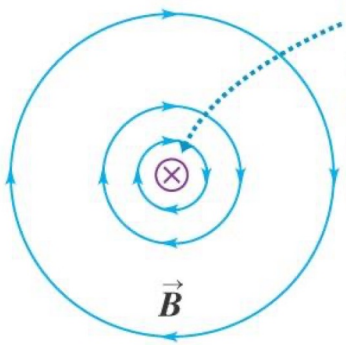
PHYS 122-Lecture 10:
Sources of Magnetic Fields

- Today: Chap 28: a) Sources of Magnetic Fields
  b) Biot-Savart Law ▶ Coulomb's Law
  c) Ampere's Law ▶ Gauss' Law

The magnetic field of a moving charge

- A moving charge generates a magnetic field that depends on the velocity of the charge.
- Figure 28.1 shows the direction of the field.

View from behind the charge

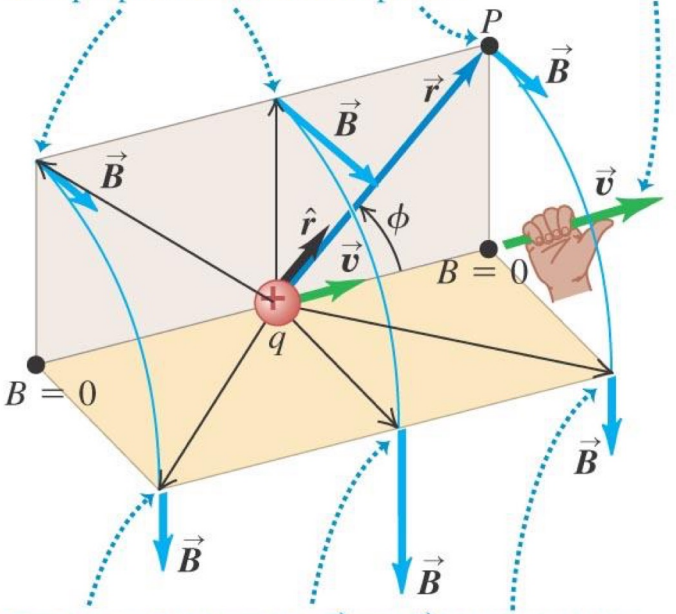


The × symbol indicates that the charge is moving into the plane of the page (away from you).

Perspective view

Right-hand rule for the magnetic field due to a positive charge moving at constant velocity: Point the thumb of your right hand in the direction of the velocity. Your fingers now curl around the charge in the direction of the magnetic field lines. (If the charge is negative, the field lines are in the opposite direction.)

For these field points, $\vec{r}$ and $\vec{v}$ both lie in the beige plane, and $\vec{B}$ is perpendicular to this plane.

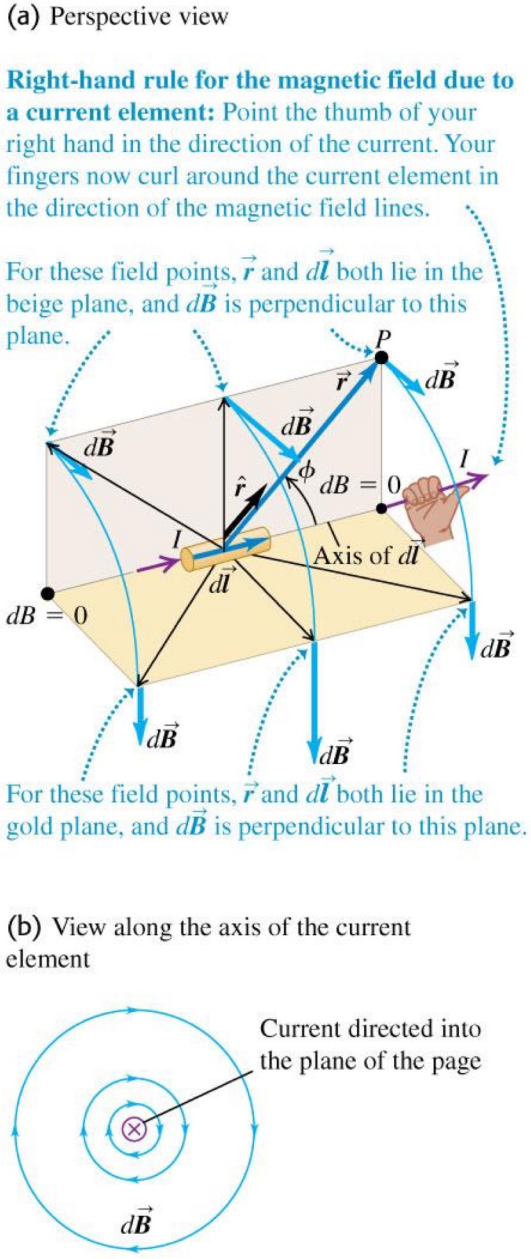


For these field points, $\vec{r}$ and $\vec{v}$ both lie in the gold plane, and $\vec{B}$ is perpendicular to this plane.

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Magnetic field of a current element

- The total magnetic field of several moving charges is the vector sum of each field.
- Follow the text discussion of the vector magnetic field due to a current element. Refer to Figure 28.3 at the right. The formula derived is called the *law of Biot and Savart*.



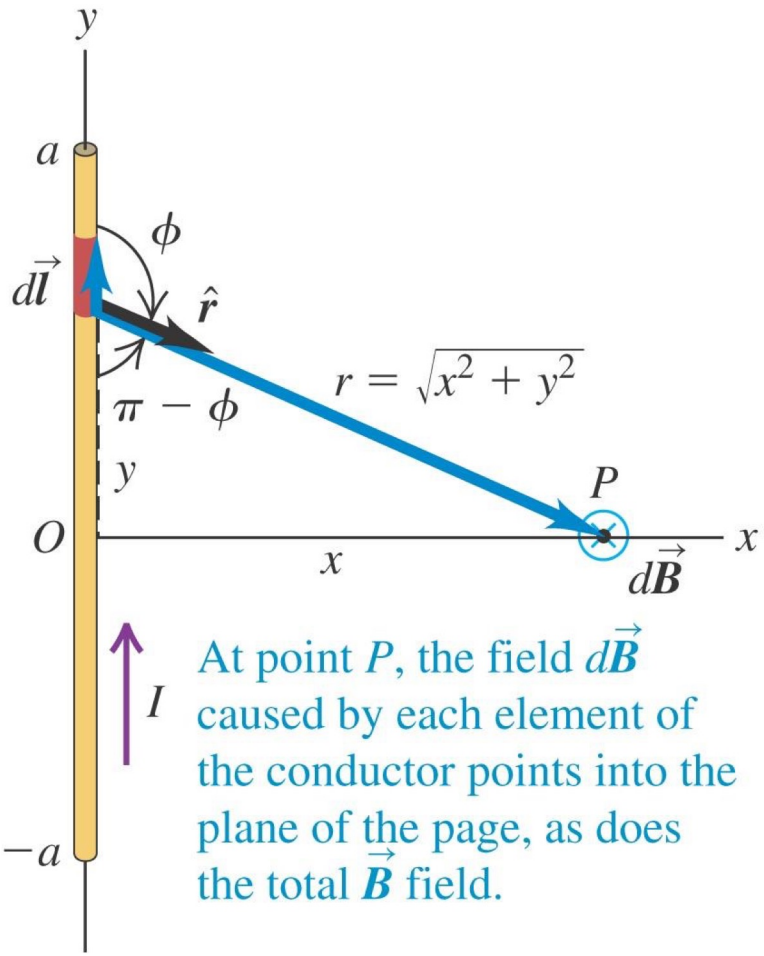
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Biot-Savart Law

$$d\vec{B} = \frac{\mu}{4\pi} \frac{i\, \boldsymbol{d\ell} \times \hat{\boldsymbol{r}}}{r^2}$$

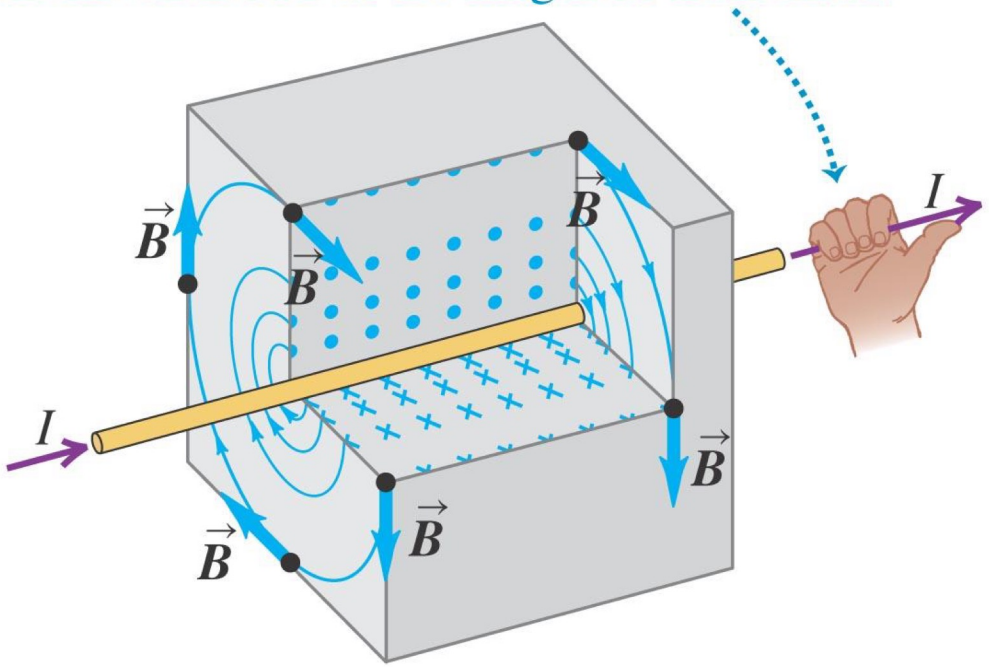
Magnetic field of a straight current-carrying conductor

- If we apply the law of Biot and Savart to a long straight conductor, the result is $B = \mu_0 I/2\pi x$. See Figure 28.5 below left. Figure 28.6 below right shows the right-hand rule for the direction of the force.



At point P, the field $d\vec{B}$ caused by each element of the conductor points into the plane of the page, as does the total $\vec{B}$ field.

Right-hand rule for the magnetic field around a current-carrying wire: Point the thumb of your right hand in the direction of the current. Your fingers now curl around the wire in the direction of the magnetic field lines.

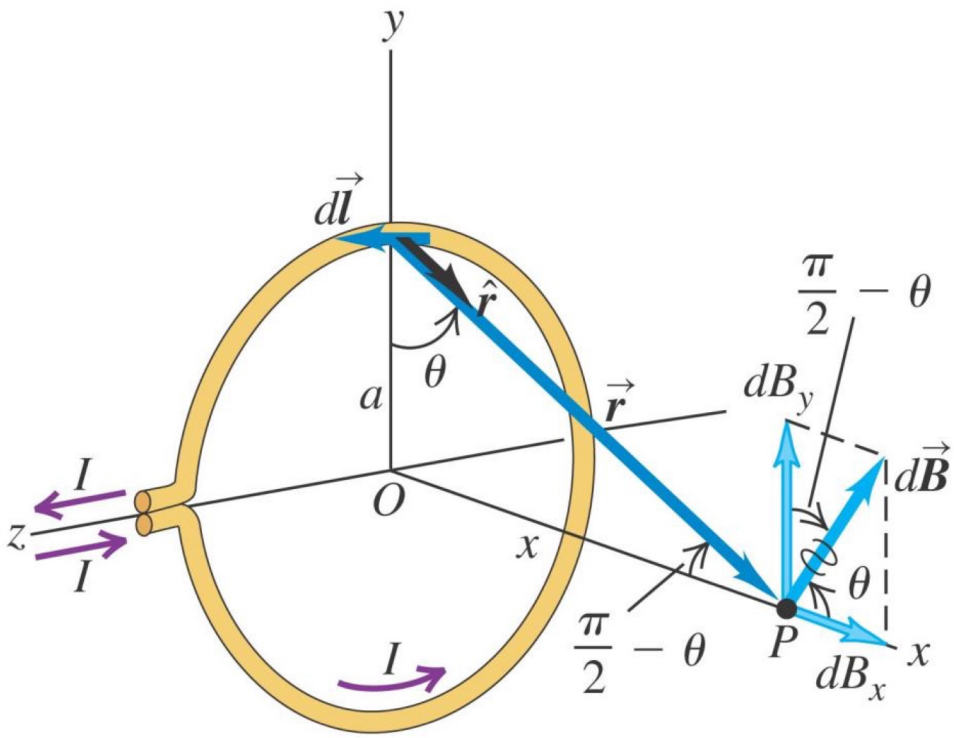


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$$B = \frac{\mu_o i}{2\pi r}$$

Magnetic field of a circular current loop

- The Biot Savart law gives $B_x = \mu_0 I a^2/2(x^2 + a^2)^{3/2}$ on the axis of the loop. Follow the text derivation using Figure 28.12 at the right.
- At the center of N loops, the field on the axis is $B_x = \mu_0 NI/2a$.



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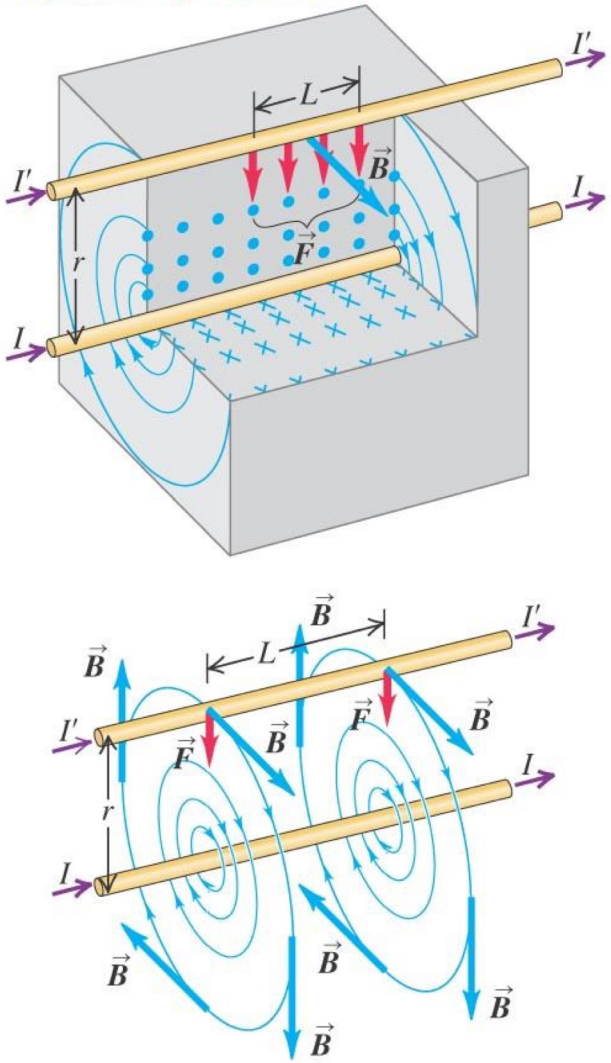
Magnetic Field of a Circular Current Loop

Force between parallel conductors

- The force per unit length on each conductor is $F = \mu_0 I I' L/2\pi r$. (See Figure 28.9 at the right.)
- The conductors attract each other if the currents are in the same direction and repel if they are in opposite directions.

The magnetic field of the lower wire exerts an attractive force on the upper wire. By the same token, the upper wire attracts the lower one.

If the wires had currents in *opposite* directions, they would *repel* each other.



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Force Between Two Wires

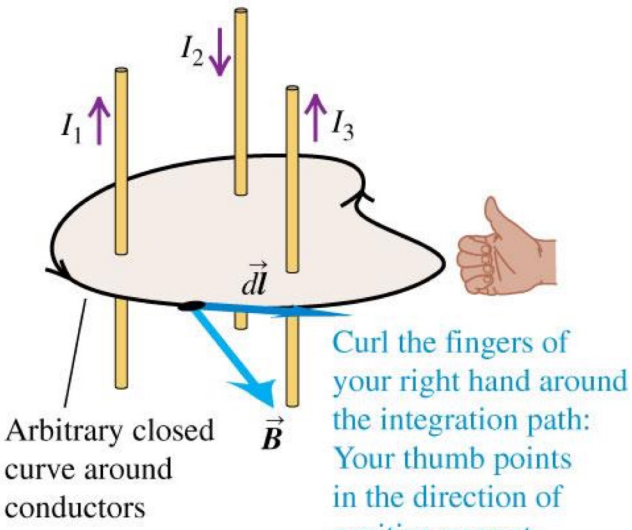
$$\overrightarrow{\mathbf{F_B}} = \mathbf{q}\left(\vec{\mathbf{v}} \times \vec{\mathbf{B}}\right)$$

$$\mathrm{F} = i' L \times \frac{\mu_o i}{2\pi r}$$

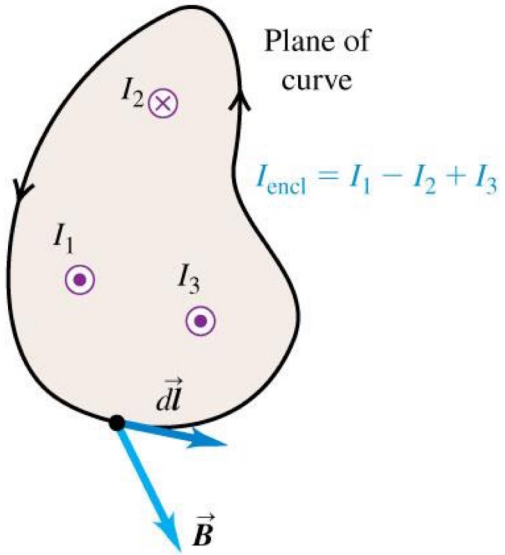
$$\frac{F}{L} = \frac{\mu_o i i'}{2\pi r}$$

Ampere's Law

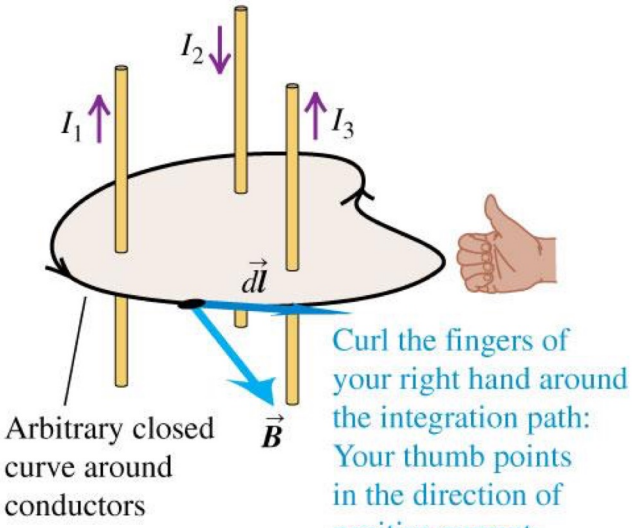
$$\oint \vec{B} \cdot d\vec{\ell} = \mu_o i_{enc}$$



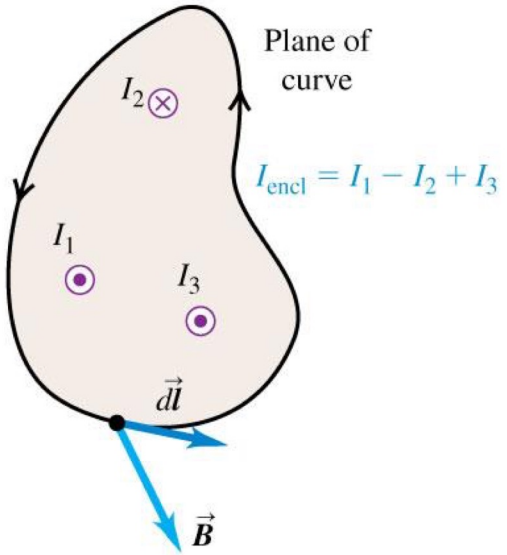
Top view



Ampere's law: If we calculate the line integral of the magnetic field around a closed curve, the result equals μ_0 times the total enclosed current: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$.



Top view



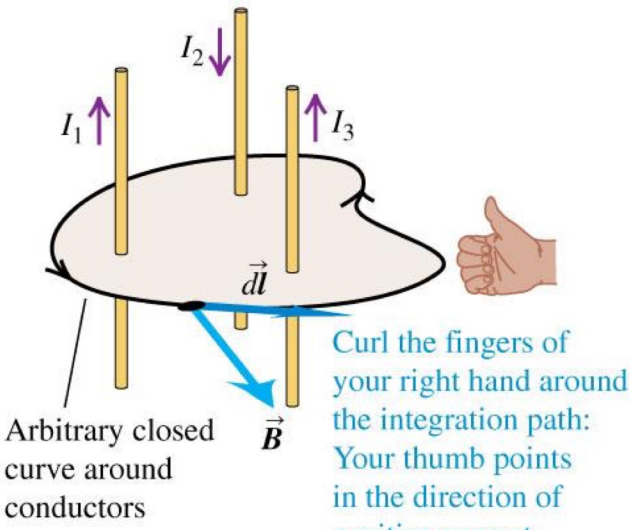
Ampere's law: If we calculate the line integral of the magnetic field around a closed curve, the result equals μ_0 times the total enclosed current: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$.

Ampere's Law

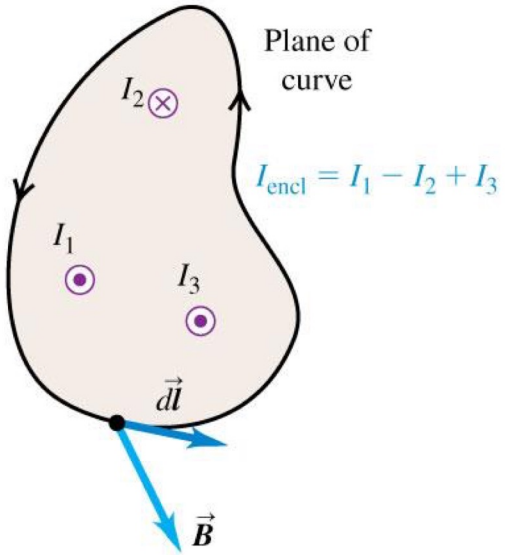
$$\oint \vec{B} \cdot d\vec{\ell} = \mu_o i_{enc}$$

$$\oint (\nabla \times \vec{B}) \cdot d\vec{A} = \mu_o i_{enc}$$

Ampere's Law



Top view



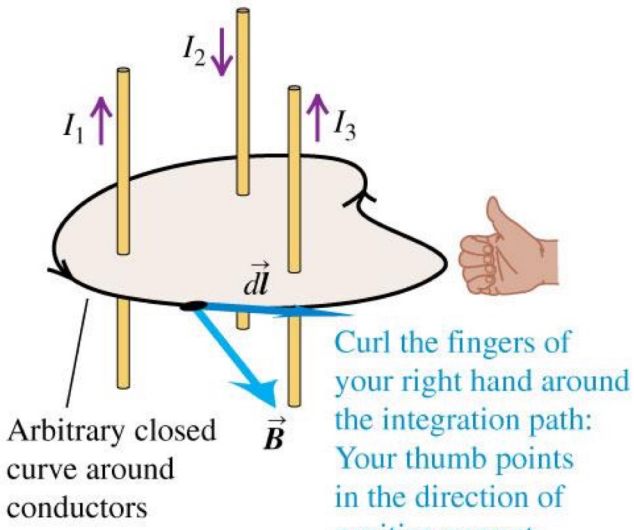
Ampere's law: If we calculate the line integral of the magnetic field around a closed curve, the result equals μ_0 times the total enclosed current: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$.

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_o i_{enc}$$

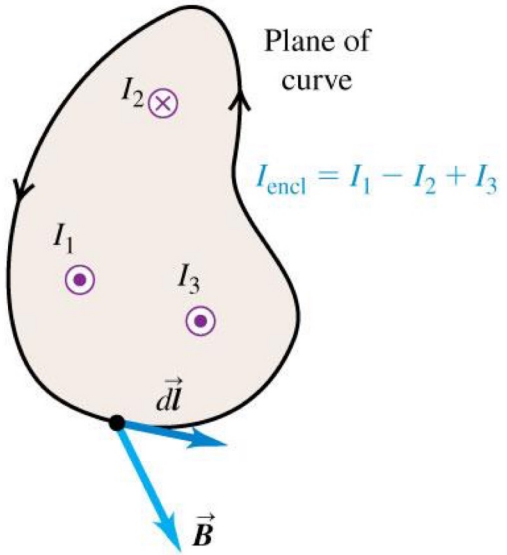
$$\oint (\nabla \times \vec{B}) \cdot d\vec{A} = \mu_o i_{enc}$$

$$\oint (\nabla \times \vec{B}) \cdot d\vec{A} = \mu_o \oint \vec{J} \cdot d\vec{A}$$

Ampere's Law



Top view



Ampere's law: If we calculate the line integral of the magnetic field around a closed curve, the result equals μ_0 times the total enclosed current: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$.

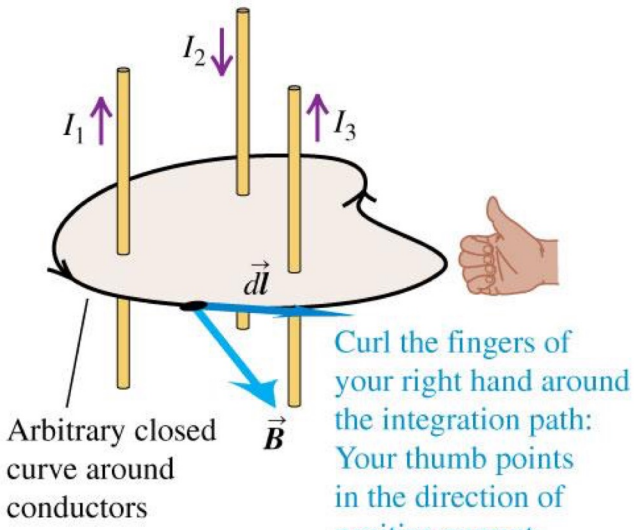
$$\oint \vec{B} \cdot d\vec{\ell} = \mu_o i_{enc}$$

$$\oint (\nabla \times \vec{B}) \cdot d\vec{A} = \mu_o i_{enc}$$

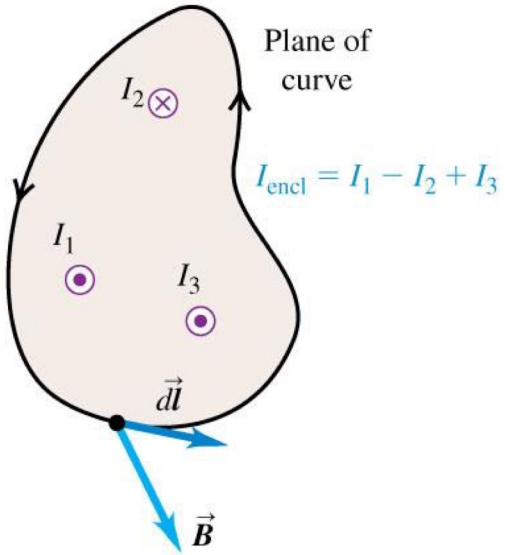
$$\oint (\nabla \times \vec{B}) \cdot d\vec{A} = \mu_o \oint \vec{J} \cdot d\vec{A}$$

$$\nabla \times \vec{B} = \mu_o \vec{J}$$

Ampere's Law



Top view



Ampere's law: If we calculate the line integral of the magnetic field around a closed curve, the result equals μ_0 times the total enclosed current: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$.

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_o i_{enc}$$

$$\oint (\nabla \times \vec{B}) \cdot d\vec{A} = \mu_o i_{enc}$$

$$\oint (\nabla \times \vec{B}) \cdot d\vec{A} = \mu_o \oint \vec{J} \cdot d\vec{A}$$

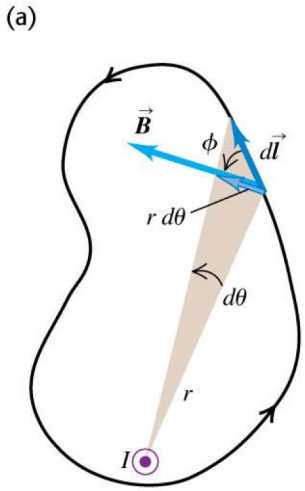
$$\nabla \times \vec{B} = \mu_o \vec{J}$$

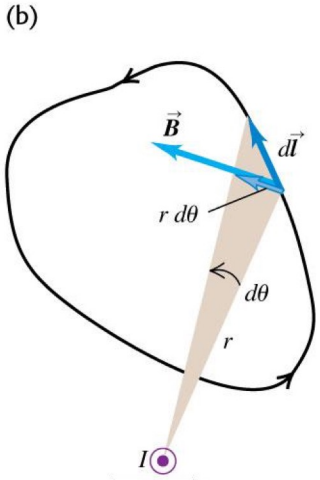
$$\nabla \times \vec{H} = \vec{J}$$

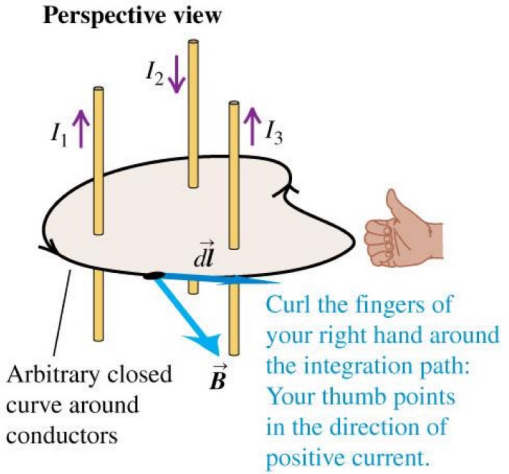
$$\vec{H} = \frac{\vec{B}}{\mu_o}$$

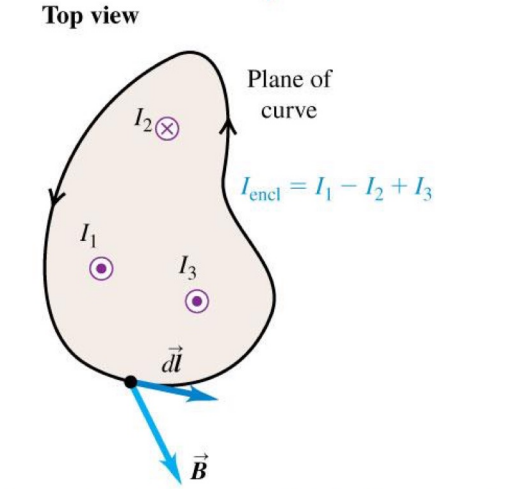
Ampere's law (general statement)

- Follow the text discussion of the general statement of Ampere's law, using Figures 28.17 and 28.18 below.







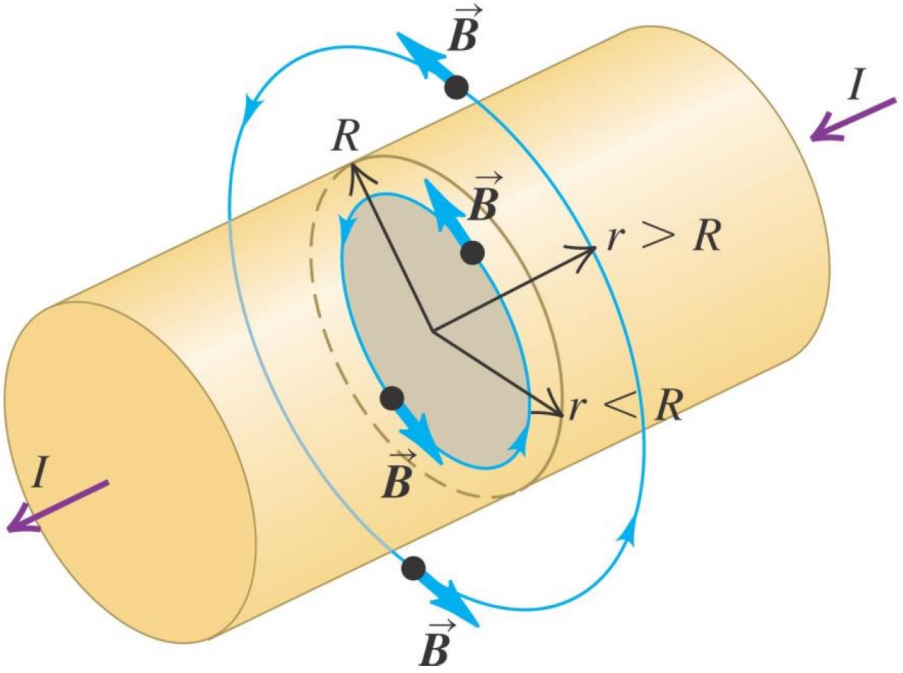


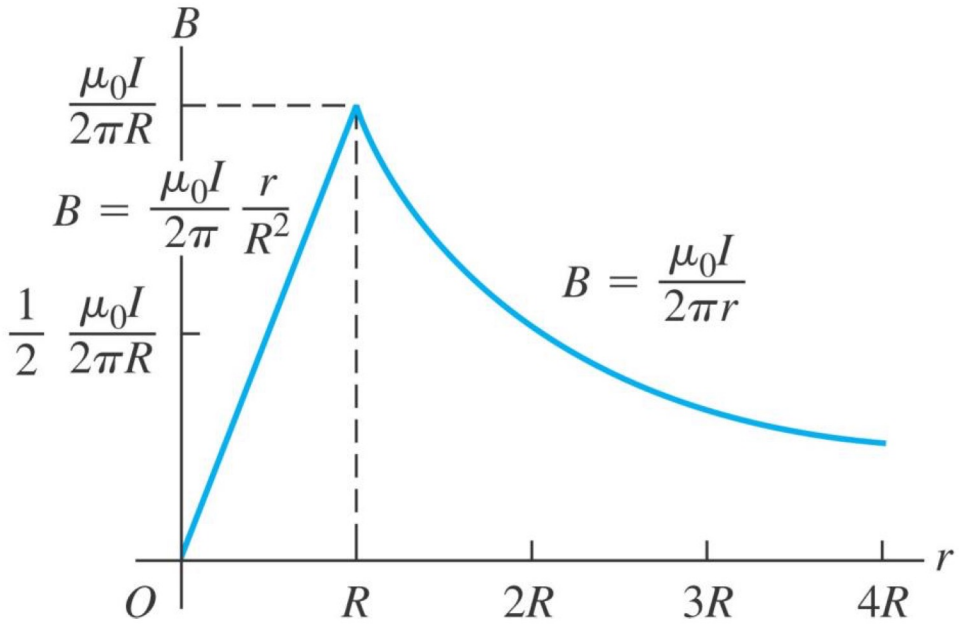
Ampere's law: If we calculate the line integral of the magnetic field around a closed curve, the result equals μ_0 times the total enclosed current: $\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{encl}$.

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Magnetic fields of long conductors

- Read Problem-Solving Strategy 28.2.
- Follow Example 28.7 for a long straight conductor.
- Follow Example 28.8 for a long cylinder, using Figures 28.20 and 28.21 below.

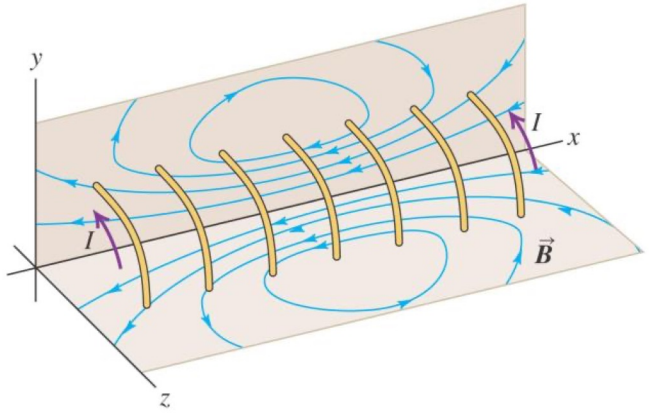


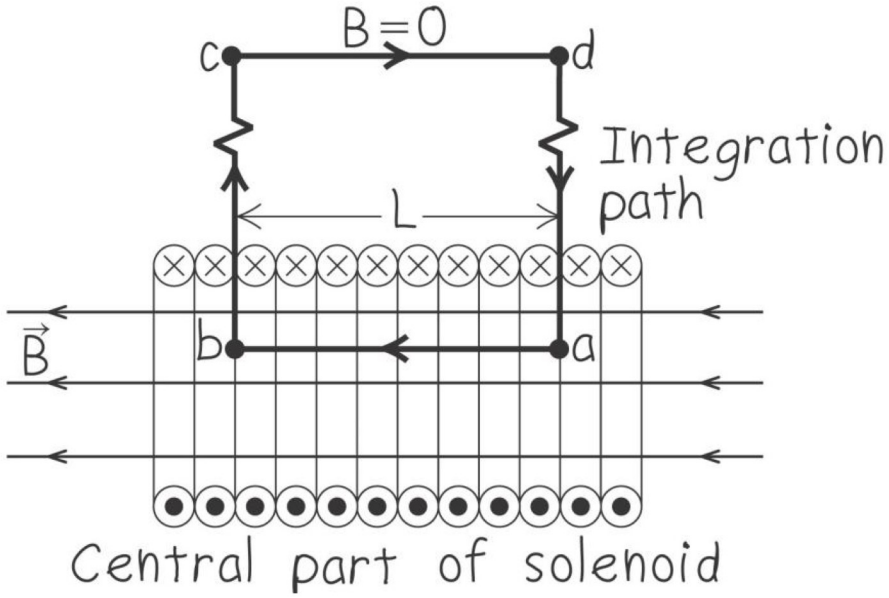


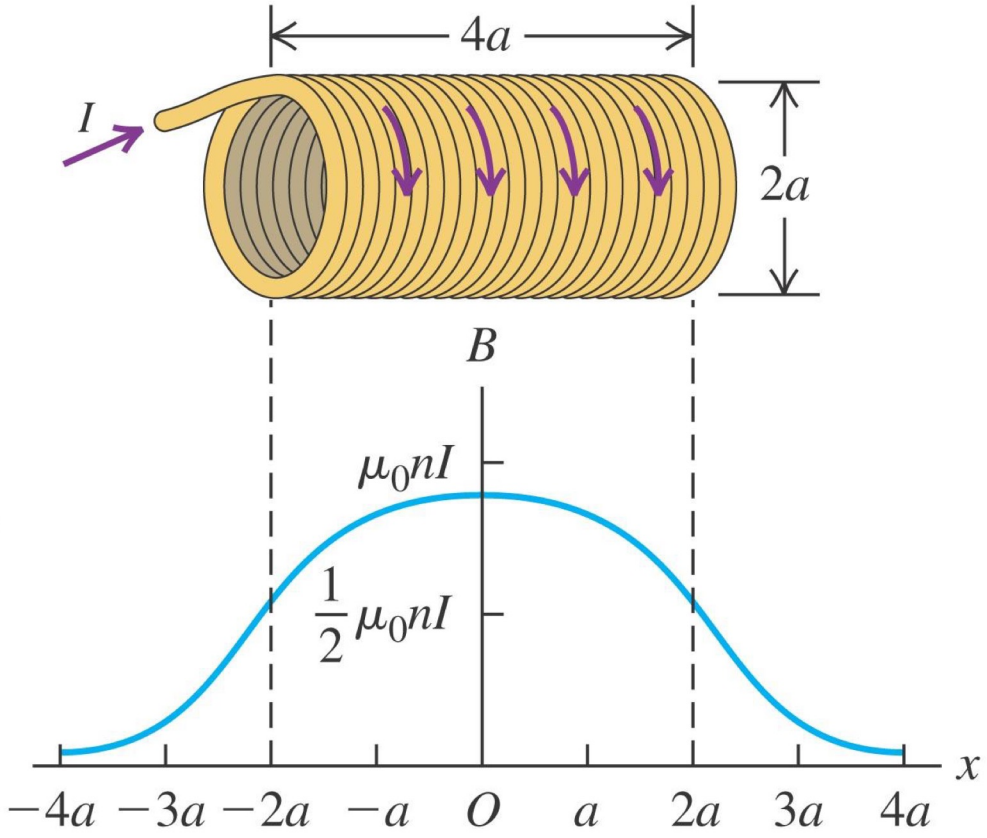
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Field of a solenoid

- A *solenoid* consists of a helical winding of wire on a cylinder.
- Follow Example 28.9 using Figures 28.22–28.24 below.



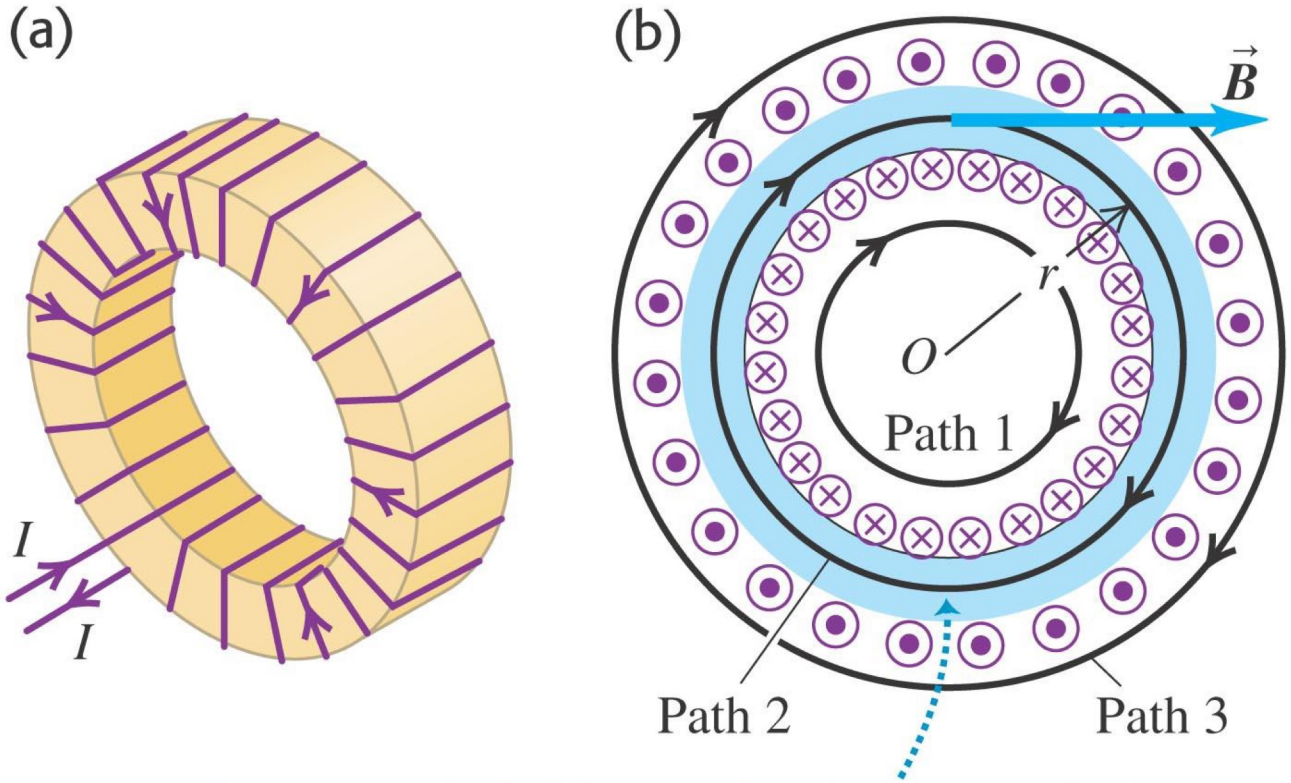




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Field of a toroidal solenoid

- A *toroidal solenoid* is a doughnut-shaped solenoid.
- Follow Example 28.10 using Figure 28.25 below.



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