PHYS 122-Lecture 4:
Gauss' Law

- Electric Flux
- Gauss' Law

Charge and electric flux

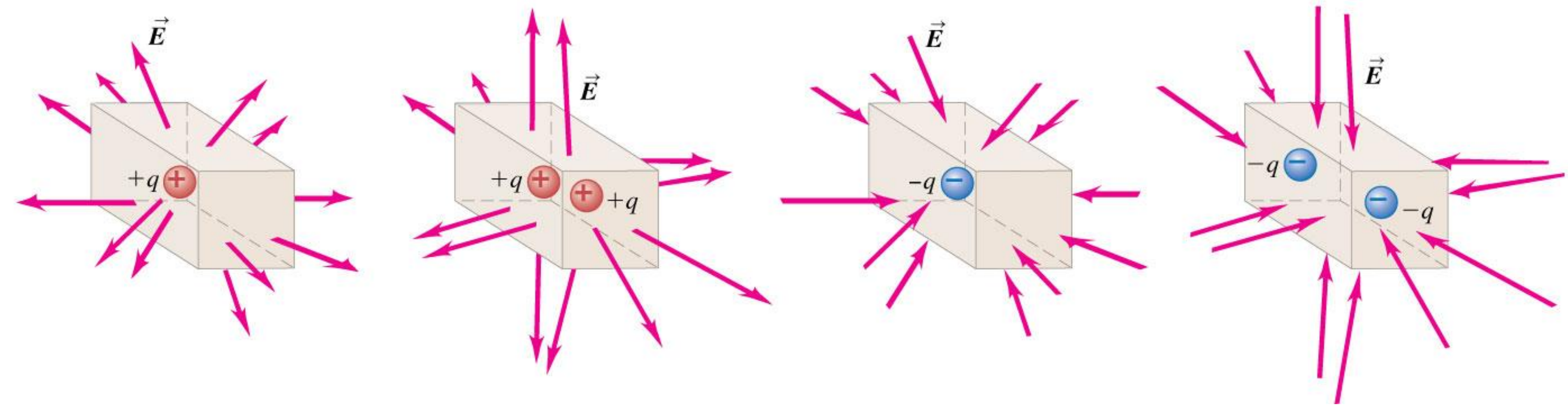
- Positive charge within the box produces outward *electric flux* through the surface of the box, and negative charge produces inward flux. (See Figure 22.2 below.)

(a) Positive charge inside box, outward flux

(b) Positive charges inside box, outward flux

(c) Negative charge inside box, inward flux

(d) Negative charges inside box, inward flux



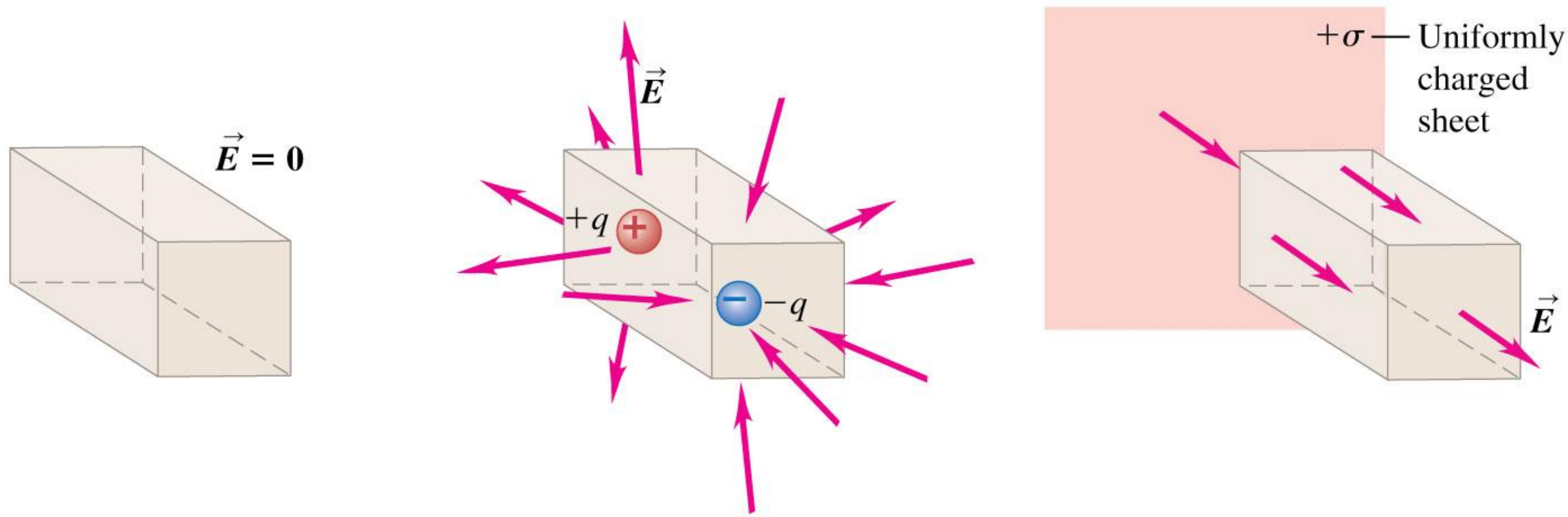
Zero net charge inside a box

- Figure 22.3 below shows three cases in which there is zero *net* charge inside a box and no net electric flux through the surface of the box.

(a) No charge inside box, zero flux

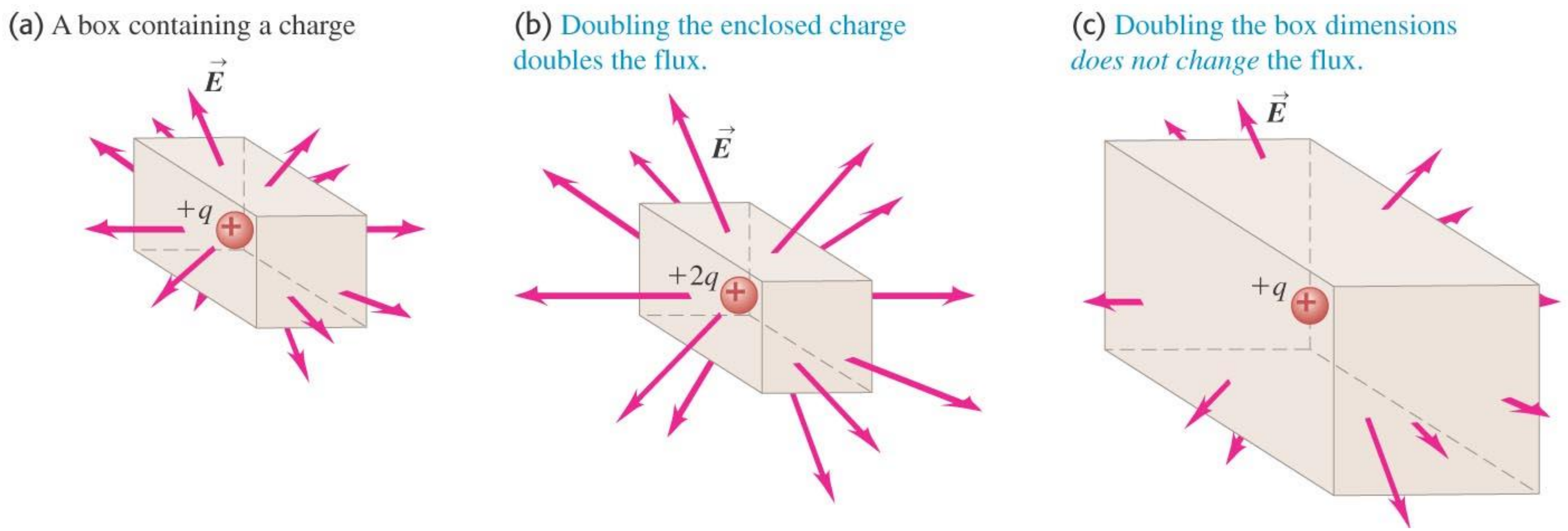
(b) Zero *net* charge inside box, inward flux cancels outward flux.

(c) No charge inside box, inward flux cancels outward flux.



What affects the flux through a box?

- As Figure 22.4 below shows, doubling the charge within the box doubles the flux, but doubling the size of the box does not change the flux.
- Follow the discussion of charge and flux in the text.



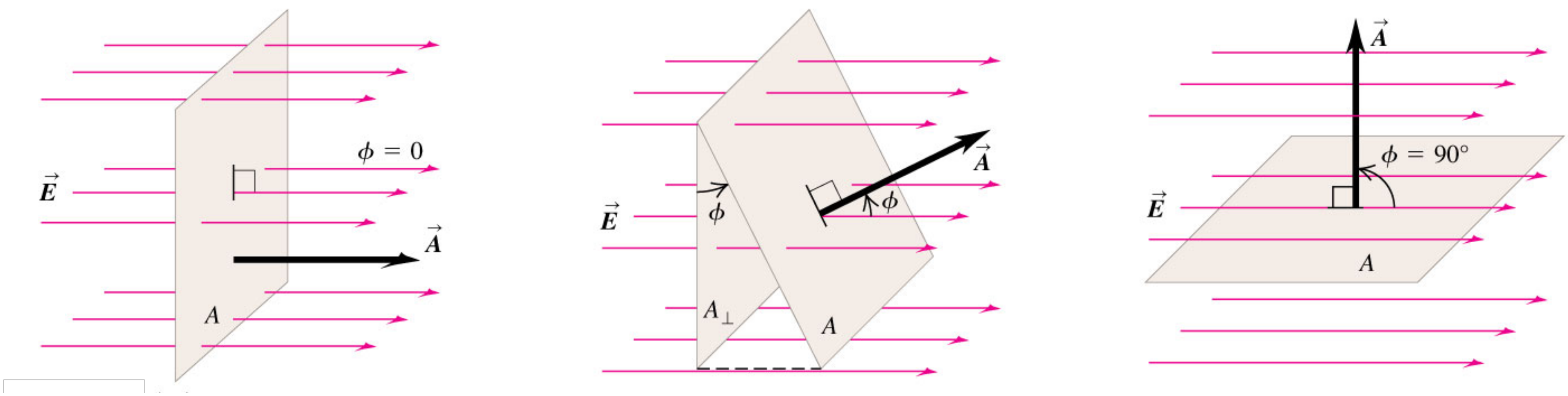
Calculating electric flux

- Follow the discussion in the text of flux calculation for uniform and nonuniform fields, using Figure 22.6 below.

(a) Surface is face-on to electric field:
- $\vec{E}$ and $\vec{A}$ are parallel (the angle between $\vec{E}$ and $\vec{A}$ is $\phi = 0$).
- The flux $\Phi_E = \vec{E} \cdot \vec{A} = EA$.

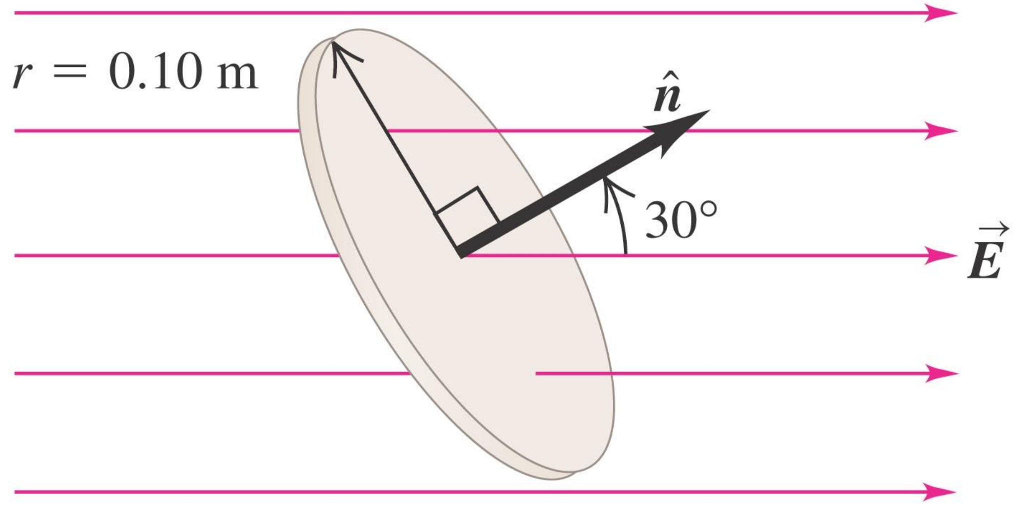
(b) Surface is tilted from a face-on orientation by an angle ϕ:
- The angle between $\vec{E}$ and $\vec{A}$ is ϕ.
- The flux $\Phi_E = \vec{E} \cdot \vec{A} = EA \cos \phi$.

(c) Surface is edge-on to electric field:
- $\vec{E}$ and $\vec{A}$ are perpendicular (the angle between $\vec{E}$ and $\vec{A}$ is $\phi = 90°$).
- The flux $\Phi_E = \vec{E} \cdot \vec{A} = EA \cos 90° = 0$.



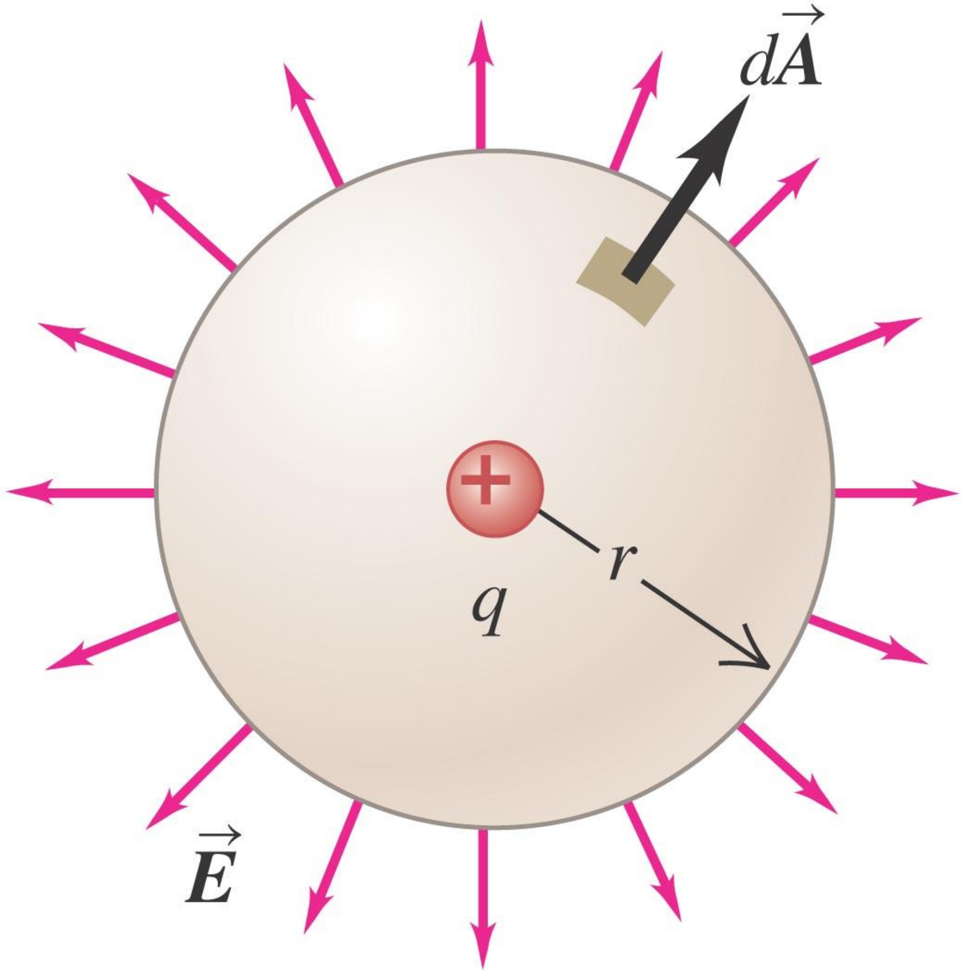
Electric Flux

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \oint E cos\phi dA$$



Electric flux through a sphere

- Follow Example 22.3 to evaluate flux through a disk. Use Figure 22.9 below.

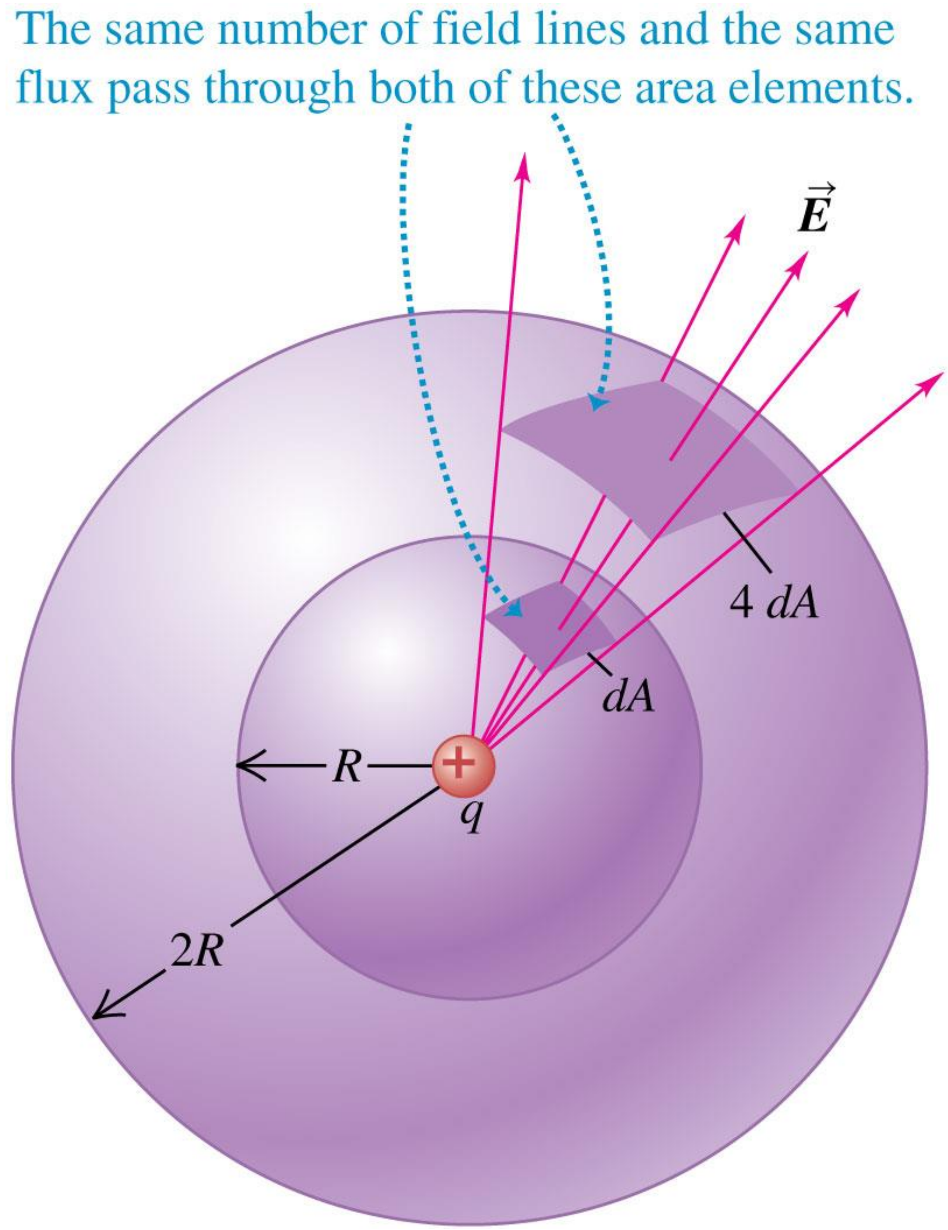


Gauss's Law

- Gauss's law is an alternative to Coulomb's law and is completely equivalent to it.
- Carl Friedrich Gauss, shown below, formulated this law.

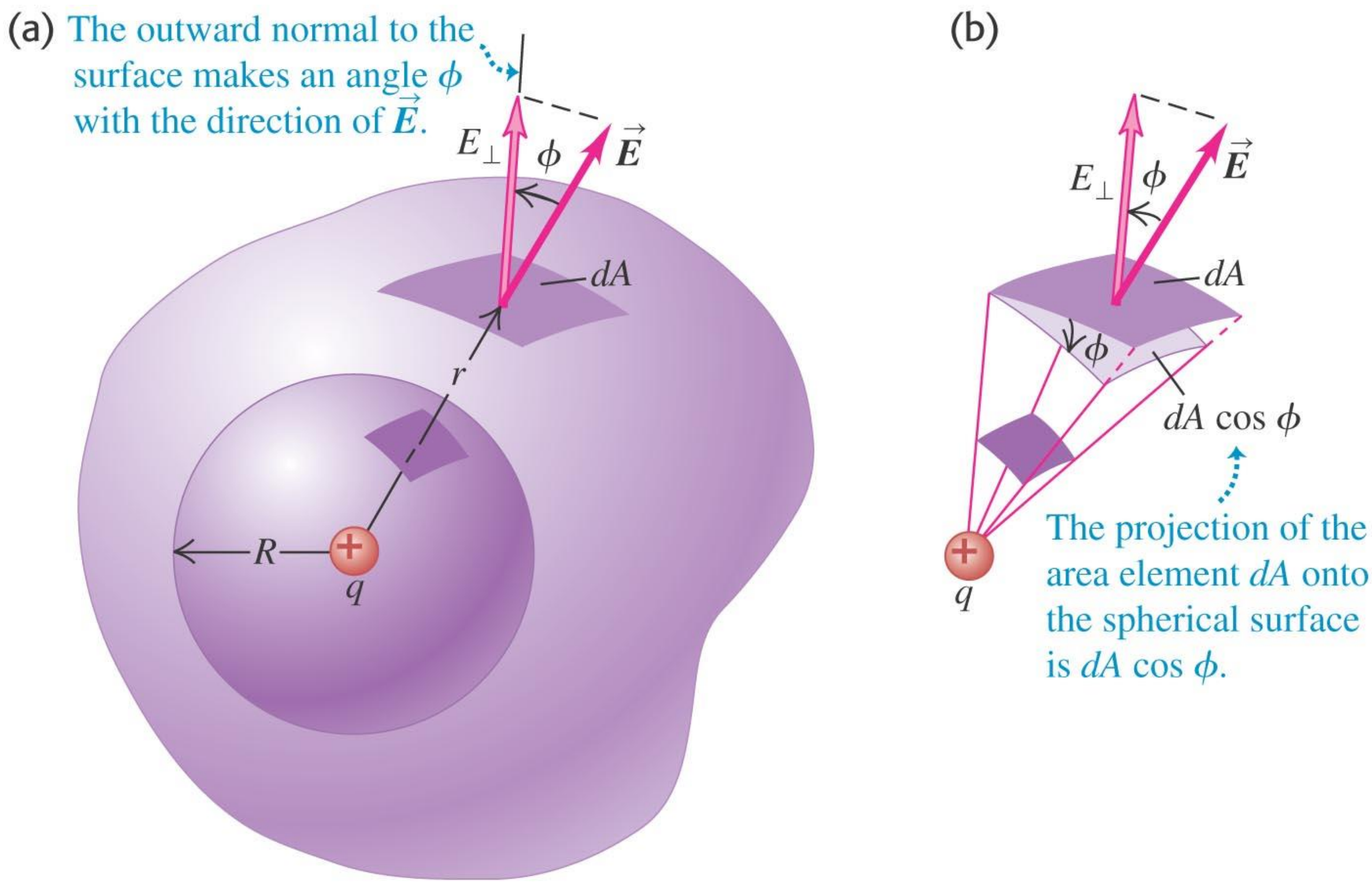
Point charge centered in a spherical surface

- The flux through the sphere is independent of the size of the sphere and depends only on the charge inside. Figure 22.11 at the right shows why this is so.



Point charge inside a nonspherical surface

- As before, the flux is independent of the surface and depends only on the charge inside. (See Figure 22.12 below.)



Gauss' Law

$$\boxed{\Phi_E = \oint \vec{E} \cdot d\vec{A} = \oint E cos\phi dA} = \frac{Q_{enc}}{\epsilon_o}$$

Gauss' Law

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \oint E cos\phi dA = \frac{Q_{enc}}{\epsilon_o}$$

$$\boxed{\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_o}}$$

Gauss' Law in Integral Form

Gauss' Law

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \oint E\cos\phi\, dA = \frac{Q_{enc}}{\epsilon_o}$$

$$\boxed{\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_o}}$$

Gauss' Law in Integral Form

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon}$$

Gauss' Law in Material

Gauss' Law

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \oint E cos\phi dA = \frac{Q_{enc}}{\epsilon_o}$$

$$\boxed{\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_o}}$$

Gauss' Law in Integral Form

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon}$$

Gauss' Law in Material

$$\oint (\nabla \cdot \vec{E}) dV = \frac{\oint \rho dV}{\epsilon}$$

(Divergence Theorem)

Gauss' Law

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \oint E cos\phi dA = \frac{Q_{enc}}{\epsilon_o}$$

$$\boxed{\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_o}}$$

Gauss' Law in Integral Form

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon}$$

Gauss' Law in Material

$$\oint (\nabla \cdot \vec{E}) dV = \frac{\oint \rho dV}{\epsilon}$$

(Divergence Theorem)

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon}$$

Gauss' Law in a Material in Differential Form

Gauss' Law

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \oint E cos\phi dA = \frac{Q_{enc}}{\epsilon_o}$$

$$\boxed{\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_o}}$$

Gauss' Law in Integral Form

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon}$$

Gauss' Law in Material

$$\oint (\nabla \cdot \vec{E}) dV = \frac{\oint \rho dV}{\epsilon}$$

(Divergence Theorem)

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon}$$

Gauss' Law in a Material in Differential Form

$$\boxed{\nabla \cdot \vec{D} = \rho}$$

$$\vec{D} = \epsilon \vec{E}$$

Gauss' Law in a Material in Differential Form for EE's

Gauss' Law

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \oint E cos\phi dA = \frac{Q_{enc}}{\epsilon_o}$$

$$\boxed{\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon_o}}$$

Gauss' Law in Integral Form

$$\oint \vec{E} \cdot d\vec{A} = \frac{Q_{enc}}{\epsilon}$$

Gauss' Law in Material

$$\oint (\nabla \cdot \vec{E}) dV = \frac{\oint \rho dV}{\epsilon}$$

(Divergence Theorem)

$$\nabla \cdot \vec{E} = \frac{\rho}{\epsilon}$$

Gauss' Law in a Material in Differential Form

EE's often call D the "Electric Field!"

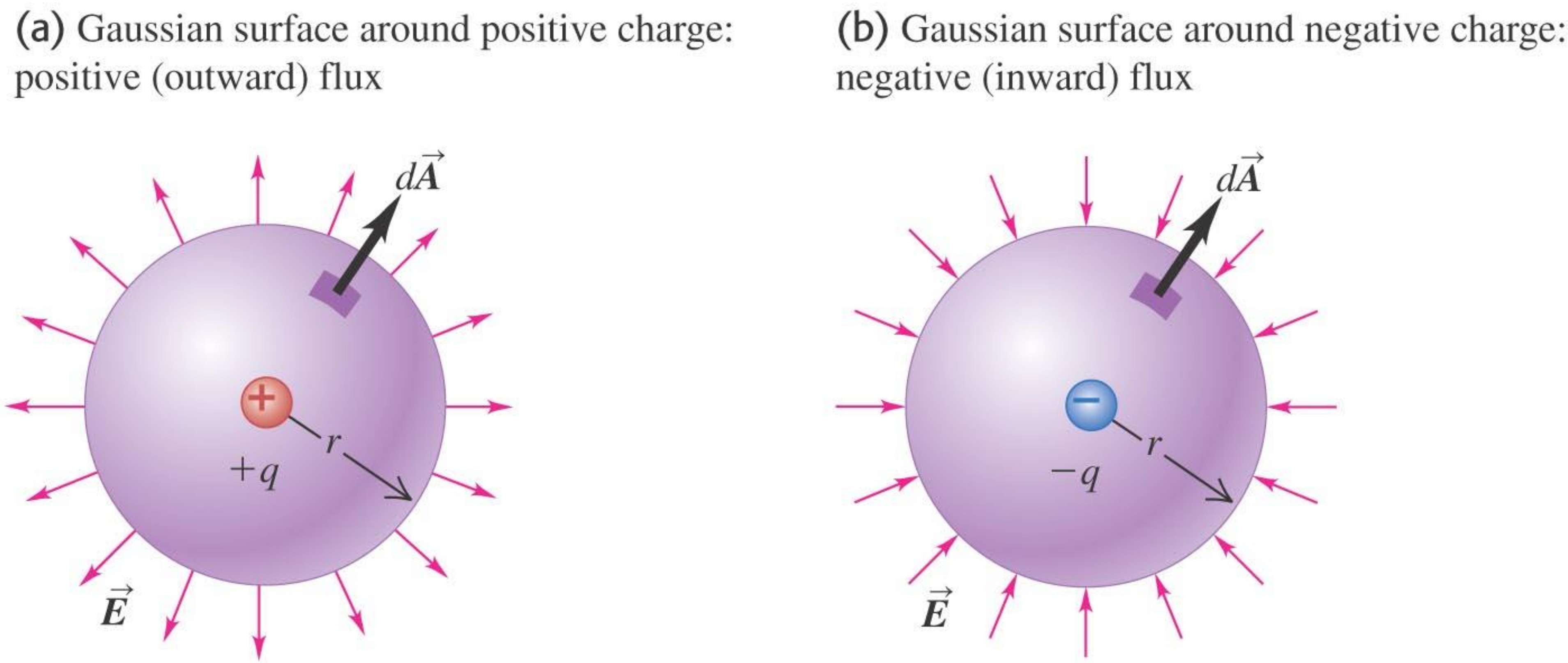
$$\boxed{\nabla \cdot \vec{D} = \rho}$$

$$\vec{D} = \epsilon \vec{E}$$

Gauss' Law in a Material in Differential Form for EE's

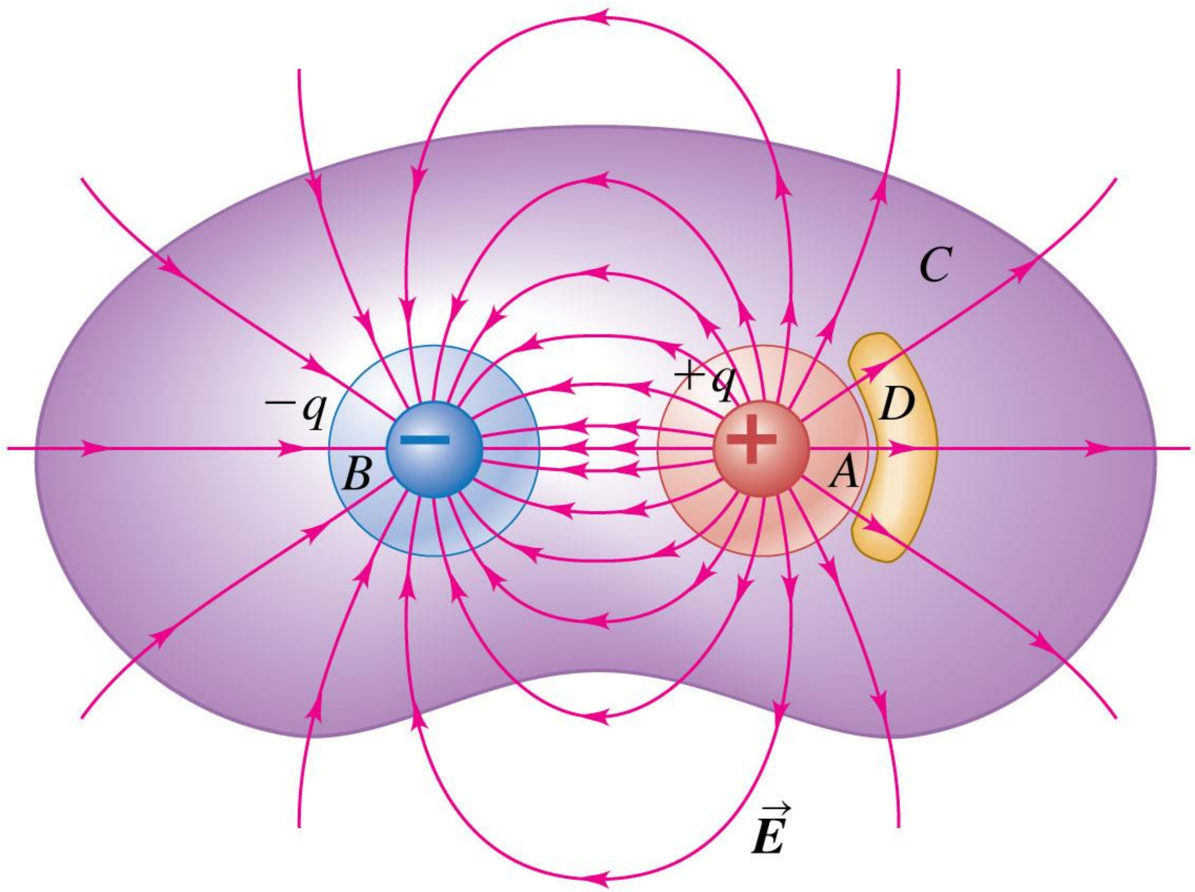
Positive and negative flux

- Figure 22.14 below shows that flux is positive if the enclosed charge is positive, and negative if the charge is negative.



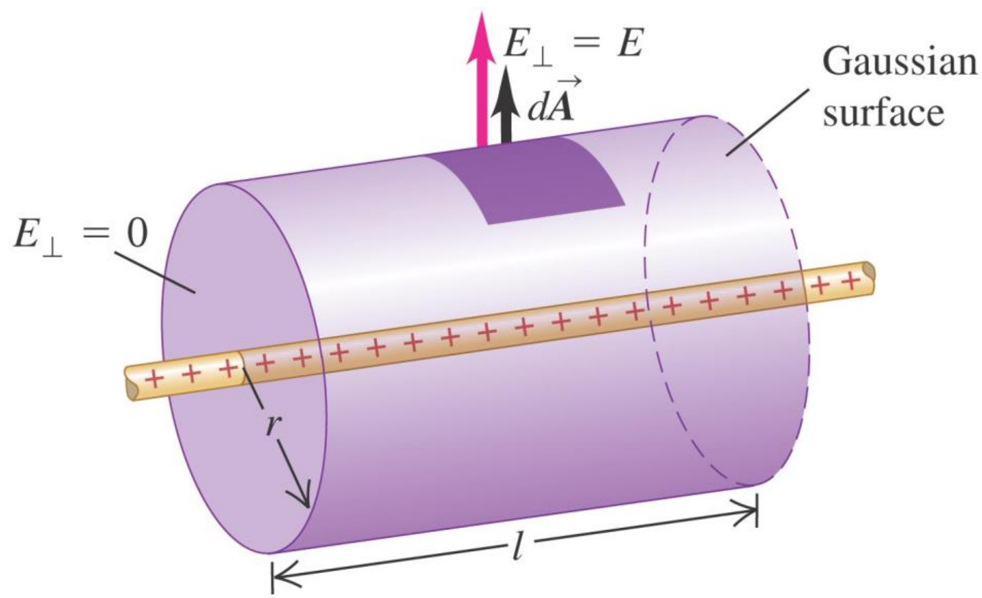
General form of Gauss's law

- Follow the text discussion of the general form of Gauss's law.
- Follow Conceptual Example 22.4 using Figure 22.15 below.



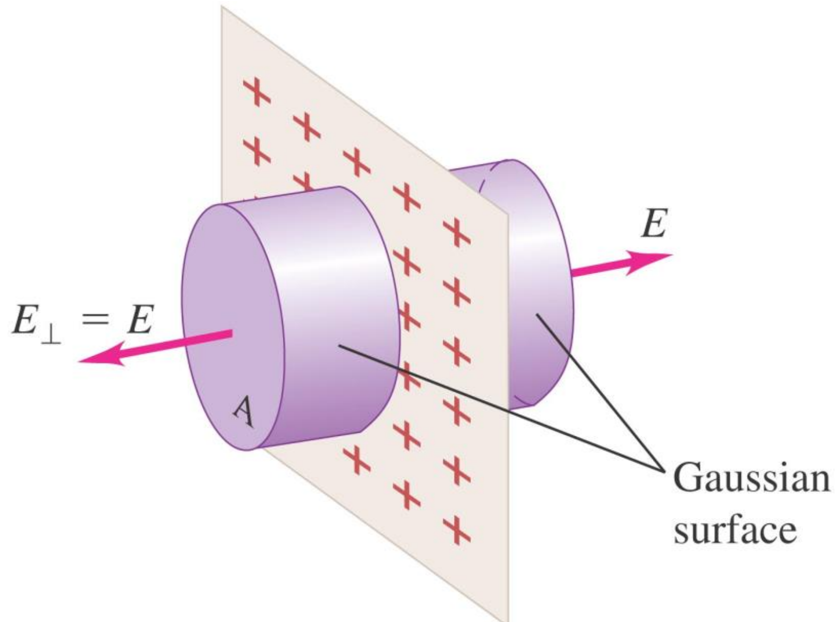
Field of a line charge

- Follow Example 22.6 using Figure 22.19 below, for a uniform line charge.



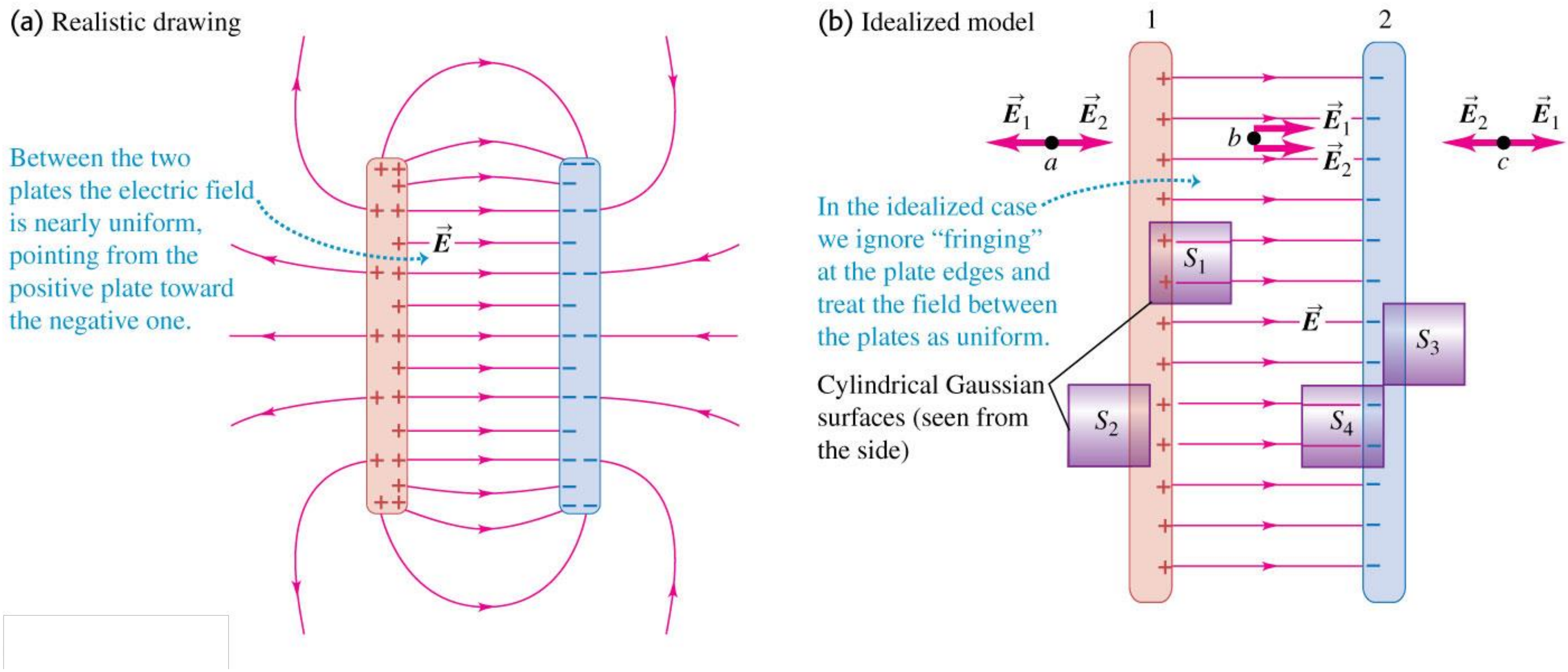
Field of a sheet of charge

- Follow Example 22.7 using Figure 22.20 below, for an infinite plane sheet of charge.



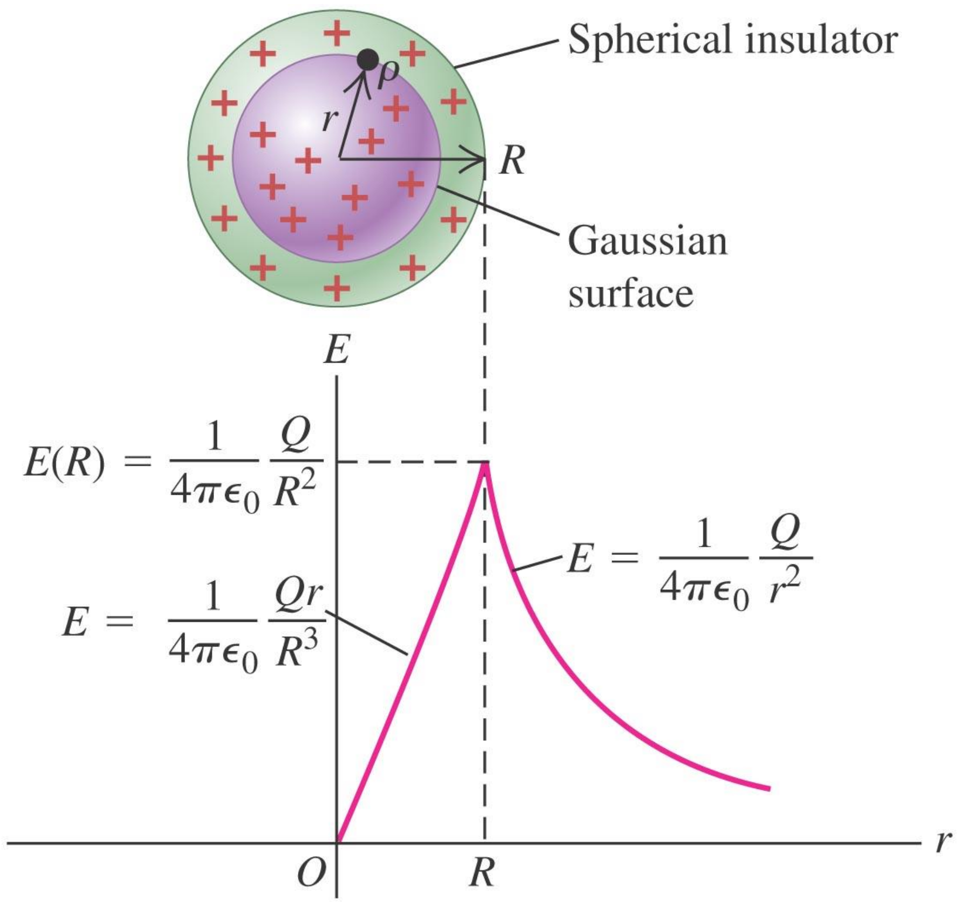
Field between two parallel conducting plates

- Follow Example 22.8 using Figure 22.21 below for the field between oppositely charged parallel conducting plates.



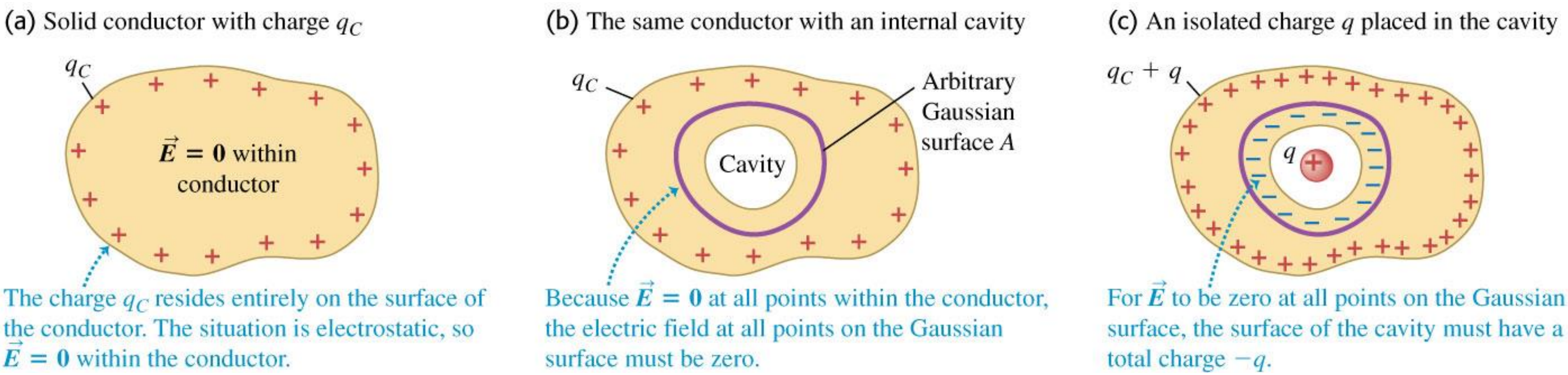
A uniformly charged sphere

- Follow Example 22.9 using Figure 22.22 below, for the field both inside and outside a sphere uniformly filled with charge.
- Follow Example 22.10 for the charge on a hollow sphere.



Charges on conductors

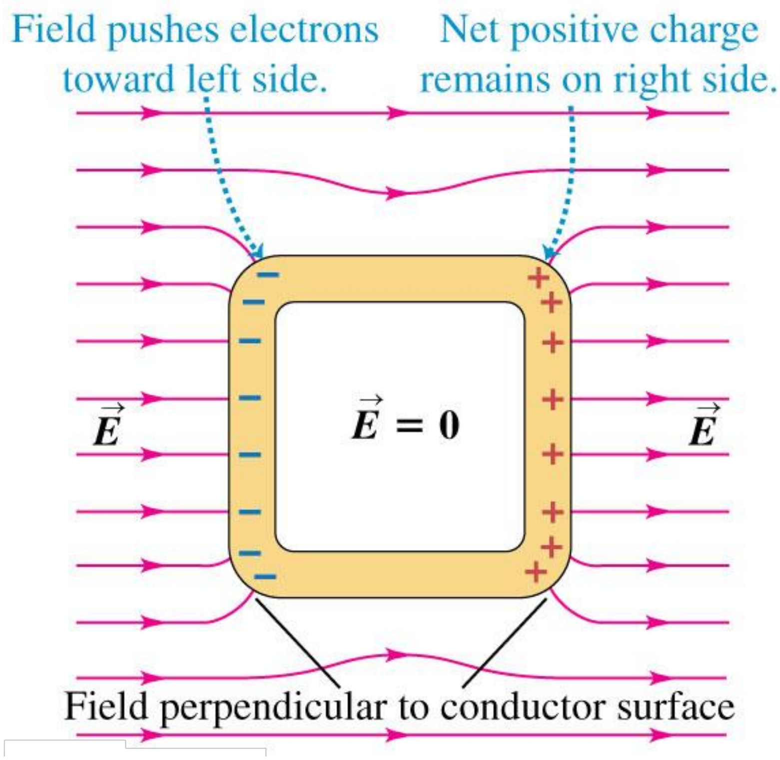
- In the text, follow the discussion on charges in a conductor having a cavity within it. Figure 22.23 below will help the explanation.



Electrostatic shielding

- A conducting box (a *Faraday cage*) in an electric field shields the interior from the field. (See Figure 22.27 below.)

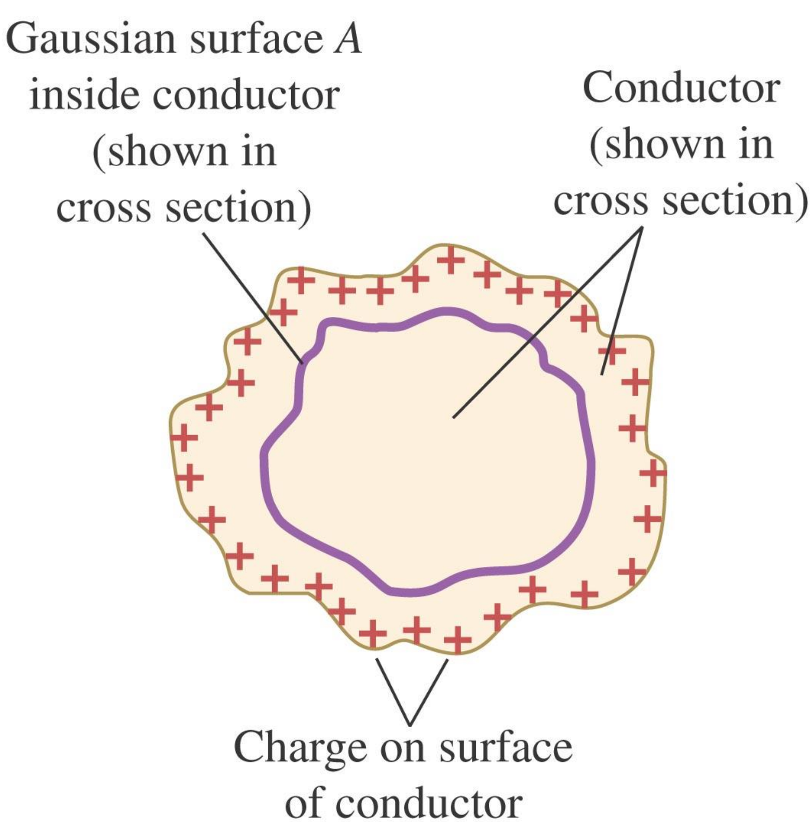
(a)

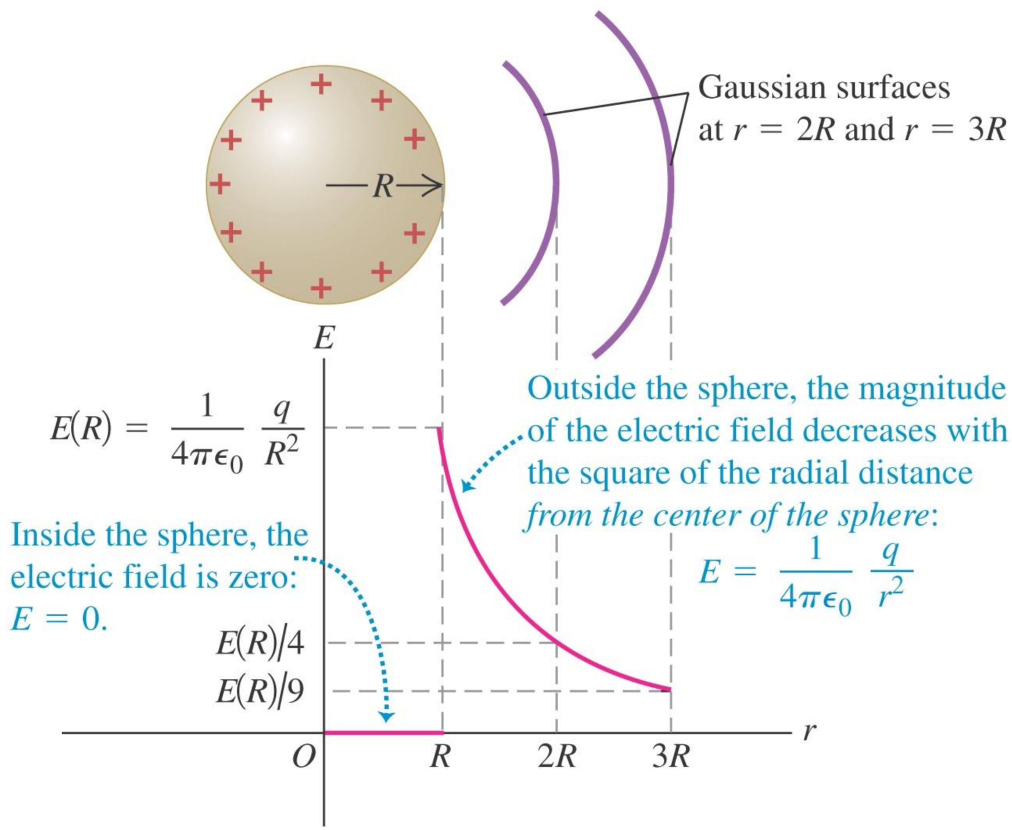


(b)

Applications of Gauss's law

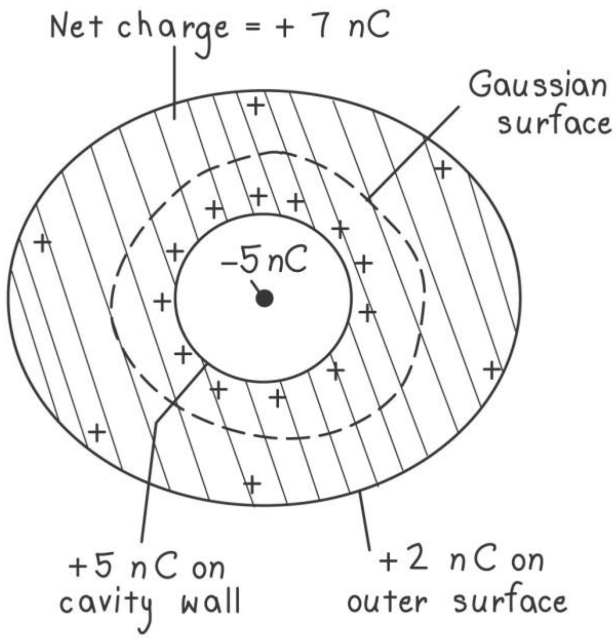
- Under electrostatic conditions, any excess charge on a conductor resides entirely on its *surface*. (See Figure 22.17 below left.)
- Follow Example 22.5 using Figure 22.18 below right.





A conductor with a cavity

- Follow Conceptual Example 22.11 using Figure 22.24 below.



Field at the surface of a conductor

- The electric field at the surface of any conductor is always perpendicular to the surface and has magnitude σ/ε_0.
- Follow the discussion in the text, using Figure 22.28 at the right.
- Follow Conceptual Example 22.12.
- Follow Example 22.13, which deals with the electric field of the earth.