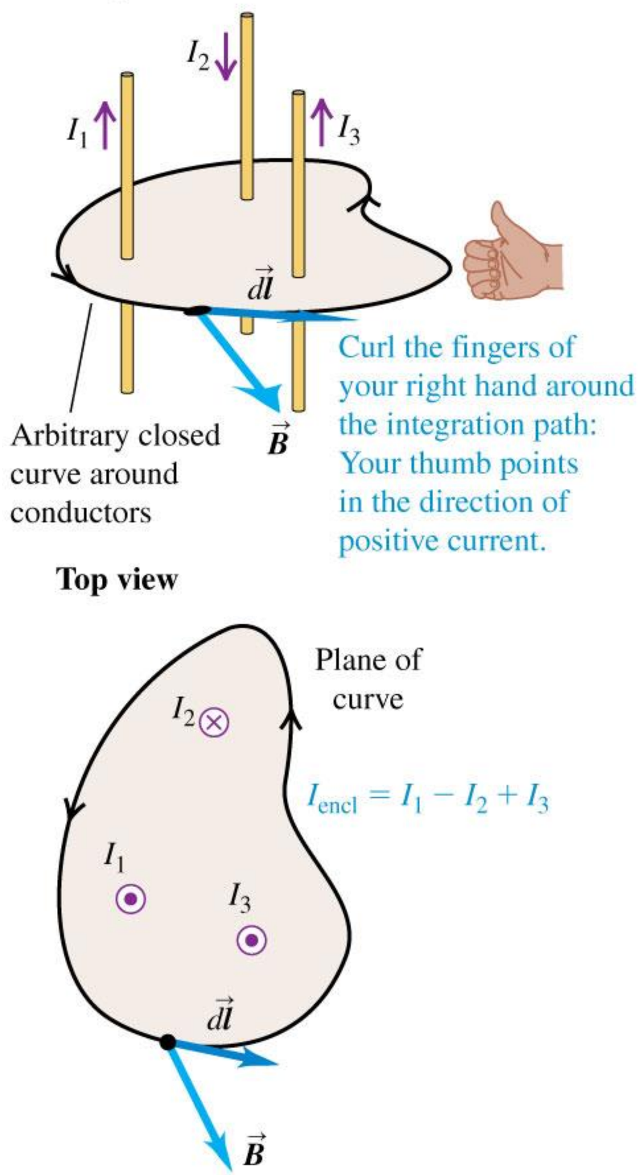
$$\frac{F}{L} = \frac{\mu_o I I'}{2\pi r}$$

Ampere's Law

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_o I_{enc}$$



Ampere's law: If we calculate the line integral of the magnetic field around a closed curve, the result equals μ_0 times the total enclosed current: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$.



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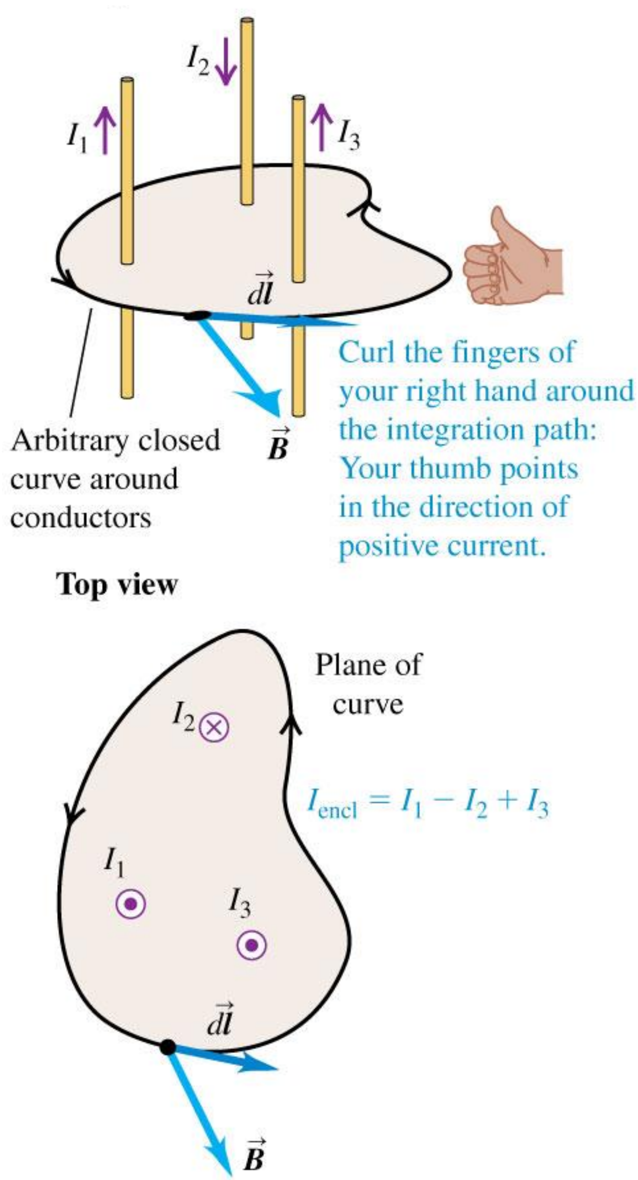
Ampere's Law

$$\boxed{\oint \vec{B} \cdot d\vec{\ell} = \mu_o I_{enc}}$$

$$\oint (\nabla \times \vec{B}) \cdot d\vec{A} = \mu_o I_{enc}$$

Ampere's Law

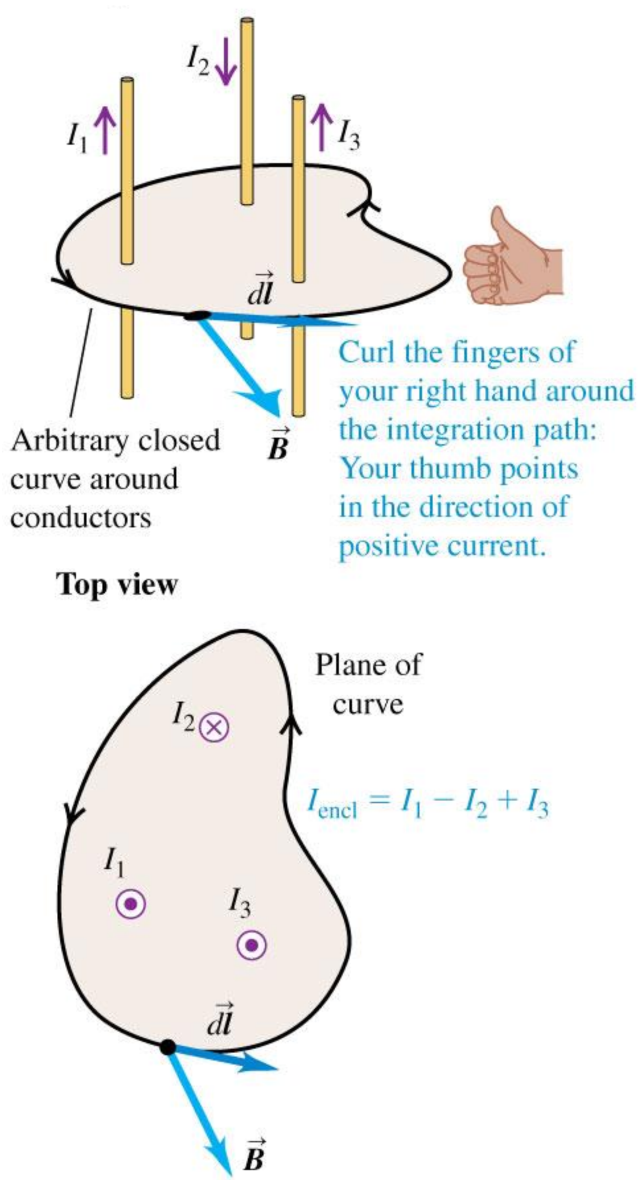


Ampere's law: If we calculate the line integral of the magnetic field around a closed curve, the result equals μ_0 times the total enclosed current: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$.

$$\boxed{\oint \vec{B} \cdot d\vec{\ell} = \mu_o I_{enc}}$$

$$\oint (\nabla \times \vec{B}) \cdot d\vec{A} = \mu_o I_{enc}$$

$$\oint (\nabla \times \vec{B}) \cdot d\vec{A} = \mu_o \oint \vec{J} \cdot d\vec{A}$$



Ampere's law: If we calculate the line integral of the magnetic field around a closed curve, the result equals μ_0 times the total enclosed current: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}}$.

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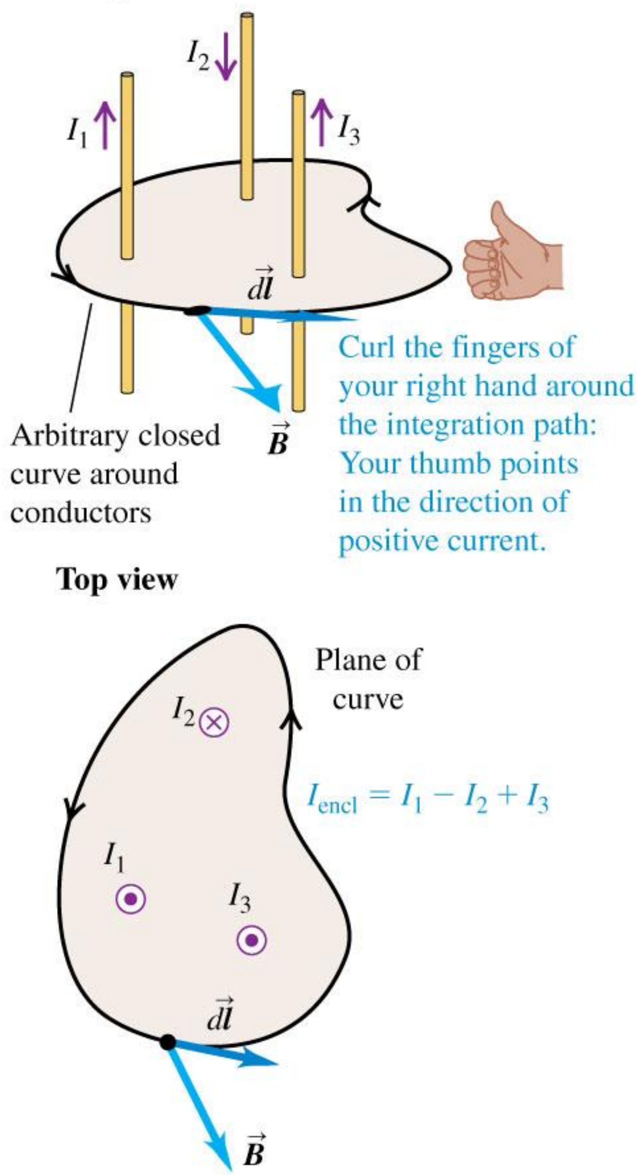
$$\boxed{\oint \vec{B} \cdot d\vec{\ell} = \mu_o I_{enc}}$$

$$\oint (\nabla \times \vec{B}) \cdot d\vec{A} = \mu_o I_{enc}$$

$$\oint (\nabla \times \vec{B}) \cdot d\vec{A} = \mu_o \oint \vec{J} \cdot d\vec{A}$$

$$\nabla \times \vec{B} = \mu_o \vec{J}$$

Ampere’s Law



Ampere’s law: If we calculate the line integral of the magnetic field around a closed curve, the result equals μ_0 times the total enclosed current: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\mathrm{encl}}$.

$$\boxed{\oint \vec{B} \cdot d\vec{\ell} = \mu_o I_{enc}}$$

$$\oint (\nabla \times \vec{B}) \cdot d\vec{A} = \mu_o I_{enc}$$

$$\oint (\nabla \times \vec{B}) \cdot d\vec{A} = \mu_o \oint \vec{J} \cdot d\vec{A}$$

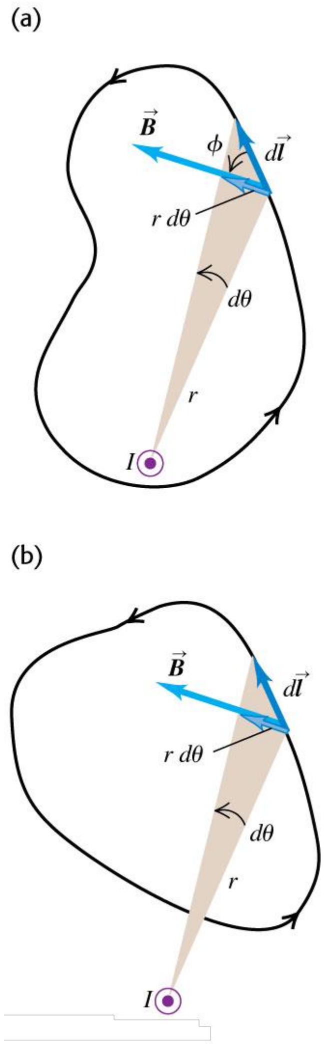
$$\nabla \times \vec{B} = \mu_o \vec{J}$$

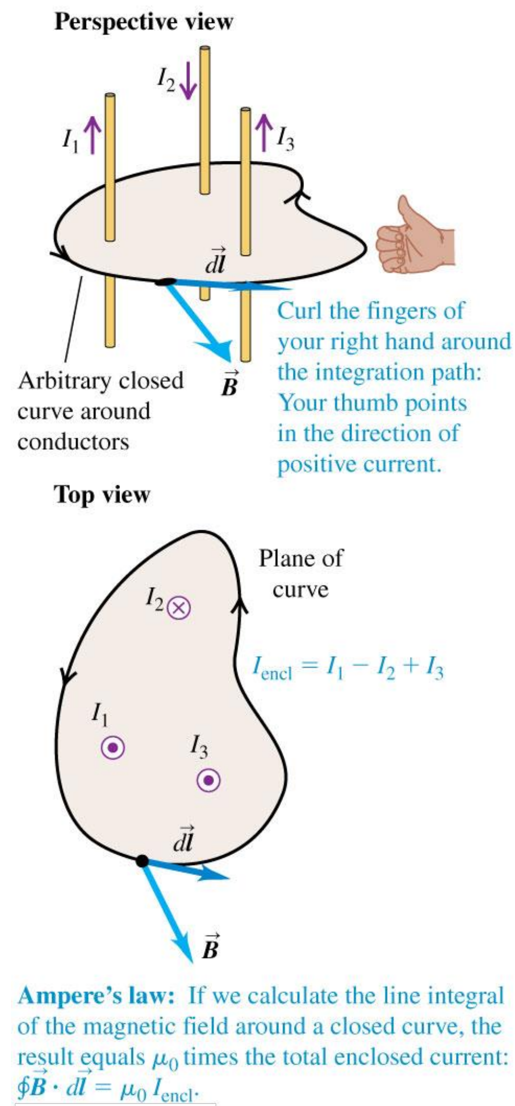
$$\nabla \times \vec{H} = \vec{J}$$

$$\vec{H} = \frac{\vec{B}}{\mu_o}$$

Ampere's law (general statement)

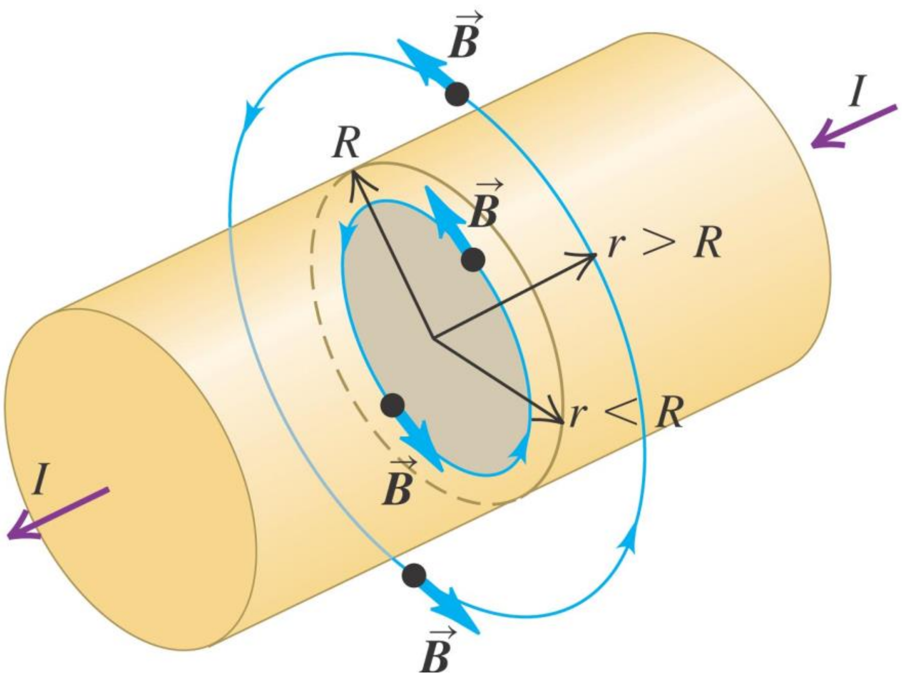
- Follow the text discussion of the general statement of Ampere's law, using Figures 28.17 and 28.18 below.

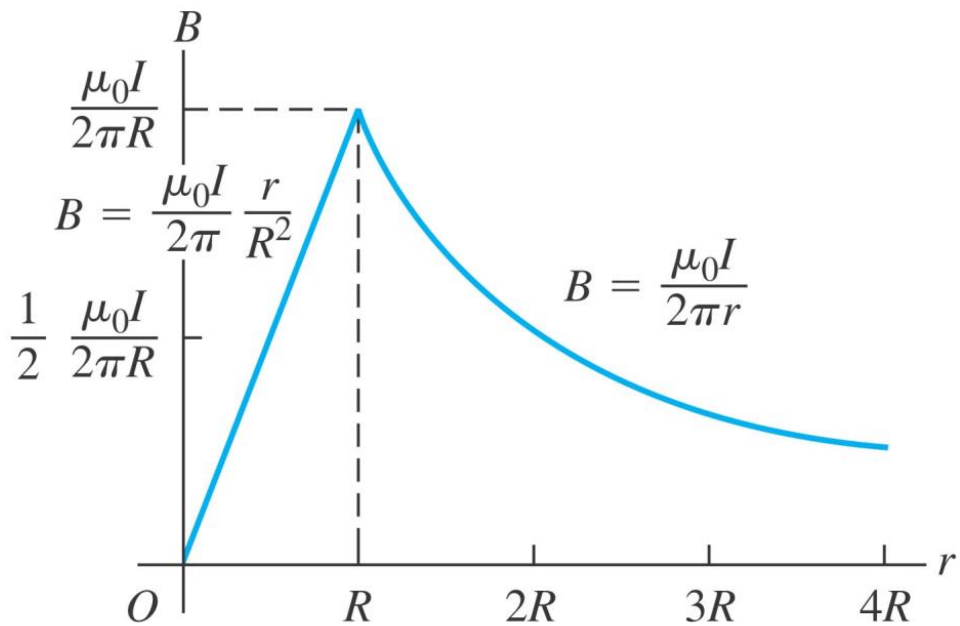




Magnetic fields of long conductors

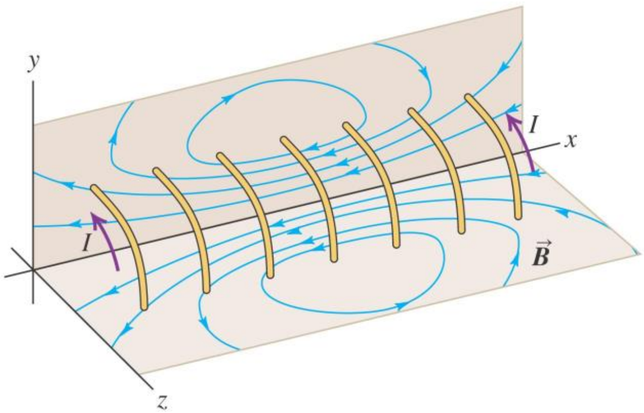
- Read Problem-Solving Strategy 28.2.
- Follow Example 28.7 for a long straight conductor.
- Follow Example 28.8 for a long cylinder, using Figures 28.20 and 28.21 below.

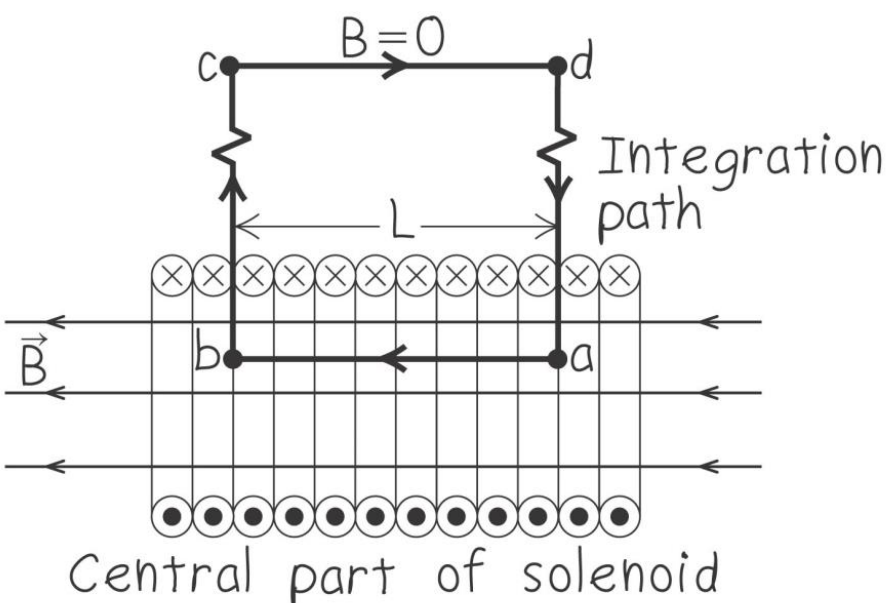


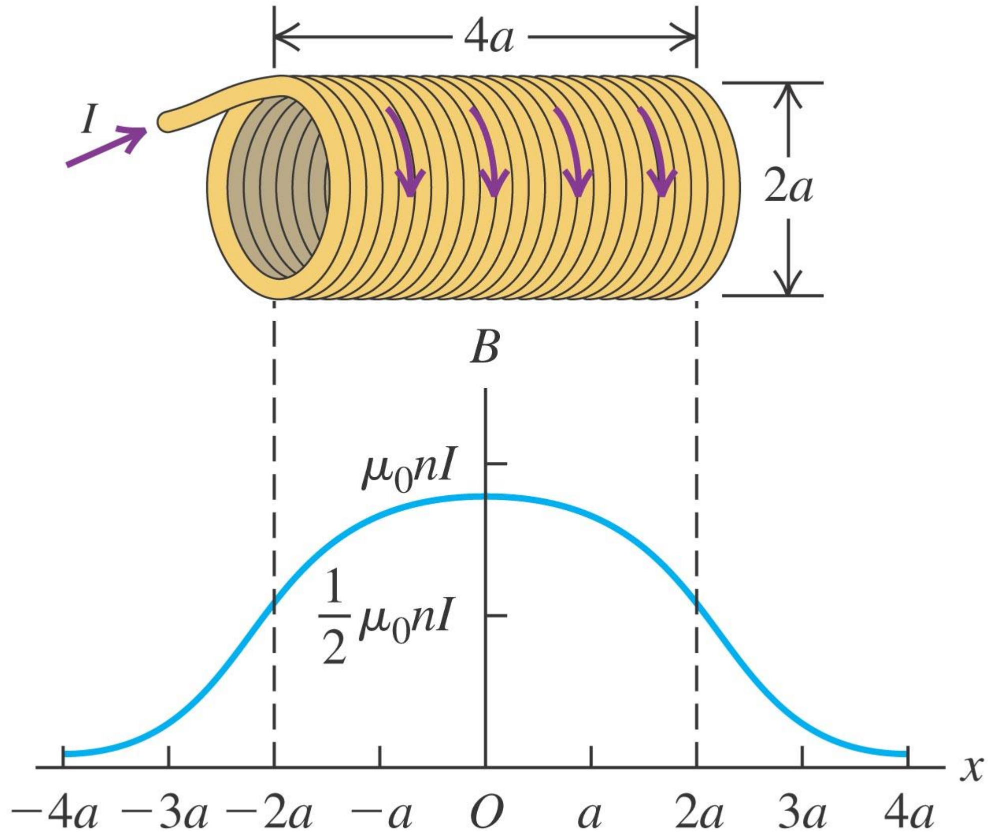


Field of a solenoid

- A *solenoid* consists of a helical winding of wire on a cylinder.
- Follow Example 28.9 using Figures 28.22–28.24 below.







Field of a toroidal solenoid

- A *toroidal solenoid* is a doughnut-shaped solenoid.
- Follow Example 28.10 using Figure 28.25 below.